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FOR

FIELD AND OFFICE

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MANUAL OF SURVEYING

FOR

FIELD AND OFFICE

BY

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PREFACE

This book is a manual of the practice of surveying designed for the use of civil engineering students in their preparation for field and office exercises. Primarily it is intended for use in conjunction with a treatise on surveying if the field and office practice is developed with the theory, or for use alone if practice is given after the theory has been studied. It is felt that the book is also adapted for use in short elementary courses in surveying in which the time devoted to field and office work is comparatively long, such as the courses usually given to mechanical and electrical engineering students.

The aim in the preparation of this volume has been to provide an efficient means of teaching students the proper use of surveying instruments, the proper procedure in making surveys, and the proper methods of computing and mapping. Attempt has been made to treat the practical phases of the subject in great detail, for the author's experience in teaching leads him to believe that superior results are attained only when the student is enabled before beginning a work to see clearly its purpose, a method of accomplishing it, and the reason for using such method. Much of the material of the book has been suggested to the author through questions asked and mistakes made by students and inexperienced surveyors over a period of several years.

The first chapter is designed to prepare the student for the practice of surveying as developed in the succeeding chapters. It deals with general instructions for field and office work, and emphasizes the more difficult and important parts of surveying, such as note keeping, precision of measurements, consistent accuracy, and efficient methods of computing.

The remainder of the book, through the explanation of problems of field and office, progressively builds up the practice of surveying from elementary work with tape to complete topographic surveys.

Attention is called to the following features:

1. The book is of a size and weight to be conveniently carried in the coat pocket.

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2. The explanation of each problem is complete in itself, unless some portion has been developed in a prerequisite problem; and therefore there is no necessity for seeking elsewhere in the book for additional information concerning methods, the use of instruments, etc.

3. The suggestions following each problem contain much information not usually included in text-books, but which is usually brought to the attention of the inexperienced surveyor either by reason of his own blunders or through oral instructions. It is believed that these suggestions will be particularly helpful in field classes where the parties are so scattered that it is impossible for each student to receive the constant attention of an instructor.

4. The precisions of measurements necessary for various surveys have been emphasized by establishing limits of error or by stating the errors allowed in practice.

5. Where there is more than one method of performing a surveying operation and the methods are not radically different, the variations follow the detailed explanation of the most common case and are compared one with another.

6. The practical applications of field and office exercises and the dependence of methods on conditions are brought to the attention of the student.

7. Emphasis is placed on methods of checking, especially those methods that require little additional labor and may be applied as the work progresses.

8. The sample pages of notes and computations—of which there are a liberal number—are examples of clear, systematic forms used in actual practice, and are not unduly elaborate.

9. The problems in the determination of latitude, longitude, and azimuth may be performed with little or no previous knowledge of astronomy. The methods explained give results sufficiently accurate for any but the most precise surveys.

10. The book contains all tables which will ordinarily be of use in plane and topographic surveying. The Polaris tables make the use of the Nautical Almanac unnecessary, and render the determination of the meridian a very simple process.

11. The method of explaining the relation between size of angle, angular error, and ratio of precision of the trigonometric functions has not, to the author's knowledge, before appeared in any text-book. This important phase of the

precision of measurements is usually not well understood, even by the experienced surveyor.

12. In the description of the adjustments of the transit and level, distinction has been made between the type of instrument having adjustable objective slide and the type in which the objective slide is permanently fixed as far as lateral motion is concerned. Also, in the description of the adjustments of the plane-table alidade, distinction is made between the type of alidade having telescope tube rigidly attached to the horizontal axis and the type having a collar in which the telescope may be turned about its axis.

13. The plane-table, as the most efficient instrument for general topographic surveying, has received due consideration, and plane-table methods for large-scale work have been given more attention than is usually accorded them.

14. The portions of the book dealing with fieldwork and office work are closely co-ordinated, problems in computing and drafting in general being the continuation of field problems.

15. The cross references to related subject matter are numerous; the running titles are suggestive of page contents; and the index is sufficiently complete to make ready reference possible.

The author is indebted to Prof. George L. Hosmer, Mr. H. H. Jordan, and Mr. W. H. Rayner for valuable criticisms of portions of the manuscript; and to Mr. A. R. Alger for a rough draft of a considerable number of the problems and notes of Chapters III and IV.

Much credit is due Prof. Ira O. Baker for his keen interest in the work, for the painstaking care with which he reviewed the manuscript, and for valuable suggestions without which the book would have been very imperfect.

Acknowledgments are also due Mr. O. H. Tittman, Superintendent of the U. S. Coast and Geodetic Survey, for furnishing data for the Polaris tables; John Wiley and Sons for permission to use Table X; and Mr. C. W. Reinhardt for the careful preparation of the sample pages of notes and computations and of the plates of letters.

R. E. D.

URBANA, ILL., *May*, 1915.

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MANUAL OF SURVEYING

FOR

FIELD AND OFFICE

CHAPTER I

FIELD AND OFFICE WORK

SEC. 1. THE FIELD NOTE-BOOK

1. Kind of Book.—The note-book should be of good quality paper, with stiff board or leather cover, made to withstand hard usage, and of a size convenient to slip into the coat pocket. There are several special field note-books sold by engineering supply companies which are intended for particular kinds of notes. For general surveying or for students in fieldwork where the problems to be done are general in character, an excellent form of note-book has the right-hand page divided into small rectangles with a red line running up the middle, and the left-hand page divided into several columns. In general, tabulated numerical values are to be written on the left-hand page; sketches and explanatory notes on the right.

2. Field Notes.—No matter what the character of the survey, all field notes should be taken in the field at the time the work is being done. Notes made later, from memory or copied from other field notes, may be useful but they are not field notes. Notes should be neat and intelligible to anyone having a knowledge of surveying. They are generally recorded in pencil, but are permanent work, and should not be regarded as memoranda to be used only for the time being or in the immediate future.

The observer should also realize that the notes will very likely be used by other persons not familiar with the locality; they must rely entirely upon what he has recorded. For this reason not only should the note-book contain all necessary information, but data should be recorded in a form which will admit of only one interpretation, and that the correct one.

A good sketch will help more than anything else to convey a correct impression to others, and for this reason sketches should be used freely. In general, sketches are not drawn to scale, but in most work it is better to construct them approximately to scale. It is to aid in accurate sketching that the right-hand page of the note-book is divided into rectangles. If the note keeper will bear in mind what use is to be made of the notes, it will aid him greatly in deciding what data are necessary and what are not. The student should understand at the beginning of his work that it is not an easy thing to take good notes. It is possibly the most difficult thing he will be called upon to do. Too many beginners seem to think that a page of neat letters, figures, and lines constitute good notes; but although neatness is one of the primary requisites, it must be borne in mind that neatness alone can not make the record clear and complete. To make the notes clear, the student should strive to put himself in the place of one who is not on the ground at the time the survey is made; and to make the notes complete, he should thoroughly understand for what purpose they are to be used. He should strive at all times to make the notes on the present problem clearer, neater, and more complete than the notes of the previous problem. Before making any survey, the necessary data to be collected should be carefully considered; and when doing the fieldwork, all such data should be obtained, but no more.

Various problems throughout the following pages are accompanied by forms of notes similar to those used by many engineers. The student should not look upon these as standard, but rather as examples of clear notes, and should feel free to introduce a form of his own if he believes his system is the better. With those problems unaccompanied by sample pages of notes, the student will be expected to exercise his own ingenuity.

3. Suggestions for Keeping Field Notes.—Use a 4H pencil and keep it well pointed. Make a neat title either on the flyleaf or on the cover, showing the owner of the note-book, the number and name of the course, and the year in which the notes are taken. Leave several pages in the front of the book for an index. Show in the index, the number, name, date, and pages of each problem done in the field. Also show whether or not the work has been accepted by the instructor. Always keep the index up to date.

Take all notes in the note-book while in the field. Leave nothing to memory or guesswork.

For each problem make a title at the top of the page, showing name and number of problem, date, weather, names of the members of the party and the duty of each, and time occupied in doing the fieldwork. Use the left-hand page for a tabulated list of measurements; the right-hand page for sketches and explanatory notes.

If a page of notes is abandoned for any reason, do not obliterate the notes or cut out the page, but write diagonally across the page in large letters "*abandoned*" and designate the page number of the continuation of the notes. Also show same in the index. Notes may be abandoned because they are illegible or because they contain such erroneous or useless data that another survey is necessary.

Do not change notes without first being *certain* that a change should be made, and above all, go at the work with a truly scientific spirit, being absolutely honest with yourself.

Do not make the notes appear *more* accurate or *less* accurate than they really are.

In general do not erase numerical data. If a number is in error, draw a line through it and write the corrected value above. Portions of sketches and explanatory notes may be erased when there is a good reason for doing so. Make the sketches freehand and of liberal size.

REINHARDT'S style of slant lettering is generally conceded to be the best form of lettering for taking notes rapidly and neatly, because of its simplicity. (See Fig. 48, page 223, for examples of form.)

4. Explanatory Notes.—The purpose of explanatory notes should be to make clear that which the numerical data and sketches fail to do. In some surveys explanatory notes entirely take the place of sketches, in which case they are placed on the right-hand page in the same line with numerical data they explain. In other surveys they are used in conjunction with sketches and numerical values, and are placed in such position as not to interfere with other data. Sketches themselves frequently contain features which are only intelligible through some note. When used in this capacity, notes should be placed in such position on the page with the sketch as not to interfere with other figures or letters. At the same time they should be placed in close proximity to that which they

explain. Certain abbreviations of the more common artificial features met with by the surveyor should be utilized by the student. A few are enumerated below:

stk.	Stake.	*c.f.*	Crow-foot.
tk.	Tack.	*c.c.f.*	Cut crow-foot.
mon.	Monument.	*C.*	Center.
S.B.	Stone bound.	℄	Center line.
d.h.	Drill-hole.	*sec.cor.*	Section corner.
tel.	Telegraph pole.	*¼sec.cor.*	Quarter section corner.
spk.	Spike.	*tnp.cor.*	Township corner.
na.	Nail.	△	Triangulation station.
c.b.	Curb.	⊡	Stadia station.
C.B.	Catch basin.	⊙	Transit traverse station.
M.H.	Manhole.	*B.M.*	Bench mark.
b.b.	Base-board.		

5. Numerical Data.—The figures used should be plain; one figure should never be written over another. If the numerical data are to be tabulated in columns on the left-hand page, place the numbers so that all figures in the tens column will be on a straight line, thus at a glance one can tell what numbers are in tens or in hundreds or in thousands. Where decimals are used, the decimal point should never be omitted; and the number should always show with what degree of precision the measurement was taken. Thus an angle of 64° when measured to the nearest minute should be recorded 64° 00′ and a distance measured to the nearest 0.1 ft, should be recorded not as 142.20 ft. but as 142.2 ft. Numbers placed on sketches should always be in such position as to clearly indicate to what they refer. Frequently this is impossible except by the aid of dimension lines or arrows.

6. Sketches.—Sketches are very rarely made to scale, but in many cases they are made approximately to scale. In other cases, however, certain distances are exaggerated for the sake of clearness. Sketches should be made large enough to prevent crowding of details. Paper is cheap, and clearness of details will often save hours of time in the office.

A sketch should be made when data can not be clearly described by *short* explanatory notes, or when it will aid in

any way in the correct interpretation of the notes. What the sketch should show must be determined by the note keeper, for the most part before a line is drawn. To make good sketches one must not only be able to judge distances and angles with a fair degree of accuracy, but one must also know what features to include. A sketch crowded with unnecessary data is often very puzzling even though all necessary features are included. Culture and many of the public and private works may be most readily shown by conventional signs (Figs. 44, 45, 46, and 47, pages 218–222).

Sec. 2. Field Problems

7. Fieldwork.—It is not possible, in the ordinary course in surveying, to develop the student into an expert instrumentman; however, it is expected that the course will give the student a working knowledge of surveying instruments and their uses. The work is not always carried out as it would be in actual practice, because of the brief time allowed for field problems and because during this time it is necessary for all students to obtain as much practice as possible in the manipulation of field instruments and the taking of notes. In elementary fieldwork no long surveys of any sort are attempted, but a number of short problems are taken up which might in practice become parts of extended surveys.

Members of the student field parties should alternately assume the various duties involved in the field problems, but during any single period should not change positions. The ability to hold the rod properly should be looked upon as being just as essential as the knowledge of how to manipulate the level, for intelligent direction of work makes necessary a thorough understanding of all its details.

8. Study the Problems.—Before going into the field, the student should understand exactly what he is to do and why he is to do it. This can be accomplished only by a thorough study of the problem for the day, noting first the object of the problem, and then conducting a critical examination of the course of procedure. In his mind he should go through the various steps involved until he has completely mastered them, so that while in the field he may spend all his time and attention in putting into practice that of which he has already learned the theory. A student who fails to do this is seriously

handicapped in his fieldwork, and, though he may work conscientiously while in the field, will have but a superficial knowledge of what he is doing. The problems should be regarded as instructions which are to be rigidly followed, and a thorough understanding of the work to be accomplished is as essential as the skill to carry it out. After making a thorough study of the problem the student should prepare a list of the equipment (instruments, stakes, etc.) necessary for its performance.

9. Habit of Correctness.—The student should aim to do his work thoroughly, introducing checks wherever possible, and no measurement should be regarded as correct until verified. In as far as it is practicable, methods of verification should differ from the methods used in original measurements. All persons are liable to mistakes, but mistakes in fieldwork become discreditable to the maker if he allows any other than himself to discover the discrepancy. When the student is tempted to neglect his work in this manner in order to save time, he should realize that habits are very easy to form but very difficult to break; and that nothing, unless it be wilful dishonesty, is so injurious to the reputation as habitual carelessness.

10. Consistent Accuracy.—On the other hand extreme accuracy should not be thought necessary at all times, but rather an accuracy consistent with the purposes of the survey. Beginners often fail to comprehend the different degrees of precision necessary for the different kinds of work, or worse yet, fail to maintain a consistent degree of accuracy throughout any one survey. There are no set rules for the relative accuracy of different classes of surveys, nor could there be, for the objects and conditions are too many and too complicated, but common sense can always be resorted to. Thus one can see the absurdity of taking the measurements of farming land, worth only a few dollars an acre, with the same precision as the measurements of small city lots, worth thousands of dollars. The student should then, before beginning a problem, establish the limit of error for himself, using his own common sense and the experience of others to guide him. The best surveyor is not the one who is extremely accurate, but the one who can make a survey just accurate enough to serve its purpose without waste of time or money.

11. Relation Between Angles and Distances.—If measurements are to be consistent, it is evident that the precision of angles should correspond to the precision of related distances—or in other words, the percentage of error in angle should equal the percentage of error in distance. Oftentimes it is impracticable to maintain an exact equality between these two percentages; but with one or two exceptions, which will be considered presently, surveys should be so conducted that the variation between precision of angle and precision of distance will not be large. Since the accuracy of a survey is often judged by the number of significant places in the recorded distances, it is always best to measure angles with a precision at least equal to the precision of the distances. It is common practice before beginning a survey or any distinct portion of a survey to fix the permissible error of linear measurement, this being expressed as a ratio, as 1/1000; sometimes it is desired to locate a given point within a specified distance of its true position. The following table shows for various angular errors the corresponding ratios of precision and the errors for a length of 1000 ft. For a length other than 1000 ft. the linear error is in direct proportion.

Angular error	Linear error in 1000 ft.	Ratio of precision
10′	2.9089	1/344
5′	1.4544	1/688
1′	0.2909	1/3 440
30″	0.1454	1/6 880
20″	0.0970	1/10 300
10″	0.0485	1/20 600
5″	0.0242	1/41 200

To illustrate the use of the preceding table suppose distances are to be chained with a precision of 1/10,000; the corresponding permissible angular error is 20″. Again, the distance from the instrument to a desired point is determined as 660 ft. with a probable error of 2 ft. For an angular error of 10′ the corresponding linear error is $0.66 \times 2.9 = 1.91$ ft. Therefore the angle need be determined only to the nearest 10′.

The *exceptions* referred to in the first part of this article are surveys in which the distances are roughly determined and the

angles are measured with more than the required accuracy without increased effort or loss of time. For example, in rough chaining the ratio of precision might be 1/1000, corresponding to an angular error of 03′. But with the ordinary transit, the angles could be determined to the nearest 01′ as quickly as to the nearest 03′.

12. **Angles Used in Trigonometric Computations.**—Very often field measurements are made the basis of computations involving the trigonometric functions, and it is necessary that the computed results be of a required precision. If the values of these functions were exactly proportional to the size of the angles—or in other words, if any increase in the size of an angle was accompanied by a proportional increase or decrease in the value of a function—the problem of determining the precision of angular measurements would resolve itself into that explained in the preceding article; but since the ratios of the rates of change to the values of the sines of small angles, the cosines of angles near 90°, and the tangents and cotangents of small and large angles are relatively large, it is evident that the precision with which an angle is determined should be made to depend upon the size of the angle and the function to be used in the computations. Usually too little attention is paid to this important phase of the precision of measurements, even by experienced surveyors, and as a consequence computed results are very often assumed to be more nearly correct than they really are. It is not practicable to measure each angle with exactly the precision necessary to insure sufficiently accurate computed values, but at least the surveyor should have a sufficiently comprehensive knowledge of the purpose of the survey and of the properties of the trigonometric functions to keep the angles within the required precision.

The curves of Plates I and II, pages 9 and 10, show the ratios of precision corresponding to various angular errors from 05″ to 01′ for sines, cosines, tangents, and cotangents.* For the function under consideration these curves may be utilized to determine the ratio of precision corresponding to a given angular error and angle, or to determine the maximum or minimum angle that for a given angular error will furnish the required ratio of precision, or to determine the precision with

* This method of representing the effect of angular error on the ratio of precision was suggested by an article in "Engineering Record," Oct. 18, 1913, by Mr. W. H. RAYNER.

which angles of a given size must be measured to maintain a required ratio of precision in computations.

The following examples illustrate the use of the curves:

1. In computations involving the use of cosines a ratio of precision of 1/10,000 is to be maintained. It is desired to know with what precision angles must be measured. On the

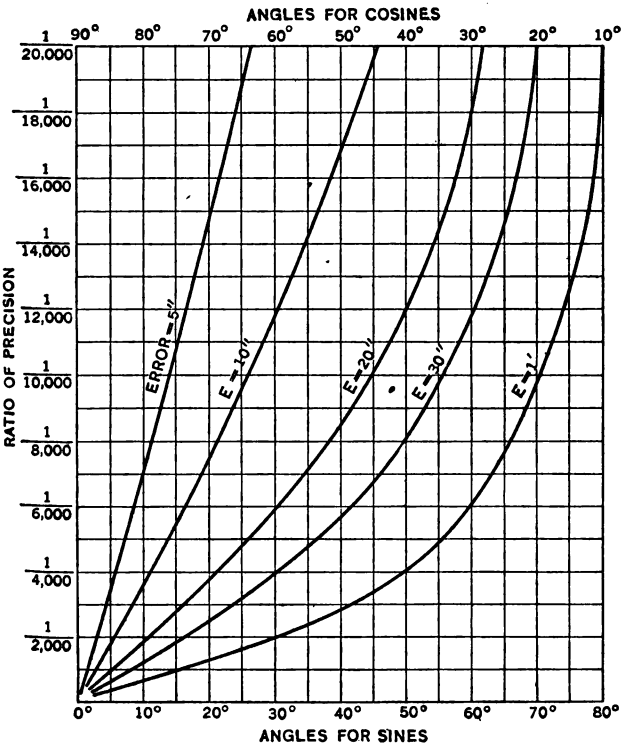


PLATE I.—RATIO CURVES FOR SINES AND COSINES.

Ratio Curves for Cosines opposite 1/10,000 it will be seen that for angles of about 76° the angular error can not exceed 05″, for angles of about 64° the angular error can not exceed 10″, and so on.

2. An angle measured with a one-minute transit is recorded as 32° 00′. The probable error of the angle is 30″. It is

desired to know the ratio of precision of a computation involving the tangent of the angle. On the Ratio Curves for Tangents it will be seen that the ratio of precision opposite the intersection of the curve $E = 30''$ and a line corresponding to 32°, is 1/3000.

3. In a triangulation system the angles can be measured with an error not exceeding 05″. Computations involving the use of sines must maintain a ratio of precision of not less

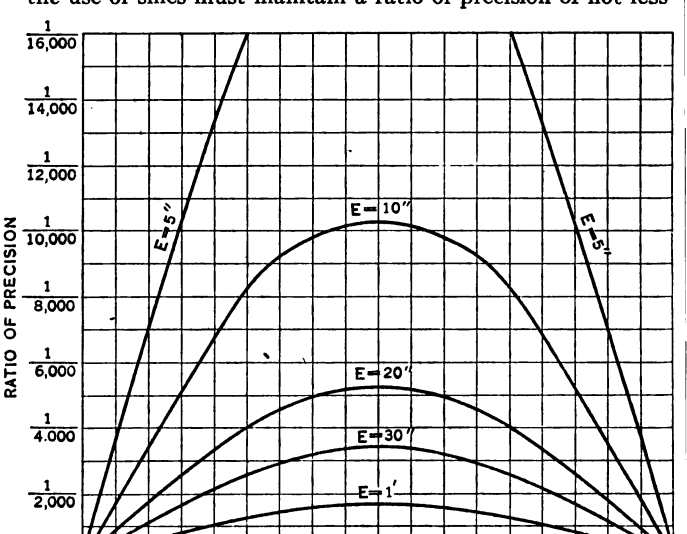


PLATE II.—RATIO CURVES FOR TANGENTS AND COTANGENTS.

than 1/20,000. It is desired to determine the minimum allowable angle. On the Ratio Curves for Sines the angle corresponding to a ratio of precision of 1/20,000 and an angular error of 05″, is 26°.

It should here be noted that linear as well as angular measurements must always be taken with the required precision of the computed results, for no result can be more nearly correct than the data from which it was obtained.

13. Speed.—Speed in fieldwork depends to a large extent upon practice in handling instruments; but no amount of

practice will secure rapid work unless the work be carefully planned, systemized, and carried out with consistent accuracy. To attain rapidity, the student should be constantly on the lookout for little ways of saving time. He must have system in every thought and move. If he is handling an instrument, he must know exactly what he is to do some time before he does it. He must acquaint himself with every part of the instrument. The turning of the right screw the right way at the right time should not require a cessation of mental activity from other things, but should be a mere automatic movement. Although rapidity is essential, when obtained at the expense of accuracy it is a thing to be avoided. The surveyor who is really quick in his work is he who never hurries beyond the speed at which he can maintain the required accuracy, but who utilizes all his time to the best advantage.

14. Signals.—Except for short distances a good system of hand signals between different members of the party makes a more efficient means of communication than is possible by word of mouth. The number of signals necessary will depend upon the character of the work and the nature of the country. A few of the more common are given below:

"*Right*" or "*Left.*"—The arm is extended in the direction of the desired movement, the right arm being extended for a movement to the right and the left arm for a movement to the left. A long, slow, sweeping motion of the hand indicates a long movement; a short, quick motion indicates a short movement. This signal may be given by the transitman in directing the chainman on line, by the leveler in directing the rodman for a turning-point, by the chief of the party to any member, or by one chainman to another chainman.

"*All Right.*"—Both arms are extended horizontally and the forearms waved vertically. The signal may be given by any member of any party.

"*Plumb the Flag*" or "*Plumb the Rod.*"—The arm is held vertically and moved in the direction that the flag or rod is to be plumbed. It is given by the transitman or leveler.

"*Give a Foresight.*"—The instrumentman holds one arm vertically above his head.

"*Establish a Turning-point*" or "*Set a Hub.*"—The instrumentman holds one arm above his head and waves it in a circle.

"*Give Line.*"—The flagman holds the flag horizontally in both hands above his head and brings it down and turns it

to a vertical position. If he desires to set a hub, he waves the flag with one end in the ground from side to side.

"*Turning-point*" or "*Bench Mark.*"—In profile leveling the rodman holds the rod horizontally above his head and then brings it down on the point.

"*Wave the Rod.*"—The leveler holds one arm above his head and moves it from side to side.

"*Pick up the Instrument.*"—Both arms are extended downward and outward, then inward and up, as one would do in grasping the legs of the tripod and shouldering the instrument. It is given by the chief of the party or by the head chainman when the transit is to be moved to another point.

Care should be taken to make the signals so clear that they may be readily understood. Where long sights are taken or where the peculiar color of the background renders hand signals indistinct, colored flags similar to those used by railroad trainmen, may be used to good advantage. Of course the color should be in contrast with that of the background. Red can be seen very well against snow, while white can be distinguished clearly against the dark green of the forest.

Sec. 3. Surveying Instruments

15. General Remarks.—As the use of the various surveying instruments is made necessary in the field problems explained in the following chapters, suggestions and precautions for the care and manipulation of these instruments are given. These should be heeded, not only because the student is responsible for the equipment he is using, but because he will establish the foundation of a very desirable qualification —that of carefulness.

Poor instruments should not be made the object of excuses for inferior work, instead a special endeavor should be made to maintain the required accuracy. The instruments used by elementary classes in surveying are rarely in good condition, and very seldom in good adjustment owing to the rough handling they receive; but if the problem is properly performed, instrumental errors will be eliminated to a great extent. The student will find later in practice that he will be called upon many times to do good work with poor equipment, and should be glad of an occasional chance of becoming acquainted with poor and defective instruments.

On taking the equipment from the locker, the student should *thoroughly examine each piece*, and in case of loss or injury, should *immediately report the loss to his instructor* so that the liability may be fixed.

16. Care of Transit, Level, and Compass.—(1) Handle the instrument carefully when removing it from its box. (2) See that it is securely fastened to the tripod head. (3) Avoid carrying the instrument on the shoulder while passing through doorways or beneath low-hanging branches. Before climbing over a fence or fallen tree place the instrument on the other side of the obstacle with the tripod legs well spread. (4) Never leave the instrument while it is set up in the street, on the sidewalk, in the vicinity of buildings in the process of construction, or in any other place where there is the slightest possibility of accident. (5) Do not set the tripod legs too close together; and see that they are firmly planted. (6) Tighten the leveling screws until they come to a firm bearing, but not enough to warp the leveling head. *The general tendency is to tighten these screws far more than is necessary.* (7) Always put on the sun shade. Do this by a screw motion in a clockwise direction, thus doing away with any possibility of loosening the objective lens. (8) *When the magnetic needle is not in use, see that it is raised off the pivot.* When the needle rests on the pivot, jars are apt to blunt the point of the pivot or to chip the jewel, thus causing the needle to be sluggish. (10) Dust caps for both the objective and the eyepiece should be put in place and the instrument wiped clean before returning it to the box. (11) If the instrument is not to be returned to its box, see that it is protected from dust with a cloth hood.

17. Care of Small Equipment.—(1) Keep the chain or tape straight when in use. (2) Do not use the flag pole as a bar to loosen stakes or stones. Such usage bends the steel point and soon renders the flag pole unfit for lining purposes. (3) The brass or steel shoe on the foot of the leveling rod should not be struck against hard objects, as this, if continued, will round off the foot of the rod, thus introducing a possible error in leveling work. (4) Long rods when not in use should be placed in an upright position or supported their entire length, for otherwise they are apt to warp. Jointed leveling rods should have all clamps loosened when not in use to allow for possible expansion of the wood. (5) To avoid

losing pins, tie a piece of colored cloth (preferably bright red) through the ring of each. (6) Remember that hard knots in the plumb-bob string indicate a slovenly or inexperienced instrument-man; learn to make a sliding bowknot that can be easily undone.

SEC. 4. OFFICE COMPUTATIONS

18. General Remarks.—All computations should be preserved in a note-book for that purpose. This book may be the same as that used for field notes, but preferably it should be larger, say $8'' \times 10\frac{1}{2}''$. This size will give considerable space for a problem without turning to a new page, and the columns of tabulated values will not have to be crowded as will often be the case with the smaller book. The pages should be divided by light lines into $\frac{3}{16}$-in. squares. With such an arrangement columns can be kept straight without additional ruling, sketches can easily be made, and it will be an easy matter to give the computations a neat, systematic appearance.

In general, computations in the office will be a continuation of some fieldwork. They will be required for purposes of obtaining areas, plotting maps, profiles, and cross-sections, calculating dimensions to be laid off in the field, or ascertaining other desired information concerning the survey. It is desirable that these computations be easily accessible for future reference, and for this reason the pages of the book should be numbered and the contents of the book should be shown by a complete index. Parts of problems separated by other computations should be cross referenced. Each problem should have a clear heading which should include the name of the survey, the kind of computations, the field book and number of page of the original notes, the name of the computer, the name of the checker, and the dates of computing and checking. Enough of the field notes should be transcribed to make computations possible without further reference to the field book.

A systematic form should be used, and all calculations should be shown. Similar quantities should be tabulated in columns, and each column should have a heading showing what it contains so that the work may be inspected at a glance. Multiplications, divisions, and logarithmic computations should

be neatly and systematically recorded, so that others as well as the computer can trace the solutions without waste of time. Similar problems should be calculated in similar form. The student will soon become accustomed to a particular form, and the likelihood of mental confusion will decrease. Such of the field notes as are needed in the computations should be transcribed to the office book as neatly as possible.

19. Checking.—In practice the necessity of checking every computation is recognized. *No confidence is placed in results that have not been checked*, and important results are checked by more than one method. The student should see the necessity for this, for from experience he knows that a computation of any considerable length is rarely made without some error. He should early form the desirable habit of checking his own work, and should feel certain that his results are reasonably correct before handing them to his instructor. *Students should depend upon themselves.* It is true that students may check results by comparing work, but to do the work together comparing each step as it is completed is not checking in the true sense of the word, and is a weakness to be avoided. Such methods would not be countenanced in practice.

Most problems can be solved by more than one exact method. Since by using the same method in checking, the same error is likely to occur, the student should check his results by a different method when this is feasible. Approximate checks may be obtained in many cases through such mechanical devices as the slide-rule, the planimeter, and the protractor. The use of the slide-rule as a means of approximately checking each step as it is computed can not be too strongly advocated. Large arithmetical mistakes are almost sure to be discovered in this way, though of course mistakes due to confusion of numbers or to wrong methods will not be shown. Graphical methods may often be used, as an approximate check, to good advantage, both for final results and intermediate steps. They generally take less time than arithmetical or logarithmic solutions, and possible wrong assumptions in the precise solution will be avoided.

20. Precision of Computations.—For the most part, data forming the basis of computations in surveying consist of distances and angles determined from measurements in the field. No measurement can be exact; therefore all measurements contain errors of unknown magnitude, though methods of ob-

servation will confine these errors within certain limits. If a measurement recorded as 100 ft. contained a probable error of 1 ft., this measurement multiplied by two would contain a probable error of 2 ft., and the ratio of precision would in each case be $\frac{1}{100}$; if the 100 ft. with a probable error of 1 ft. were multiplied by 10 ft. containing a probable error of 1 ft. the result could not have a greater ratio of precision than $\frac{1}{10}$. From these examples it is obvious that in computations involving multiplication or division the result can not have a higher ratio of precision than that of the factor having the lowest ratio of precision. The computer should therefore bear in mind that results can not be more accurate than the data from which they are obtained. The general tendency among students is to disregard this most important phase of computations by making their work unnecessarily refined and by leaving in their final results an unwarranted number of significant figures. In lengthy computations it is sometimes necessary to carry out the intermediate results to one extra place but final results should show only the consistent number of significant figures. Not only is much time wasted in these useless refinements, but the final results are deceptive as well. The student should bear in mind that computations, like field measurements, should maintain an accuracy consistent with their purpose, and that though the precision of computations may be less than the precision of field measurements, it can never be greater.

Before beginning a computation, the purpose of its results should be known and the data upon which it is based should be critically examined to determine the precision of field measurements. Without this preliminary knowledge and study, the computation is likely either to indicate that the results are of a higher precision than the original data or it is likely to be unnecessarily refined. It has been before pointed out that field measurements should be taken with a precision dependent upon the purpose of the survey and that angles should have at least as great precision as related distances; nevertheless to presume such consistent accuracy would be unwarranted, for often data taken in the field serves more than one purpose, and very often the surveyor has paid little attention to securing consistent measurements. The following examples illustrate the value of examining the given data and of knowing the purpose of the computation:

1. Notes of a land survey indicate that the distances were measured with a precision of 1/10,000, while angles were measured with a probable error of 01′. It is desired that computations for area (involving sines and cosines) be as precise as the data warrants. Referring to the *Ratio Curves for Sines and Cosines* (Plate I, page 9), it will be seen that such an angular error for angles of average size corresponds to a precision of about 1/3000. It is this precision which will govern the accuracy of the computed area, and the computed area should contain four significant figures. Had it been presumed that the precision of angles was consistent with that of distances, doubtless five significant figures would have been shown in the result.

2. The notes of the preceding example are to be used in plotting a map of the area. The scale of the map is to be 100 ft. = 1 in. and the computations will involve sines and cosines. Investigation shows that at the scale mentioned distances can not be plotted closer than to the nearest foot, therefore the computations should contain but three significant figures. Had four significant figures been used, the length of computations would have been nearly doubled.

21. Trigonometric Tables.—The number of places to be used in trigonometric tables will depend upon the ratio of precision of the particular value of the function involved rather than upon the angular error. The number of significant figures for a particular angle, angular error, and function may be readily determined by inspecting either the Ratio Curves (Plates I and II, pages 9 and 10), or trigonometric tables. *Assuming that the precision of angle is the governing precision*, for tangents and cotangents three places will be sufficient for angles in error more than 02′; four places, for angles in error less than 02′ but more than 10″; and five places, for angles in error less than 10″ but more than 01″.

For sines and cosines of angles of *average size* it will be seen that four places are sufficient for angles in error not less than 20″; five places for angles in error less than 20″ but more than say 05″; and six places for angles in error less than 05″ but more than ½″. Since angles are likely to be other than average size, the preceding limits should in general be doubled, though for sines of very large angles and for cosines of very small angles the necessary number of places should be ascertained by first determining the ratio of precision corre-

sponding to the given angular error. For example the ratio of the tabular difference for 10″ to sin 80° (or cos 10°) is about 1/120,000, therefore six places will be necessary for angles as small as 10° when the error is 10″; or from the Ratio Curve for Sines it will be seen that for angles greater than 70° when the angular error is 01′, five places will be required.

22. Computations for Angles and Distances.—When, as is often the case, computations for angles or distances may be made in one of several ways, each of which depends upon data of like precision, it is best to compute angles by using functions which change rapidly, *i.e.*, by using tangents or cotangents; and to compute distances by using functions which change slowly, *i.e.*, by using sines or cosines.

For example the Ratio Curves for Tangents and Cotangents (Plate II, page 10) show that if the precision of data is 1/10,000 the maximum error of angles computed by tangents or cotangents is about 10″; the Ratio Curves for Sines and Cosines (Plate I, page 9) show that with data of a similar precision, angles between 27° and 63° could not have been computed by either sines or cosines with a precision as great as 10″.

23. Logarithms *vs.* Natural Functions.—As to whether or not logarithms shall be used will depend upon the computations under consideration. It takes less time to multiply two numbers of three digits each by arithmetic than by logarithms; possibly these numbers might be extended to four digits each, if it were only for a single computation. But for multiplying, dividing, squaring, cubing, or taking the roots of numbers, it is doubtful if arithmetic can ever be used to advantage beyond four significant figures. Where there are a number of similar computations, as is quite often the case in surveying, there is a *decided advantage* in using logarithms for *four* significant figures, for not only will less time be consumed in computing by logarithms but the liability of mistakes will be lessened and the mental strain on the computer will be decidedly decreased. Short arithmetical methods of multiplication and division are useful at times, but more often numbers that have enough significant figures to make these methods economical are large enough to make the use of logarithms more so.

Students in general either fail to appreciate the advantages of logarithms or feel that they are not sufficiently versed in the use of logarithms to use them with facility. Facility in the use of logarithms is not hard to acquire provided the work

is carefully planned. Before attempting any computations a form should be prepared which will show an orderly, systematic arrangement of all logarithms and numbers that are to enter the computations, and the given data should be recorded in this form. Like or similar operations of computing should be grouped. The order of the work should be as follows: (1) determination of logarithms of all known numbers; (2) determination of the logarithms of the functions of all known angles; (3) addition or subtraction of known logarithms; and (4) determination of numbers corresponding to computed logarithms. In using the logarithmic tables the work should be so planned that the least possible number of pages will be turned at one time. For example, if logarithms of 5623, 1213, 6157, and 2142 were to be found, it would be a waste of time to look them up in the order written. Obviously a better way would be to look them up in the order 1213, 2142, 5623, 6157. Logarithms are frequently arranged so that sines, cosines, tangents, and cotangents for the same angles appear on the same page. If, as is often the case, more than one function of the same angle is to be used, the corresponding logarithms should be determined at the same time even though these functions may be some distance apart in the computations.

The method here outlined may not seem to be the natural course of procedure to the student, because he has been used to working with arithmetic where each step is completed before another is begun; but if he will systematize his work so as to change the train of his thought as seldom as possible, he will find that his efficiency has increased, and he will be loath to use arithmetic in lengthy computations.

24. Graphical and Mechanical Methods.—Though graphical and mechanical methods are chiefly used as checks on precise methods of computing, the surveyor will find that through their use many results may be determined with sufficient precision and in much less time than they could be obtained by computation.

By plotting, areas, omitted measurements, etc., may be found graphically. Often the data given in tables may be shown by curves or diagrams.

The more usual mechanical devices are the slide-rule (of which there are several kinds for different purposes) which will in general give values within less than 1 per cent., the planimeter for determining areas of plats, the pantograph for

reducing drawings, and the protractor for measuring or laying off angles.

Sec. 5. Maps

25. General Remarks.—Maps may be divided into two classes: those that become a part of public records of land division, and those that form the basis of a study. The best examples of the former are the plats filed as parts of deeds in the county registry of deeds (in most states); and good examples of the latter are the preliminary maps along the proposed route of a railroad of which the location is to be projected. It is evident that the dividing line between these two classes is very indistinct, since many maps might serve both purposes. The requirements of a map will depend upon its purpose.

In general the things that should appear on a map that is to become a part of a public record are:

1. The lengths of all lines shown.
2. The bearings of all lines shown, or the angles between intersecting lines.
3. The location of the tract either with reference to principal meridian and base-line, if the tract is within the boundaries of the United States public land surveys; or with reference to co-ordinate axes, if within a city having a co-ordinate system.
4. The number of each block and lot, if the plat is of a city, town, or addition.
5. The position and kind of each monument set, with distances to reference marks.
6. The positions and names of all roads, streams, landmarks, etc.
7. The names of adjacent property owners, and the bearings of all property lines intersecting lines of the tract mapped.
8. The direction of the meridian (true or magnetic or both).
9. A legend or key to symbols shown on the map.
10. A diagram of the scale used with a corresponding note stating the scale. (The latter is usually shown in the title.)
11. A full and continuous description of the boundaries of the tract by bearing and length of sides; and the area of the tract.
12. The witnessed signatures of those possessing title to the tract mapped; and if the tract is an addition to town

or city, a dedication of all streets and alleys to the use of the public.

13. A certification by the surveyor that the plat is correct to the best of his knowledge.

14. A neat and explicit title showing the name of the tract, or its owner's name, its location, the scale of the drawing, the surveyor's name, and the date.

Of maps made the basis of studies, there are so many varieties and the requirements are so varied that a definite statement of all that each should include would be impossible. In general, maps of this kind show very few dimensions (often not any), the value of the map depending upon the correct representation of the position of features of the land rather than directly upon field measurements or computed values. Maps of this class may be divided into two kinds: (1) those that graphically represent such natural and artificial features as condition and culture of land, and public and private works (often called flat topographic maps); and (2) those that include some or all of the preceding features, but also represent the relief or contour of the ground (called topographic maps). Other things which should *always* appear on these two kinds of maps are:

1. The direction of the meridian.

2. A legend or key to symbols used, if they are other than the common conventional signs (Figs. 44–47, pages 218–222).

3. A diagram of the scale of the map with a corresponding note stating the scale.

4. A neat and appropriate title generally stating the kind or purpose of the map, the name of the tract mapped or the name of the project for which the map is to be used, the location of the tract, the scale of the drawing, the name of the engineer or draftsman or both, and the date.

26. Meridians.—The direction of the meridian is indicated by a needle or feathered arrow pointing north, of sufficient length to be transferred with reasonable accuracy to any part of the map. A meridian needle 4 in. or 5 in. long will be sufficient for large maps; 2 in. or 3 in. long for small maps. Preferably it should be near the middle of the drawing, but it should not interfere with features of the map. The true meridian is usually represented by an arrow with *full* head; the magnetic meridian by an arrow with *half* head. When both are shown the angle between them should be indicated.

The requisites of a meridian needle or arrow are that it shall be simple, graceful, and of such a size and in such location on the map that it will readily catch the eye; but it should not be of such proportions that it will be the *first thing* that strikes the eye. The general tendency is to make needles and arrows too large, blunt, and heavy. It is a waste of time to construct elaborate, ornate needles; moreover such needles do not add to the value of the maps which they adorn, but on the contrary often make such a marked contrast with the rest of the drawing as to detract from its general appearance.

27. Lettering.—All lettering should be free-hand. The style of lettering to be used will largely depend on the purpose of the map.

For office drawings or drawings which are not to become more or less public property, the REINHARDT style of letters, of which there are two kinds—inclined and vertical (Fig. 48, page 223), are used almost exclusively (except for titles). Although not as beautiful as some other styles of letters, this style is extremely practicable, since the letters may be rapidly constructed and easily read. Frequently the two kinds with their variations (extended and compressed forms) may be advantageously combined on a single drawing. Thus the names of streets and streams may be shown in extended form, inclined capitals; the names of property owners in normal form, lower-case vertical letters; and notes in compressed form, lower-case inclined letters.

Italic, *Roman*, and *Gothic* letters (Fig. 49, page 224) are more beautiful in appearance than the REINHARDT letters, and although more difficult of construction their use is often justifiable *on drawings to be used by the general public.* Of these three styles, Roman letters are the most difficult to construct on account of their fine lines and shadings; and slight deviations in their form are most quickly apparent, possibly on account of familiarity with perfect forms through printed pages. Gothic letters, both inclined and vertical, are most readily constructed; and unless the draftsman through considerable practice first familiarizes himself with the forms of letters of the other two styles he had best stick to Gothics. Italics are very little different from Roman letters except that they are inclined, but for this reason deviations in their form are not so apparent. Frequently all these styles may be used on a single drawing. For example the maps of the U. S.

Geological Survey show the names of civil divisions in Roman letters, hydrography in Italics, public works in inclined Gothics, and hypsography in vertical Gothics.

28. Titles should be so constructed that they will readily catch the eye, and the sizes of different parts should be in proportion to their importance. The best position for the title is the lower right-hand corner of the sheet. In general each line should be centered and the distance between lines should be such that the title as a whole will appear well balanced. The space occupied by the title should be in proportion to the size of the map. The general tendency is to make the title too large. One author has expressed the opinion that the length of the rectangle enclosing the title should not exceed $\frac{1}{5}$ that of the longest border-line of the sheet, and that the width of the enclosing rectangle should not exceed $\frac{1}{4}$ the length of the shortest border-line. These limits for most maps seem to be high. The different parts of the title should be in straight lines, and only the common styles of capital letters should be used. The scrolls and flourishes to be seen in the titles of many old plans are now obsolete, and few well-designed titles contain lower-case letters. It is best to construct the title of all vertical or all inclined letters, but a change of style of lettering between different parts is permissible when such change will serve to accentuate the important parts of the title. Thus the principal part may be constructed in Roman letters, while the remainder is in Gothics. It is bad practice to change the style for each line, for it conveys the impression that the draftsman was trying to show what he could do. In general, titles should be constructed free-hand, as well as the letters on the body of the map. The use of REINHARDT letters for the important parts of titles, even those of office plans, is not prevalent, though they may be appropriately used for the less important parts where small letters are used. The use of Gothic capitals is most common, for not only are they effective but they are easily made.

CHAPTER II

ELEMENTARY FIELD PROBLEMS

Problem 1. Pacing

Object.—To determine the length of normal pace, to test the reliability of an assumed 3-ft. pace, and to determine an unknown distance by pacing.

Procedure.—(1) Walk over an assigned course of known length ten times at an ordinary gait, counting the paces each time. Record the observed number in the field note-book as shown in accompanying form. Compute the average length of the natural pace and average number of paces for 100 ft. (2) With a tape mark a course 30 ft. long, every 3 ft. Walk over this course several times to obtain the proper stride; then pace the assigned course with this stride, recording the data and making computations as previously explained. (3) Walk over a course of unknown length several times at a natural pace, then several times at an assumed 3-ft. pace. Estimate the distance by each method; and then find the true distance with a steel tape. Note the error.

Hints and Precautions.—(1) In attempting to walk at a natural rate, avoid the general tendency to exceed that rate. (2) Count the paces carefully, estimating to the nearest one-tenth pace at the end of the course. (3) Reject observations that vary from the mean by more than 3 per cent. (4) Remember that field notes are a permanent record and should show clearly all the work done in the field. If an observation is rejected, draw a line through it, but do not erase. (5) Never fail to include in the notes of any survey, a complete description or sketch of the work accomplished.

Practical Applications.—Pacing is employed to a considerable extent in reconnaissance surveys, since for that sort of work approximate distances are in accord with the other rough methods used. It is also used by topographers and map sketchers on small-scale work.

A small pocket instrument, called a pedometer, automatically registers the number of paces. It is used to avoid the

1.

Prob. 1. Pacing.

Standardization of Pace.

Natural Pace.			3 ft. Pace.	
Trial.	Paces. per 300 ft.		Trial.	Paces. per 300 ft.
1	116.5		1	98.5
2	117.3		2	101.4
3	117.8		3	102.4
4	115.9		4	101.6
~~5~~	~~121.4~~		5	97.1
6	118.6		6	100.2
7	117.1		7	101.6
8	117.2		8	103.0
9	119.7		9	100.1
10	118.5		10	102.4
11	115.0		Mean	100.8
Mean	117.4		Paces per 100 ft = 33.6	
Paces per 100 ft = 39.1			Length of Pace = 2.98'	
Length of Pace = 2.56'				

Estimating Length. Line 251-252

Trial	1	2	3	Mean	Length in ft.
Nat. Paces	371	377	375	374	957
3 ft. Paces	318	326	324	323	969
Tape					948

Error in Natural Pace 1 in 100

" " 3 ft. " 1 in 45

F.F. Smith.
Oct. 1, 1913.
Fair & Cold.

Measurements taken with Steel Tape from Locker No. 46 by F.F. Smith, M.C. and J.C. Brown, M.C.

300 Ft. Course over Smooth Ground. Strong Wind at Right Angles with Course.

Line 251-252 over Slightly Rolling Ground. Crosses Highway and two Rail Fences.

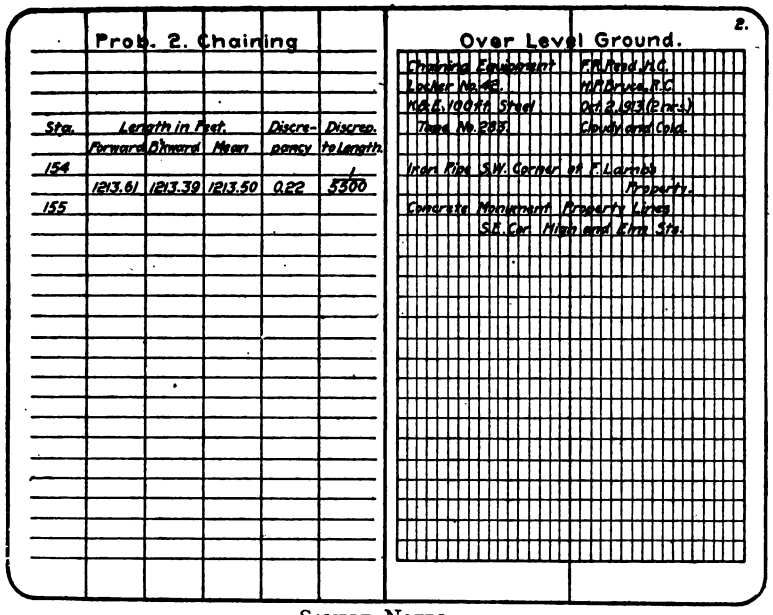
SAMPLE NOTES 1.

necessity of counting paces, which for a long distance is not only very monotonous, but absorbs the mind of the observer as well. This little device can often be used to good advantage.

Problem 2. Chaining over Level Ground with 100-ft. Steel Tape

Object.—To chain over an approximately level course about 1200 ft. long, and to check the distances by chaining in the opposite direction.

Procedure.—(1) Having set a flag pole at one end of the line *B*, the rear chainman goes to the other end *A*, taking one pin. (2) The head chainman with a flag pole, ten pins, and the zero end of the tape moves toward *B* until the one hundred-foot end of the tape approaches *A*. The rear chainman then calls out "*chain*," whereupon the head chainman with the flag pole in a perpendicular position at the zero end of the tape (or with a pin at the zero end if the ground is smooth and a clear sight can be obtained) allows himself to be put on line by the rear chainman. (3) When this condition has been fulfilled the rear chainman holds the one hundred-foot mark at *A* until the head chainman has stuck a pin at the zero point. The rear chainman, when ready for the measurement, calls "*stick*," and when the pin is firmly placed and the distance checked the head chainman replies "*stuck*." (4) The rear chainman then drops his end of the tape, calls "*forward*," pulls the pin at *A*, and both men move rapidly forward, the head chainman keeping approximately on line by sighting over the flag pole at *B* to some advanced point. (5) As the rear end of the tape approaches the pin in the ground the rear chainman calls "*chain*," a signal for the head chainman to stop, and the operation of setting the pin is repeated as before. (6) As the eleventh pin is stuck the head chainman calls "*out*" and waits until the rear chainman comes up. The chainmen then count the pins to make sure that none are lost and record the tally. (7) In observing fractions at the end of the course, first note the number of feet, and with the zero end of the tape held at the end point estimate the number of tenths; then slide the tape forward until, with a footmark at the last pin set, the station falls within the finely graduated foot at the end of the tape.



SAMPLE NOTES 2.

The distance should then be read to tenths of feet and estimated to hundredths. Record the data as shown on page 27. (8) Compute the ratio of discrepancy to length. Measurements should check within one in five thousand.

Hints and Precautions.—(1) The number of pins held by the rear chainman at any time indicates the number of hundreds of feet chained since the last tally. *The pin in the ground should not be counted.* (2) The rear chainman should refrain from holding onto the tape as he moves from station to station, for, if he moves too slowly, the head chainman is retarded, and if he moves too fast the chain is liable to become kinked. (3) Be careful not to disturb the "*stuck*" pin by allowing the tape to press against it. (4) Avoid injury to the tape by always keeping it straight while in use. (5) Avoid inconsistent errors by checking every measurement. (6) In order not to obstruct the view of the rear chainman in lining, the head chainman should stand beside the tape facing the line. This position also enables the chainmen to better withstand the pull on the tape. (7) If a reel is not provided for winding the tape, do it up in 5-ft. lengths in the form of a figure eight. To do this, stand beside one end of the tape, take the end of the tape in the left hand, and, allowing the tape to slide loosely through the right, extend the arms. As the 5-ft mark comes along, grasp it with the right hand, and, bringing the hands together, lay it in the fingers of the left hand without permitting the tape to turn over. Then grasp the loop with the left hand and again extend the arm for another 5-ft. length. When the last mark is reached, tie the loop lightly where the ends of the tape come together. By properly twisting the tape it may then be put in circular form.

Practical Applications.—Linear measurement by tape is the basis of all surveying in which a considerable degree of accuracy is desired. This method of chaining is the one used in general engineering practice, such as land, city, railroad, and highway work.

Problem 3. Standardization of Tape and Chaining over Uneven Ground

Object.—To standardize the tape before going into the field and on returning from work; to find the horizontal length of an

assigned course about 800 ft. long over uneven ground; and to correct for the error in length of tape as determined by the standardization tests.

Procedure.—(1) Standardize the tape by comparing it with the official standard. Maintain the required pull by means of a spring balance attached to one end of the tape. Determine the error with a finely divided scale. (2) Chain the course as in the previous problem, except that, since the ground is uneven, the tape must be held horizontally and the point transferred to or from the ground at the low end of the line by means of a plumb-bob. Maintain a pull estimated to be equal to that used during standardization. (3) For measurements in which the line slopes faster than 5 or 6 ft. in a hundred, it will be necessary to "*break chain;*" that is, to choose some intermediate point on the tape at which it will be possible to hold the tape horizontal. (4) Correct for the error in length of tape by adding to or subtracting from the observed distances, the observed error multiplied by the number of tape lengths in the course. *The correction should be added when the tape is longer than the standard, and subtracted when it is shorter.* Data should be recorded as shown in sample notes 3, page 30.

Hints and Precautions.—(1) The general tendency is to hold the downhill end of the tape too low. Avoid this by comparing the tape with some level line, or if possible, have the two ends in line with the horizon. (2) In "breaking chain" the head chainman should first pull the tape forward its *full length*, then go back to the footmark necessary to hold the tape level. After this point has been projected to the ground and marked with a pin obtained from the rear chainman, he again moves forward (without moving the tape) to the footmark necessary to make the tape level from the intermediate point just set. The rear chainman arriving at the point, holds on the footmark previously held by the head chainman. In this way the chain is always advanced its full length each time it is moved, except at the end of the course, and thus there is no chance of making a mistake of a fraction of a tape length. The intermediate points, being marked by pins obtained from the rear chainman introduce no opportunity for a mistake in tally. (3) It is usually more accurate to chain downhill than to chain uphill, since in chaining downhill the rear end may be held firmly on the ground.

Prob. 3. Chaining

Sta.	Length in Feet. Forward	Backward	Mean	Correction.	Cor. Length
351					
	311.73	311.65	311.69	+0.07	311.76
352					
	~~198.60~~ ~~198.40~~	198.62	198.61	+0.04	198.65
353					
	213.84	213.76	213.81	+0.05	213.86
354					

Standardization.

Before	100.021
After	100.023
Mean	100.022

Over Uneven Ground.

5

Chaining Equipment — J. Pratt, H.C.
Locker No. 35 — S. Fulton, H.C.
Chicago 100 Ft. Steel-Tape — Oct. 3, 1913 (2 hrs.) Cold and Windy.

Tk. in Stk. Ctr. Line M.C.R.R. at Haskell St.
East on Ctr. Line Haskell St. up steep hill.
Nail in pavement.
North over Creek and R.R. Embankment.
S.B. S.E. Corner Jones' Lot.
North up hill (Rough).
¾" Iron pipe N.E. Corner Jones' Lot.

Tape compared with Standard at City Hall.

SAMPLE NOTES 3.

Problem 4. Survey of Field with Steel Tape.

Object.—To collect sufficient data for calculating the area of the field by two sides and included angle, by three sides, and by base and perpendicular methods. See Sample Notes 4, page 32. (The data may be used in Problem 69, page 180.)

Procedure.—(1) Divide the field into triangles, avoiding as far as possible any construction which will result in forming a very acute angle; that is, make the triangles as nearly equilateral as the shape of the field will readily permit.

(2) Measure the sides and altitude of each of the triangles. Erect perpendiculars by one of the following methods:

3 : 4 : 5 *Method.*—To erect a perpendicular to AB that shall include point C in the triangle ABC (Fig. 1) by the 3 : 4 : 5 method. Assume point a to be on the perpendicular.

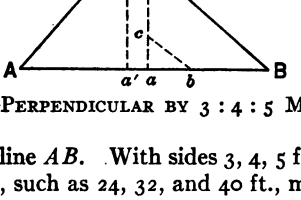


Fig. 1.—Perpendicular by 3 : 4 : 5 Method.

Set a pin at a on line AB. With sides 3, 4, 5 ft., or a multiple of these numbers, such as 24, 32, and 40 ft., make a right triangle abc with the tape as follows: Set a pin at b on line AB 24 ft. from a. Loop the tape over the pins at a and b so that the 32-ft. mark on the tape coincides with a, and the 56-ft. mark coincides with b. The head chainman with the zero and 96-ft. marks of the tape grasped in one hand, moves to c setting a pin at the point where the zero and 96-ft. marks coincide. The rear chainman at a or b prevents the chain from slipping over the set pins as the head chainman sets the pin at c, and sights along ac to set C' by C. If C does not lie on this prolongation, measure the perpendicular distance from the point C to the line aC' and move the foot of the perpendicular a along the line AB by an equal amount, to the point a'. If the trial perpendicular fails to include the point C by several feet, repeat the process for a', the new point. If, however, the distance $C'C$ is small, the new foot of the perpendicular may be set by making aa' equal to $C'C$ and it may be assumed as correct without further test.

4.

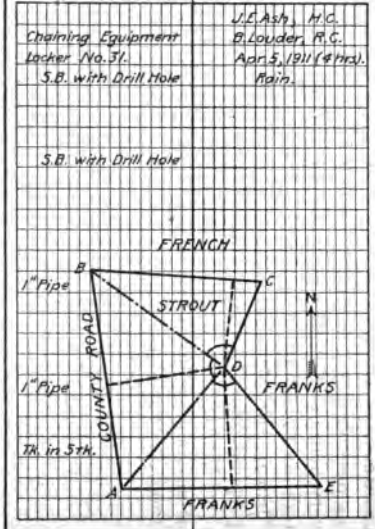
Survey of Field. with 100 ft. Steel Tape.

Sta.	Dist.	Perpendicular	Chord Length	Sin ½ Angle	Angle
A					
	321.43				
D			168.00	0.8400	114°20'
	286.74				
B					
	511.08	164.32			
A					
B					
D			135.92	0.6796	85°38'
	184.01				
C					
	352.57	149.07			
B					
E					
	310.85				
D			139.64	0.6982	88°34'
A					
	441.50	226.80			
E					
D			116.94	0.5847	71°34'
C					
					360°06'

of P. S. Strout. Prob. 4.

J. E. Ash, H.C.
B. Louder, R.C.
Apr. 5, 1911 (4 hrs.).
Rain.

Chaining Equipment
Locker No. 31.
S.B. with Drill Hole



SAMPLE NOTES 4.

Chord-bisection Method.—Estimate the position of the perpendicular from C upon AB (Fig. 2), and set a pin at d on this estimated perpendicular, less than one tape length from the line AB. With d as center and length of tape as radius,

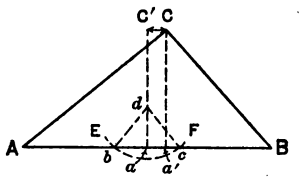


FIG. 2.—PERPENDICULAR BY CHORD-BISECTION METHOD.

the head chainman describes the arc of a circle. The rear chainman stationed at A or B determines the position of pins at b and c, the points of intersection between the line AB and the arc of the circle EF. The point a bisects the line bc. Prolong ad, place C' by C, and move point a as described in the 3 : 4 : 5 method.

(3) Determine one angle of each triangle by the chord method as follows: With the vertex of the angle as the center,

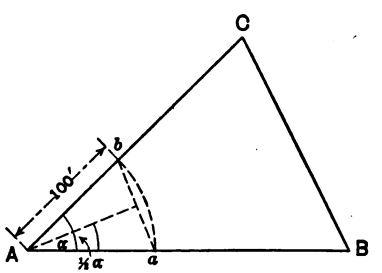


FIG. 3.—ANGLE WITH TAPE.

swing the tape, setting pins a and b (Fig. 3) at points where the arc intersects the lines AB and AC. Measure the chord distance ab.

$$\sin \tfrac{1}{2}\alpha = \frac{ab}{200}$$

Hints and Precautions.—(1) The chord-bisection method of erecting perpendiculars is the more accurate, while the

3 : 4 : 5 method requires less time. (2) If the perpendicular and the segments of the base on each side of its foot are measured, sufficient data will have been obtained for determining the angles by the tangent method. However, the chord method of measuring angles is more accurate, and the tangent method should be used only as a check. (3) Considerable care should be taken in lining in points, and intersections should be determined as closely as the eye of the observer will allow. (4) A very good way of finding the approximate position of the perpendicular is to stand on the line AB with arms extended horizontally in the directions of A and B. Now, with eyes closed, bring the arms to the front, palms together, and opening the eyes, see if the vertex of the triangle lies in line with the hands. If it does not, the observer should change his position to some other point along AB nearer the desired perpendicular.

Practical Applications.—It is obvious that surveying by this method would be too slow and inaccurate to be utilized to any great extent except in surveys covering small areas; hence this method is, in general, restricted to rough estimates or checks. In this problem the elementary steps are the more important, since the construction of perpendiculars and the measuring of angles is often necessary when the transit is not at hand. Angles measured by the tape method are usually thought of as being less accurate than those measured with the transit; but for small angles, the reverse is likely to be true.

Problem 5. Survey of Field with Irregular Boundary

Object.—To collect sufficient data for calculating the area and for plotting the assigned field. (The data may be used in Problems 70, 71, and 72, pages 180 and 182.)

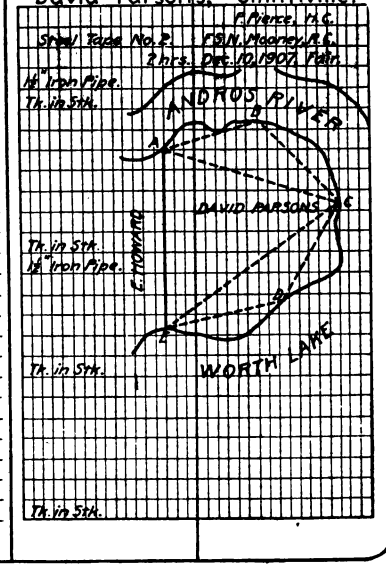
Procedure.—(1) Divide as much of the field as its shape will permit into triangles as in Problem 4, having in view simplicity of field operations. Lay out the triangles so that no long offsets will be necessary. (2) Collect data for finding the area of the triangles by one of the methods explained in Problem 4. (3) From the triangle sides nearest the boundary, take offsets at such intervals as will insure a sufficiently accurate plat. Record the data as shown in sample page of notes.

Hints and Precautions.—(1) To take the offsets accurately and quickly, set several pins at the desired intervals along the

Prob. 5. Survey of Land of David Parsons, Smithville. 3.

With 100 ft. Steel Tape.

Sta.	Dist.	Offset L	Sta.	Dist.	Offset L
A	0	0			
	25	21.4	D	0	0
	50	37.8		25	10.0
	75	44.6		50	26.2
	100	27.2		75	34.6
	111	0		100	34.5
	125	15.5		125	25.2
	140	16.7		150	14.0
B	159 86 0	0		175	4.1
	30	21.6	E	288.64 0	0
	50	42.0	A	275.98	
	90	51.4	AC	281.14	
	140	27.8	EC	273.61	
	165	13.0			
C	187 88 0	0			
	14	0			
	45	16.3			
	55	31.6			
	75	61.0			
	90	14.2			
	100	7.0			
D	109 89	0			



SAMPLE NOTES 5.

side of the triangle before measuring the offsets. If the distance to the boundary is not more than 50 ft. and the boundary is fairly regular, erect perpendiculars by estimation with the eye. (2) If the boundary changes abruptly at any point, be sure to take an offset at that point. (3) Do not measure the offsets with unnecessary accuracy. For example, if the sides of the triangles are measured to the nearest 0.01 ft., measure the offsets no closer than the nearest 0.1 ft. (4) Do not record offsets as being on the right of the line when they are on the left.

Practical Applications.—This problem is frequently made use of in land surveying where the boundary line is a stream or lake. Lines are run by compass or transit as near to the stream or lake as is practicable. Where the boundary changes direction frequently, offsets are taken at short intervals; but as long as the boundary continues as one straight line, offsets need not be taken. However, because of the greater ease with which the areas can be computed, offsets are frequently taken at regular intervals.

Problem 6. Obstructed Distance with Chaining Equipment

Object.—To determine the distance between two points with an obstacle on line.

Procedure.—(1) On an assigned course about 800 ft. long, assume that there is some obstacle which makes direct measurement and intervision impossible, and find the distance by the swing-offset method. (2) Find the distance by constructing a line parallel to the given line. (3) Assume that the points are intervisible but direct chaining is impossible, and determine the distance by methods other than those used above. (4) As a check, measure the distance directly. Compare the results.

Swing Offsets.—To find the distance by the swing offset method, the head chainman attaches the end of the tape to one end of the line as B (Fig. 4), and describes an arc of a circle the radius of which is 100 ft. and the center of which is the point B. The rear chainman stationed at A lines in the end of the tape with some distant object as O, and sets points a and b where the head chainman crosses line AO. A point C midway between a and b lies on the perpendicular CB. Bi-

sect *ab*, set a pin at *C*, and measure the distance *AC*, to obtain the necessary data for calculating *AB*.

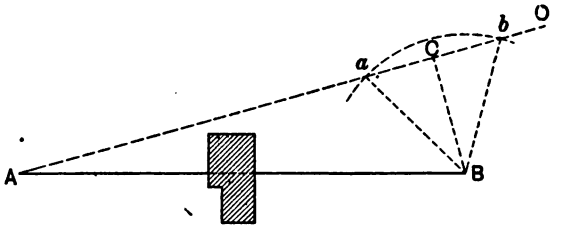


FIG. 4.—SWING OFFSET WITH TAPE.

Parallels.—Parallels may be laid off with a tape by one of the following methods: 1. By equal distances; 2. by alternate angles; 3. by similar triangles.

Method 1.—If points are intervisible, as will be the case when the obstruction is a stream or lake, erect perpendiculars at *A* and *B* (Fig. 5) by one of the methods of Problem 4,

FIG. 5.—INACCESSIBLE DISTANCE.

chain distance $AA' = BB'$ to clear the obstacle, and measure the distance $A'B'$. If the perpendiculars have been carefully erected and the distance $AA' = BB'$ is not great, the measured distance should agree closely with the length of the line *AB*. If the points are not intervisible, this method will be impossible.

Method 2.—If the offset necessary to avoid the obstacle is considerable, the preceding method will become inaccurate because of the uncertainty of right angles measured with tape. If the points are intervisible, establish *AC* (Fig. 6, page 38), so that the estimated value of α will be less than 45°. Determine the chord length of α for radius of 100 ft. (or greater if precision makes it necessary). Measure *CD*, it being roughly perpendicular to *AB*. At *C* lay off an angle equal to the angle at *A* so that *CE* will be parallel to *AB*. Measure *CE*, *E* being any convenient point such that β will be less than

45°. Measure *BE* and *EF*. $AB = AD + CE + FB$. Solve the right-angle triangles *ADC* and *BFE* for *AD* and *BF*, and compute the length of the line *AB*.

The precision of this method over that of the preceding method is due to the fact that the angles laid off are small and that the distances *AD* and *BF* are *computed* rather than measured. The reason for computing these distances will

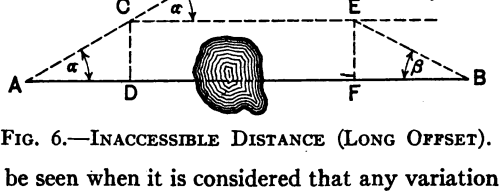


FIG. 6.—INACCESSIBLE DISTANCE (LONG OFFSET).

readily be seen when it is considered that any variation of the line *DC* from the true perpendicular will make little difference in its length as compared with the corresponding change of length of *AD*. To illustrate, suppose α (Fig. 6) equal 45°, that the true length of *AD* and *CD* is 200 ft. and that *D*, supposedly on the perpendicular through *C*, is 10 ft. in error. As *computed* from *AC* and *CD*, *AD* is about 0.3 ft. in error, but its *measured* length is 10 ft. in error.

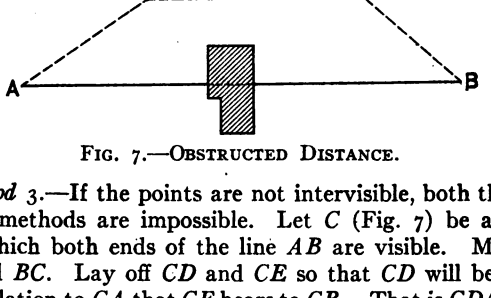


FIG. 7.—OBSTRUCTED DISTANCE.

Method 3.—If the points are not intervisible, both the preceding methods are impossible. Let *C* (Fig. 7) be a point from which both ends of the line *AB* are visible. Measure *AC* and *BC*. Lay off *CD* and *CE* so that *CD* will bear the same relation to *CA* that *CE* bears to *CB*. That is $CD/CA = CE/CB$. (It will generally be convenient to make this ratio a simple one such as ½ or ⅓.) The triangles *ACB* and *DCE* are similar. Measure *DE*, and compute *AB*.

CHAPTER III

COMPASS AND TRANSIT PROBLEMS

PROBLEM 7. ADJUSTMENTS OF THE SURVEYORS' COMPASS

1. **Bubble Adjustment.**—*To make the axes of the bubble-tubes perpendicular to the vertical axis of the instrument.* Level up the instrument, and turn 180° about the vertical axis. If the bubbles move from the center, bring each *halfway* back by means of the adjusting screws. Level the instrument, and repeat the operation to verify the adjustment.

Detection of Errors of Adjustment in Needle or Pivot.—Note the readings of both ends of the needle for various positions of the compass. If the end readings always differ by the same amount, then the needle only is out of adjustment, but if the difference in the end readings varies for different positions of the needle, there may be an error in adjustment of both needle and pivot. In making these adjustments it is necessary to *first adjust the needle*, since its error can be detected independently of the error in the pivot.

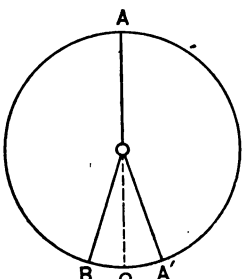


FIG. 8.—ADJUSTMENT OF NEEDLE.

2. **Adjustment of the Needle.**—*To make the middle of the needle lie in the same vertical plane with the ends.* With the instrument set up and the needle free, remove the glass cover.

Steady the needle with a piece of wood or copper or brass wire, and read both ends, A and B (Fig. 8, p. 39). Turn the compass needle until the end B takes the position previously occupied by A. If the needle is out of adjustment, the end A will fall at A'. BA' in the figure is *twice* the error. Remove the needle from the compass box, and with a pair of pliers bend it by an amount estimated to be equal to its error. When it is replaced on the pivot with one end at A, the other end should fall on C, a point midway between A' and B. Repeat the operations until the desired condition is realized.

3. Adjustment of the Pivot Point.—*To make the pivot point lie exactly in the center of the graduated circle.* Having already straightened the needle, turn the compass box until the ends of the needle are 180° apart (AB in Fig. 9). Now

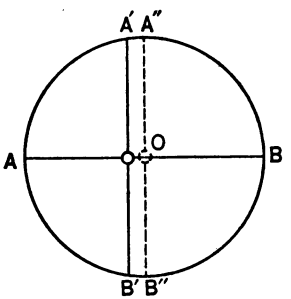


FIG. 9.—ADJUSTMENT OF PIVOT.

turn the needle through 90° to the position $A'B'$ in Fig. 9, and read both ends. The difference between the end readings will be the maximum and equal to *twice* the distance of the pivot from the true center. Remove the needle, and with a pair of pliers bend the pivot in a direction perpendicular to the last position of the needle, to the point O. The ends of the replaced needle will take the positions A'' and B'', but will read the same. Repeat the operations until adjustment is obtained.

4. Adjustment of the Sights.—*To make the plane of the sights perpendicular to the plane of the bubbles.* Level up the instrument *very carefully*, and sight first through one vane and then through the other at a suspended plumb-bob string. If the slit in either vane makes an angle with the plumb line,

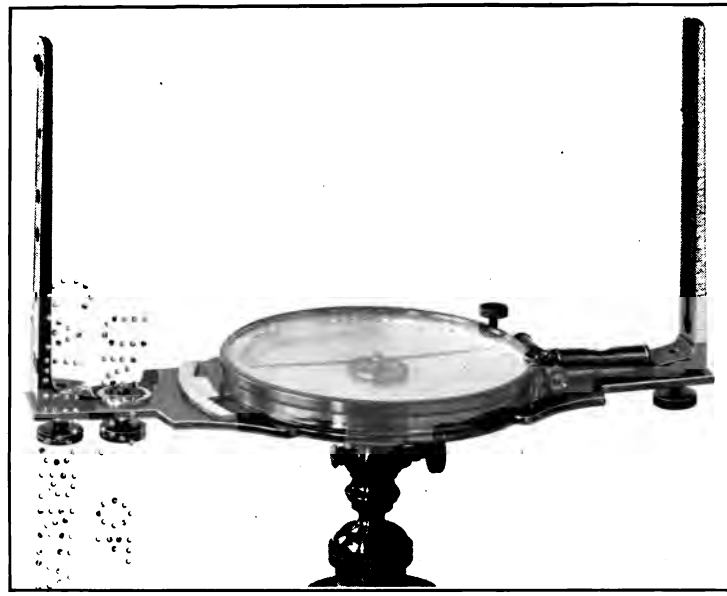


PLATE III.—SURVEYORS' VERNIER COMPASS.
(From photograph furnished by W. & L. E. Gurley.)
(*Facing page* 40.)

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the sights are out of adjustment and should be corrected by filing off the base of the vane or by putting strips of paper under one side of the base sufficient to make the slit truly vertical.

Problem 8. Determination of Magnetic Declination

Object.—To determine the magnetic declination with the surveyors' compass. See Sample Notes 8, page 42.

Procedure.—(1) See that the compass is in good adjustment; if it is not, adjust as explained in Problem 7. (2) Set the compass over one end of a meridian that has been determined by astronomical observations, sight along the line, and clamp the compass in that position. (3) By means of the tangent-screw, move the compass circle until the needle reads zero. (4) From the declination arc read the declination to the nearest minute; record the declination and the time of observation. (5) Take observations as above every five minutes over a period of half an hour or more, resetting the line of sight and compass circle for each observation. (6) If possible take a series of observations at about the same time on each of several days. (7) Determine the most probable value of the declination for the hour of observation and the probable error of a single observation and of the mean. (See Problem 34, p. 98.) (8) Determine the mean declination by adding to or subtracting from the observed declination the average solar-diurnal variation (Table VI, p. 252) for the place nearest the place of observation.

Hints and Precautions.—(1) Care should be taken to keep sources of magnetic disturbance away from the instrument. *Great care should be taken not to produce static charges of electricity by rubbing the glass.* A moistened finger pressed against the glass will remove such charges. (2) At each observation the compass box should be tapped lightly as the needle comes to rest that there may be no adhesion between the jewel of the needle and the pivot. (3) Between 6 and 7 p.m. is the best time for taking magnetic observations, because at this time the magnetic declination reaches approximately its mean value for the day, as will be seen by looking at Table VI. Between 10 and 11 a.m. the declination also reaches its mean value, but the rate of change of declination is more rapid at

6.

Prob. 8 Determination of Declination at Philips, Mass.
With Vernier Compass.

M.I.T. Tech.
G. Bolton.
May 4, 1912.
Fair and Warm.

Gurley Compass
Point Locker E.

Obs. No.	Time A.M.	Vernier Read.	V	V^2	
1	8h 15m	12°13′	4	16	
2	20	22	5	25	
3		24	7	49	
4		23	6	36	
5		12	5	25	
6		19	2	4	
7		10	7	49	
8		18	1	1	
9		14	3	9	
10	9 00	12°15′	2	4	
Mean	8h 37m	12°17′		218	$= \Sigma V^2$

Meridian Determined by Observations on Polaris. Probable Error ± 0°00′02″.

Vernier Arm and Graduated Circle turned Counter-Clockwise.

$E_c = \pm 0.67\sqrt{\frac{\Sigma V^2}{n-1}} = \pm 0.67\sqrt{\frac{218}{9}} = \pm 03.'2$

$E_m = \pm \frac{E_c}{\sqrt{n}} = \pm \frac{032}{\sqrt{10}} = \pm 01.'0$

s.d. Variation (Table XI) = ± 03′

Mean Mag. Declin. = 12° 20′ W

SAMPLE NOTES 8.

this time than between 6 and 7 p.m. (4) The values given in Table VI to correct observations taken at other times of day are approximate, since this diurnal change varies from year to year, and is greater in summer than in winter. However such approximation is in accord with other rough methods of the problem. (5) If the compass circle has been turned in a clockwise direction the declination is east. If the diurnal variation is positive for the time of the observation, the north end of the needle is deflected more to the eastward than when the declination is at a mean. Hence, the mean is determined by algebraically subtracting the diurnal variation from the observed declination, east being considered as positive and west as negative. (6) The probable error shows the precision with which the measurements were taken, and may be taken as indicative of the precision of future observations under similar conditions.

Practical Applications.—In re-running old surveys or in making new surveys the lines of which are referred to the true meridian, it is necessary that the declination be known. An isogonic chart will furnish the surveyor with an approximate value of the declination say within ten or fifteen minutes, but this will be hardly close enough for the average survey, nor is such a chart always available. The above method is the one in common use among surveyors and should furnish a determination within 03′ of the true declination.

Problem 9. Survey of Field with Surveyors' Compass and 66-ft. Chain

Object.—To find the lengths and bearings of the sides of an assigned field. (These data may be used in Problem 73, page 183.)

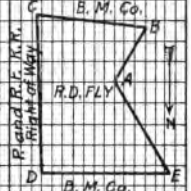
Procedure.—(1) Set off the magnetic declination for the place where the survey is to be made, on the declination arc of the compass. (2) Set up at one corner of the field, A, and sight along the line AB, being careful to have the south end of the compass box nearest the eye. Lower the needle, and as it comes nearly to rest, tap the compass lightly with the end of a lead pencil. When the needle becomes stationary read the north end, estimating the bearing to the nearest 05′. The direction is indicated by the meridianal and longitudinal letters on the compass circle between which the north end of the needle stands. (3) Take a back bearing from A. (4)

Prob. 9 Survey of Wood-Lot of R.D. Fly, Bemis, Me. 11.

With Surveyors' Compass and 66 Ft. Chain.

Sta.	Obj.	Dist.	Bear.	Int. Ang.	Cor. Bear.
A	E		N 26°00'W		
	B	24.93	S 30°40'W	238°40'	S 30°40'W
B	A		N 30°40'E		
	C	37.56	S 83°30'E	65°30'	S 83°45'E
C	B		N 84°30'W		
	D	48.42	N 2°00'W	82°30'	N 1°10'W
D	C		S 2°15'E		
	E	35.26	S 89°30'W	91°45'	N 89°20'W
E	D		East		
	A	25.77	S 26°30'E	61°10'	S 28°05'E
				539°35'	

Surley Vernier Compass. No. 89. Declin. 15° 20W. Set off with Vernier.
N.E. Dunn, Jr. F. Arseneault, Chain. Nov. 13, 1906. Snow.
Spruce Tree 18"ø Blazed B.M.Co./R.D.F.
Cedar Stump 12"ø " " / "
Rough Stone, d.h.
Ledge, d.h. and cross.
Cedar Stake 4"ø 4' high.



Note: Bearings are Referred to the True Meridian.

SAMPLE NOTES 9.

Chain the line AB, and record the distance in chains, to the nearest link. (5) Set up the compass at B. Observe the back bearing of the line AB and the forward bearing of the line BC. Chain the distance BC. (6) Continue in the same manner around the field, taking both back and forward bearings from each point. (7) Calculate the interior angles of the field from the back and forward bearings measured at the vertex of each of the angles, and correct the observed bearings for local attraction, if such exists. See Sample Notes 9.

Hints and Precautions.—(1) The upright sight vanes are usually unlike, the one to be attached nearest the north point of the compass box being marked for reading vertical angles, and the one nearest the south point being fitted with "peep" sights. It is well to bear this in mind, both when assembling the compass and when using it in the field. (2) Since the accuracy with which angles can be read depends upon the delicacy of the needle, special care should be taken to avoid any concussion between the jewel bearing of the needle and the pivot point. *Never, under any condition, move the instrument without first making certain that the needle is clamped.* (3) Keep sources of attraction such as chaining pins, axe, and pocket knives away from the instrument when bearings are being observed. (4) Be sure to set off the declination in the right direction. If the declination is east, the magnetic meridian lies to the east of true north; and since the point of zero bearing must lie on the magnetic meridian, the graduated circle must be turned in a clockwise direction. (5) In order to balance the needle or to counteract the magnetic dip, a fine brass or copper wire is wound about the southern end of the needle (in the northern hemisphere). With the instrument level if the needle does not remain horizontal when it is free to swing on the pivot, the glass should be removed and the wire moved along the needle until it balances perfectly. (6) In order not to confuse the north and south ends of the needle when taking bearings, always note the position of the counterbalancing wire. (7) The test for local attraction is to observe the forward and back bearing of each line. If these bearings fail to agree by an appreciable amount, local attraction exists at one and possibly both points from which observations were taken. To correct for local attraction, start from some line in the traverse, the forward and back bearings of which have the same angular value (that is, the ends of

which are free from local attraction) and calculate the bearings of the other sides by means of the interior angles at each point. (8) The interior angles will be correct regardless of local attraction, since at any one point the needle will be deflected as much when the compass box is in one position as when it is in any other. If n is the number of sides of the field and $\Sigma\Theta$ the sum of the interior angles, then $\Sigma\Theta = (n - 2)\ 180°$. (9) In chaining, observe the precautions stated in Problems 2 and 3, pages 26 and 28.

Practical Applications.—Formerly the compass was the most important of surveying instruments, being utilized in nearly all classes of plane surveying. At the present time, however, the transit has in a great measure taken its place, and the compass is little used except in the rougher class of surveys in which speed and not accuracy is the first requirement. Some land surveyors of the "old school" still retain the compass, but since land in this country, which a few years ago was worth but little, is now in many cases quite valuable, more accurate methods of measurement are required. For the same reason the old link chain is rapidly giving way to the light but more accurate steel tape. However, the compass is not without some advantages. It is light, and is easily and quickly set up, and an error in the bearing of one line does not affect the bearings of other lines in the same survey. It is especially suitable for preliminary surveys on lines through wooded country. Where clear sights are obtainable, only alternate stations need be occupied; and if a tree is on line, it is only necessary to set up on the other side of it, finding by trial the position which, with the backsight on the tree, will give the back bearing of the line prolonged. In retracing old lines where no greater degree of accuracy is required than was obtained in the original survey, and where there is a possibility of undetected local attraction in the old bearings, the compass may be used with economy.

Problem 10. Retracing Survey with Compass and 66-Ft. Chain. Two Adjacent Corners Known

Object.—To retrace property lines from the notes of an old compass survey. The bearings given in the original notes are magnetic, and the declination at the time of the original survey is unknown. Two adjacent corners of the plot can still be identified.

Procedure.—(1) Measure the length of the known side and compare with the original. (2) By proportionate distances compute the lengths of the other sides in terms of the re-survey chain. (3) Set up the compass at one end of the known line, sight along the line, and clamp the compass. (4) Release the needle, and as it comes to rest move the compass circle by means of the tangent-screw until the original bearing is read. (5) Proceed to lay out the field, chaining distances in terms of the re-survey chain as computed above and laying off the original bearings. Examine the ground for rotten stakes or other evidence that would have precedence over bearings and lengths of sides. (6) Reference the new corners by bearings and distances to nearby permanent objects.

Hints and Precautions.—(1) Examine the ground well; careful digging with a spade will often reveal unmistakable evidence of the location of the original corner. Discolored earth may mark the former position of a stake long since rotted away. (2) If the original corners were witnessed, the witness points should be sought for as carefully as for the original corners. If such points are found, the corner should be re-established from them rather than from bearings and distances of the sides. (3) Observe the hints and precautions of the previous problem.

Practical Applications.—The above method of re-running a survey is in common use by land surveyors. It is the duty of the surveyor to re-establish corners as near to their original position as possible, regardless of whether the old survey was correctly made or not. If he discovers errors in the original survey he may point them out, but the re-adjustment of property lines is not within his scope.

Problem 11. Adjustments of the Engineers' Transit

Note.—In the description of the following adjustments it is assumed that the objective slide does not admit of adjustment, but is permanently fixed in the telescope tube so far as lateral motion is concerned; and that the maker has done his part by so constructing the instrument that the optical axis and the axis of the objective slide coincide and are perpendicular to the horizontal axis. This ideal construction

is never exactly attained; but in most modern instruments the departure is so slight that it need not be considered in ordinary transit work, and in precise surveys the resulting errors are eliminated by methods of procedure.

For transits with adjustable objective slides, the adjustments are the same as for those with permanently fixed objective slides except as noted on page 51.

(1) To Make the Vertical Cross-hair Lie in a Plane Perpendicular to the Horizontal Axis.

Test.—Sight at a well defined point about 200 ft. from the instrument. Swing the telescope through a small vertical arc, so that the point will appear to traverse the length of the vertical cross-hair. If the point appears to move continually on the hair, the cross-hair lies in a plane perpendicular to the horizontal axis.

Correction.—If the point does not appear to move always on the cross-hair, loosen two adjacent reticule screws and turn the reticule in the telescope until the point appears to traverse the entire length of the hair. Tighten the *same* two screws.

(2) To Make the Axes of the Plate Bubbles Perpendicular to the Vertical Axis.

Test.—Turn the alidade until each bubble-tube is parallel to a pair of foot-screws. Bring the bubbles to the centers of the tubes by means of the foot-screws. Reverse the alidade 180° in azimuth. If the bubbles remain in the centers of their tubes, the axes of the bubble-tubes are in a plane perpendicular to the vertical axis.

Correction.—If the bubbles do not remain in the centers of the tubes, bring them *half way* back by means of the adjusting screws. Level the instrument again, and repeat the process to verify the results.

(3) To Make the Line of Sight Perpendicular to the Horizontal Axis.

Test.—Set up and level the instrument. Sight on a point *A* (Fig. 10) about 500 ft. away, plunge the telescope and set another point *B*, on the line of sight and about the same distance away on the opposite side of the transit. Turn the instrument in azimuth and again sight at *A* (with telescope inverted). Plunge the telescope; if *B* is on the line of sight, the desired relation exists.

Correction.—If *B* is not on the line of sight, set a point *C*, beside it on the line of sight. Mark a point *D*, *one-quarter*

PLATE IV.—ENGINEERS' COMPLETE TRANSIT.
(From photograph furnished by C. L. Berger & Sons.)
(Facing page 48.)

of the distance from C to B, and adjust the reticule by means of the two opposite horizontal screws, until the new point is on the line of sight.

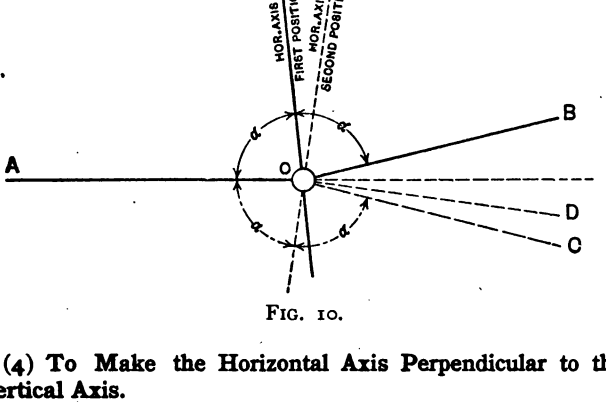


FIG. 10.

(4) To Make the Horizontal Axis Perpendicular to the Vertical Axis.

Test.—Set the transit near a building, flag staff, or other structure upon which is some well defined point at a considerable angle of elevation. Level the instrument very carefully, thus making the vertical axis truly vertical. Sight at the high point A (Fig. 11, page 50); and with the horizontal motions clamped, depress the telescope and set a point B on the ground. If the horizontal axis is truly horizontal, A and B will be in the same vertical plane, since the line of sight has been made perpendicular to the horizontal axis. Plunge the telescope, reverse in azimuth, and again sight on A. Depress the telescope as before; if the line of sight falls on B, the horizontal axis is perpendicular to the vertical axis.

Correction.—If the desired relation does not exist, set C on the line of sight beside B. A point D *one-half* the distance from B to C will lie in the same vertical plane with the high point. Loosen the screws of the bearing cap, and raise or lower the adjustable end of the horizontal axis until the line of sight will pass through both A and D when the telescope is swung in a vertical arc. *The high end of the axis is always on the same side of the vertical plane through the high point as the point last set.* In re-adjusting the bearing cap, care should be taken not to bind the horizontal axis, but it should not be left

so loose as to allow the objective end of the telescope to drop of its own weight when not clamped.

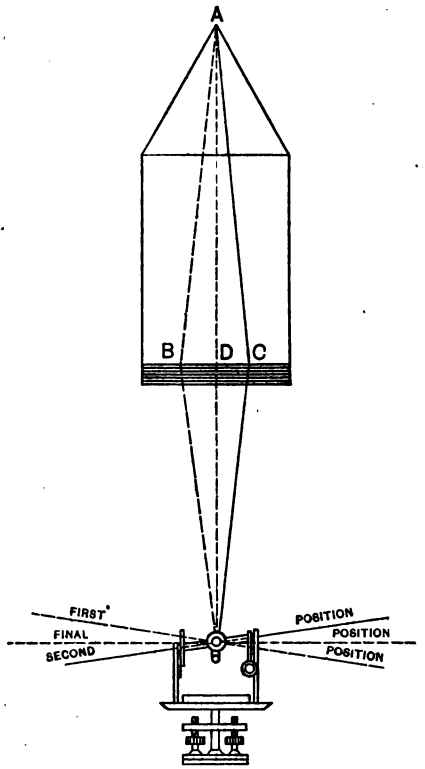


FIG. 11.—ADJUSTMENT OF HORIZONTAL AXIS.

(5) To Make the Line of Sight Coincide with the Optical Axis.

Test.—Set two stakes, one about 25 ft. and the other 300 or 400 ft. from the transit. With the horizontal axis clamped, take a rod reading on the distant point, and without disturbing the horizontal axis read the rod on the near point. Plunge the telescope, turn through 180°, and set the horizontal cross-hair at the last rod reading with the rod held on the near point.

Sight to the distant point. If the desired relation exists, the first and last readings will be the same.

Correction.—If there is a difference between the rod readings and it is considerable, move the horizontal cross-hair by means of the upper and lower reticule screws until it has apparently traversed over several times the apparent error. Repeat the process, gradually reducing the movement of the cross-hair as the rod readings to the distant point approach each other, until by successive approximations the error is reduced to zero. The rod when held on the near point should be read with great care, for a small difference in the position of the cross-hair on the near rod will be sufficient to indicate a considerable error on the distant rod.

(6) To Make the Axis of the Telescope Bubble Parallel to the Line of Sight.

Test and Correction.—Same as the peg adjustment of the dumpy level (Problem 25, p. 79) except that the correction is made by moving the bubble-tube until the axis is horizontal after having set the line of sight on the rod reading for a level line.

(7) To Make the Vertical Circle or Arc Read Zero when the Telescope Bubble is in the Center of the Tube.*

Test.—With the plate level, bring the telescope bubble to the center, and read the vernier of the vertical circle.

Correction.—Loosen the vernier, and move it until the desired relation is established.

(8) To Adjust the Auxiliary Bubble on the Vernier of the Vertical Circle so that its Axis is Parallel to the Axis of the Telescope Bubble when the Vernier Reads Zero.

Test.—Bring the telescope bubble to the center of its tube, and move the vernier by means of the tangent-screw until it reads zero.

Correction.—By means of the capstan-headed screws, move the bubble attached to the vertical vernier until it is in the center of its tube.

Transit with Adjustable Objective Slide.†—Some transits have objective slides which move in adjustable rings, and it is claimed by the manufacturers of such instruments that the optical axis will require no further attention after the factory adjustment has been made unless through accident some of

* This adjustment is not applicable to a transit having a bubble attached to the vertical vernier.

† See Note on page 47.

the parts of the telescope become bent. However, it is well to test the adjustment occasionally, and the instrumentman should be able to make corrections if necessary. Having performed the adjustment of the vertical cross-hair (Adjustment (3)) for an average length of sight, focus the vertical cross-hair on a distant point. Move the objective out, and bring it to a focus on some very well-defined point near the instrument. Plunge the telescope, turn 180° about the vertical axis, and again set the vertical cross-hair on the near point. Sight toward the distant point. If the objective slide is in adjustment, the line of sight will strike the first point sighted. If the line of sight does not strike the distant point, move the ring controlling the objective slide by means of the screws on the sides of the telescope until by estimation the line of sight has moved over one-half the apparent error at the distant point. The relation between the adjustments of the vertical cross-hair and optical axis is such that the adjustments must be repeated alternately until both are found to be correct. The vertical adjustment of the objective slide may be performed in a similar manner with reference to the horizontal cross-hair, but it is usually best to make corrections with the horizontal cross-hair as described in Adjustment (5) unless one is reasonably sure that the factory adjustment, through accident or otherwise, has been altered.

To Center Eyepiece Slide.—A transit telescope which shows objects erect may have an eyepiece slide one end of which moves through an adjustable ring. When the transit has otherwise been adjusted, the cross-hairs may not appear in the center of the field of view owing to the axis of the eyepiece slide not being coincident with the optical axis. This is a convenient relation but is unnecessary as far as the proper working of the transit is concerned. The adjustment is made by loosening and tightening the screws which project from the telescope tube between the eye end of the telescope and the screws controlling the cross-hair ring.

Hints and Precautions.—(1) The adjustments of the transit are more or less dependent one upon another. For this reason, if a transit is badly out of adjustment, time will be saved by first making the corrections roughly for those adjustments which are dependent one upon another until all the tests have been tried, and then repeating the tests and corrections (beginning at the first). The plate bubbles will not be disturbed by

other adjustments and may be exactly corrected before other adjustments are tested. The adjustments of the telescope bubble and vertical circle depend upon the correct position of the line of sight and for this reason should not be tested until the adjustments of the cross-hairs and standards have been tested and found correct. (2) In making the corrections be sure that no screw is left loose or unduly strained. (3) Always have the tripod shoes planted firmly in the ground. (4) *Repeat the test of each adjustment after the correction has been made.* (5) If the transit has an erecting eyepiece which is permanently centered, Adjustment (5) may usually be made with sufficient precision for ordinary spirit or trigonometric leveling by simply moving the horizontal cross-hair until it appears to be in the middle of the field of view. If the transit has an inverting eyepiece, the cross-hair ring will limit the field of view and the cross-hair will appear in the middle whether or not it intersects the axis of the telescope. (6) In place of performing Adjustment (5) as previously described, some instrumentmen prefer to remove the telescope from the standards and to revolve it about *its* axis (not the horizontal axis) in improvised wyes. With the telescope in these wyes and the bubble down, the position of the horizontal cross-hair on some object is noted. The telescope is then carefully revolved about its axis until the buble is over the telescope. If the horizontal cross-hair no longer remains on the point, it is moved, by means of the reticule screws, over a distance estimated to be equal to one-half the displacement, and the test is then repeated. The "wyes" may be V-shaped notches in two thin boards, and they may be kept in position by nailing the boards to the edge of a third board. With some transits it will be necessary to remove the telescope bubble before the telescope can be placed in the wyes. This method presupposes that the geometrical axis through the centers of the two sections of the telescope tube over the improvised wyes coincides with the optical axis, and that this geometrical axis does not change position as the telescope is turned in the wyes. Instruments with coatings of nonconducting material over the metal would be least likely to fulfil these conditions. (7) Remember that the threads of the reticule screws *are in the reticule* and not in the barrel of the telescope; hence the reticule will be drawn toward the screw that is tightened. (8) In making Adjustment (4) it is not sufficient to adjust until the mid-

point on the ground is sighted, as in so doing the plane of sight is moved away from the high point at the same time.

Problem 12. Angles of a Triangle with Transit. Linear Ties

Object.—To determine the value to the nearest half minute of each angle of a triangle, and the angular error of closure, and to witness the corners.

Procedure.—(1) Drive stakes at the vertices of a triangle with sides paced about 300 ft. Tack each stake. (2) Set the transit over one stake, level the plate bubbles by means of the foot-screws; set the horizontal vernier at 0°, and clamp the upper motion. (3) Sight at the flag pole held vertically over one of the other points, by turning the transit on the lower motion. When the vertical cross-hair is nearly on the flag pole, clamp the lower motion and bisect the flag pole by use of the lower motion tangent-screw. (4) Unclamp the upper motion and turn the telescope to the other point. Clamp and bisect the flag pole making the final setting with the tangent-screw controlling the upper motion. (5) Read and record the angle turned off on the horizontal limb. (6) By the same process determine the value of the other angles. (7) Determine the angular error of closure. (8) Reference the corners by two or more linear ties to nearby permanent objects of very definite character.

Hints and Precautions.—(1) To set up the transit, first set it approximately over the point, then adjust the legs to make the tripod head nearly level. Now pick up the instrument bodily, set it carefully over the point, and press each shoe firmly into the ground, at the same time adjusting each leg separately until the plumb-bob falls within ½ in. of the tack and the instrument is approximately level. Finally loosen two adjacent foot-screws, and slide the instrument on the adjusting head until the plumb-bob is over the tack. See that the adjusting head is well centered before beginning to set up. (2) Attach the plumb-bob to the instrument by a sliding bowknot if an adjusting device is not provided. Hard knots should never be tied in the plumb-bob string. (3) The plumb bob should be suspended at such a height as to just clear the tack. (4) Protect the plumb-bob from the wind while completing the "set up." (5) The flagman should

stand directly behind the flag pole, holding it lightly with the fingers of both hands at the height of the chin, and allowing it to balance on the tack head. (6) The flag pole should never be stuck behind the point, in place of being held upon it, if accurate results are desired. (7) Before taking a sight, be sure that the eyepiece is accurately focused on the cross-hairs. The test for parallax is to move the head slightly from side to side while looking through the telescope. The eyepiece slide should be adjusted until the cross-hairs are very distinct and the objective should be moved until there is no apparent motion of the cross-hairs on the image. (8) When sighting, focus the objective carefully upon the flag pole so that it appears very distinct in outline. Do not allow the hand to rest upon the instrument or the clothing to brush against the tripod. (9) The vertical cross-hair should be sighted at the middle of the flag pole (not at the side) and as near to the ground as conditions will permit. (10) If it is necessary to sight at the top of the flag pole, the flagman should be extremely careful to hold it vertical. (11) Do not forget to clamp the motion before using the tangent-screw. (12) Be sure the right tangent-screw is being used (a common source of error with beginners). (13) In reading the angle, be sure to read all of the scale reading before adding the vernier reading (that is do not omit a 30′ space or a 20′ space). When reading the vernier, *have the eye directly over the coinciding graduations.* The reason for this is that, if the eye is not held directly over the coinciding graduation marks and the upper surfaces of the two plates are not in the same plane (as may be the case through wear), the graduations will appear to coincide when in reality they do not. Observe that the graduations on either side of those coinciding, fail to concur by the same amount. (14) The least reading, or fineness of reading of a vernier, is determined by dividing the value of the smallest division on the scale by the number of divisions of the vernier on one side of the index. (15) The plate bubbles should be leveled before measuring an angle, *but the foot-screws should not be disturbed between initial and final settings of the line of sight.* (16) The instrumentman should carefully avoid coming in contact with the tripod legs while he is sighting. (17) Tie lines should intersect at angles between 30° and 150°. If the lines were to intersect at lesser or greater angles, the re-location of the obliterated corner

would become uncertain. (18) Make clear and complete notes. Tabulate measurements on the left-hand page; and in the sketch on the right-hand page show the location of the triangle with reference to important surroundings, such as buildings and roads, and show the exact length of reference ties.

Practical Applications.—The transit is almost universally used by engineers and surveyors for measuring angles on work requiring precision, such as city, railroad, mine, and highway surveys. On such surveys it is important that angles be read quickly and accurately. The vernier makes accurate readings possible, and the principle on which it is based should be understood.

Problem 13. Prolongation of a Line by Double Sighting with Transit

Object.—To produce a straight line with precision, setting stakes at intervals of approximately 300 ft.

Procedure.—(1) Set two points about 300 ft. apart in such position as to afford an open field for a thousand feet or more in advance. (2) Set the instrument on the forward point with the telescope inverted, using the lower horizontal motion, the upper motion being clamped. (3) Plunge the telescope and set a stake on the line a hundred paces in advance. Mark a point on the stake exactly on line. (4) Take a second backsight upon the rear stake in the same manner as before, with the telescope normal. Plunge the telescope again, and mark a point on the advance stake. (5) If this point is not coincident with the first point set, a point midway between them is on the line. (6) Set the transit over this point, and advance by the same process, backsighting upon the next point in the rear. Continue in this way for the desired distance. (7) Check the work by setting the instrument upon the rear point, sighting carefully upon the next point, and then noting the linear error of points set by double sighting, without moving either horizontal motion.

Hints and Precautions.—(1) Be sure that one backsight from each station is taken with the *telescope inverted* and one with the *telescope direct.* (2) Tacks should be set in all stakes, and after being set should be checked. A finely divided scale should be used for bisecting the distances. (3)

Observe all precautions suggested under Problem 12 which apply to this problem. In the field notes tabulate the approximate distances (pacing) between points set by double sighting and also the deviation of these points from a straight line.

Practical Applications.—The method of double sighting is used when it is desired to set a point in advance accurately on line. The process of double sighting eliminates instrumental errors. It is used in prolonging lines of a considerable length, such as railroad tangents. Frequently a line prolonged by simply plunging the telescope with a transit supposed to be in perfect adjustment has later been found to be not a straight line but a curve of large radius.

Problem 14. Intersection of Lines with Transit

Object.—To bisect the three angles of a triangle, and to mark the point of concurrence of the bisectors.

Procedure.—(1) Drive three stakes, *A*, *B*, and *C*, at the vertices of a roughly equilateral triangle, with sides 300 or 400 ft. in length. Tack each stake. (2) Set up the transit and measure the angle at *A*. Lay off one-half of the measured amount, thus establishing the bisector of angle *A*. On the bisecting line of sight and on an estimated bisector of angle *B* drive a stake *o*, and partially drive a tack. Set two more tacked stakes, *m* and *n*, on the bisecting line of sight about 10 ft. from and on opposite sides of *o*. (3) Set the transit over *B* and locate the position of the bisector as at *A*. Drive a stake on this line and under a cord stretched from *m* to *o* or *n* to *o* as the conditions require. Tack the exact point of intersection, *p*. (4) Set the transit on *C*, measure the angle, and bisect as at *A* and *B*. (5) Measure the discrepancy between this bisector and the point of intersection of the first two bisectors at *p* to hundredths of feet. Also measure the angular discrepancy; it should not exceed two minutes. See accompanying sample notes, page 58.

Hints and Precautions.—(1) In setting the tacks on stakes *o*, *m*, and *n*, it is convenient to leave them projecting a little so that the cord may be tied in place. (2) To estimate the bisector of angle *B*, stand on the point and extend the hands toward *A* and *C* respectively. Now, with the eyes closed, bring the hands together. The point of meeting will be approximately on the bisector. (3) Observe all precautions

Prob. 14. Intersection of Lines;

Sta.	Ang.	½ Ang.
A	67°51'	33°55'30"
B	58°24'	29°12'
C	53°44'	26°52'
	179°59'	89°59'30" ck.

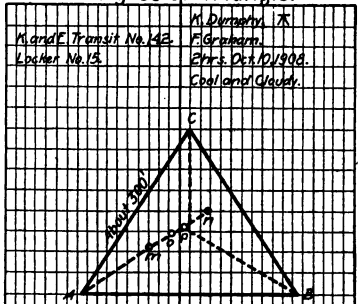
Dist. from p to Bisector from C = 0.02 ft.
Angular Discrepancy = 30"

7.

Angles of Triangle.

M. and F. Transit No. 142.
Locker No. 15.

M. Dunaway, K
F. Graham.
2 hrs. Oct. 10, 1908.
Cool and Cloudy.



Tacks in all stakes.

SAMPLE NOTES 14.

mentioned under Problems 12 and 13 that apply to this problem.

Practical Applications.—The method here used for determining the point of intersection of two lines is used in practice whenever it is necessary to find the meeting point of two lines produced at an angle with each other. For example, to measure the angles between the walls of a building it is necessary to construct lines parallel to each wall and to find their point of intersection over which the transit is to be set to measure the angle. (See Problem 15.)

Problem 15. Measurement of Angle when Transit Can Not be Set at Vertex

Object.—To measure the angle between two walls of a building.

Procedure.—(1) Drive and tack stake *a* (Fig. 12) at a convenient distance from the wall. Determine the perpendicular distance from the wall by holding a point of the tape

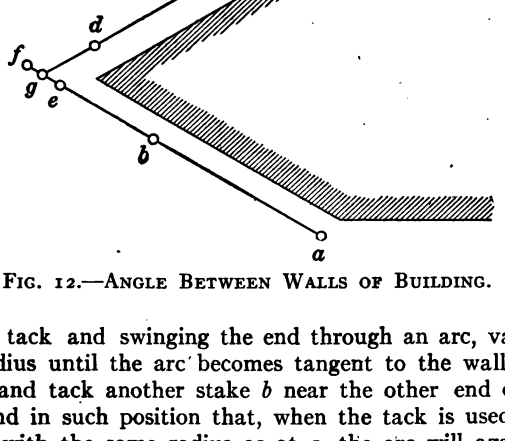


Fig. 12.—Angle Between Walls of Building.

on the tack and swinging the end through an arc, varying the radius until the arc becomes tangent to the wall. (2) Drive and tack another stake *b* near the other end of the wall and in such position that, when the tack is used as a center with the same radius as at *a*, the arc will again be tangent to the wall. The line *ab* will be parallel to the wall.

(3) Set stakes *c* and *d* in the same manner, making *cd* parallel to the other wall. (4) Set the transit at *a* and sight on *b*, then set and tack two stakes, *e* and *f*, on the line and on opposite sides of line *cd* prolonged. Stretch a cord between the points marked on *e* and *f*. (5) Set the transit on *c*, sight at *d*, and set point *g* on line and under the cord. (6) Move the transit to the intersection of the two lines, and measure the angle *agc*, which is equal to the angle between the walls.

Hints and Precautions.—(1) Some well-defined straight line of the wall should be selected as the working line. (2) Set the transit at *a* and *c* to prolong the lines to *g* instead of at *d* and *b*, to avoid any error that might result from plunging the telescope.

Practical Applications.—This problem is of frequent occurrence in structural work. It is necessary to know accurately the angle between two walls wholly or partly in place in order to detail the structural fabrics.

Problem 16. Prolongation of Line Past Obstacle (Offset Space Limited)

Object.—To prolong a line, determined by two fixed points, past an obstacle when the conditions are such as to limit the lengths of the offsets.

Procedure.—(1) Let *A* and *B* (Fig. 13) be two points on the same side of an obstacle. Set up the transit at *A*, sight on

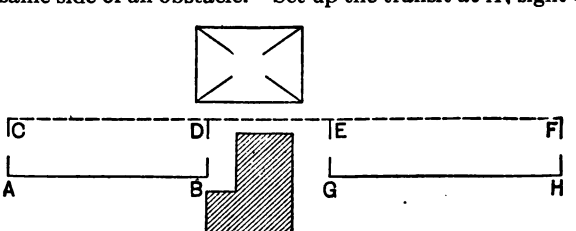


Fig. 13.

B, and turn off a right angle. Set and tack a stake at *C* on this perpendicular at a convenient distance from *A*, say 20 ft. (2) Set the transit at *B*, turn off 90° as before, and set another point *D* at the same distance from *AB*. (3) Set the transit on *D*, backsight on *C*, and by method of double sight-

ing (Problem 13) set two stakes E and F in advance, one just beyond the obstacle and one at a considerable distance in advance. (4) From E and F offset to points G and H, using the same offset distance as for setting C and D. G and H are on the line AB prolonged, and from these points the line may be continued at will. (5) If the obstacle is imaginary, set the instrument on A and locate G and H directly for a check. Measure and record the errors at G and H.

Hints and Precautions.—(1) The lengths of offsets should be measured very carefully. (2) If the instrument is in good adjustment the points E and F may be set by a single reversal, or if a clear sight can be obtained the points may be set without reversal with the transit at C.

Practical Applications.—This problem is of very frequent occurrence in land and highway surveys. It is necessary within confined limits to offset in order to prolong a line which is obstructed by fences, hedges, brush, etc. The shortness of the offset distance necessitates taking two offsets and measuring them very carefully, because turning a right angle from the short backsight DB will not give an accurate long foresight. A short backsight should never be depended upon to give an accurate foresight.

Problem 17. Determination of an Inaccessible Distance

Object.—To obtain the distance between two points on opposite sides of a river.

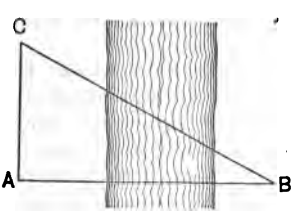


Fig. 14.

Procedure. *First Method.*—(1) Set the transit on A (Fig. 14), sight on B. Turn off an angle of 90°, and set C at a distance nearly as great as AB. (2) Measure angle ACB and length AC. Then $AB = AC \tan ACB$.

Second Method.—(1) Set the transit on A (Fig. 15), sight on B. Turn off 90°, set C at a distance estimated to be nearly equal to AB, and measure AC. (2) Set transit at C, sight on B, and turn off 90° to set D which must also be on line AB.

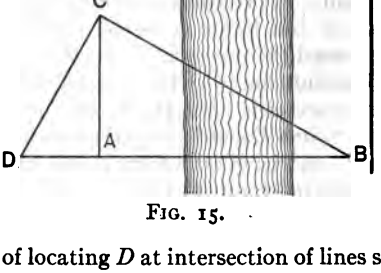


FIG. 15.

(For method of locating D at intersection of lines see Problems 14 and 15.) (3) Chain the distance AD. (4) Triangles ABC and ACD are similar. Hence $AB = \frac{\overline{AC}^2}{AD}$. If the river is imaginary, measure the distance AB for a check.

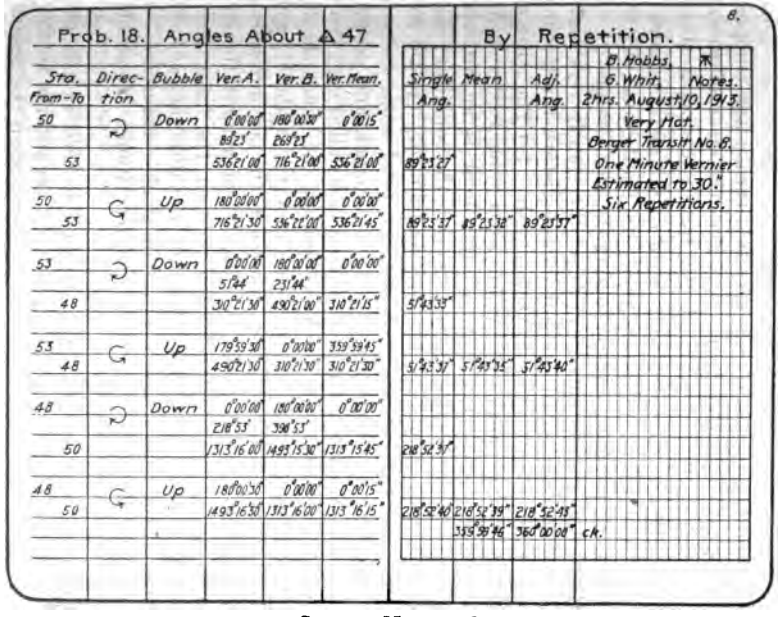
Hints and Precautions.—(1) The distance AC in each case should equal at least one-half of AB. (2) If good results are expected, distances must be carefully measured.

Practical Applications.—This problem is of use in measuring the distance between any two intervisible points where the distance can not be directly chained. The second method permits a solution when trigonometric tables are not at hand.

PROBLEM 18. ANGLES BY REPETITION

Object.—To obtain a more accurate determination of the angles between various stations about a point than would be possible by a single measurement. See Sample Notes 18.

Procedure.—(1) Set the transit very carefully over the point. (2) Set the A vernier at zero, read the B vernier, and record the readings. (3) With the telescope in its normal position, measure one of the angles in a clockwise direction, and record both vernier readings to the smallest reading of the vernier. (4) Leaving the upper motion clamped, again set on the first point and again measure the angle in a clockwise direction (thus doubling the angle). (5) Continue until six repetitions have been secured. Record both vernier read-



Prob. 18. Angles About Δ 47

By Repetition.

8.

Sta. From–To	Direction	Bubble	Ver. A.	Ver. B.	Ver. Mean.	Single Ang.	Mean	Adj. Ang.
50		Down	0°00'00"	180°00'30"	0°00'15"			
			89°23'	269°23'				
53			536°21'00"	716°21'00"	536°21'00"	89°23'27"		
50		Up	180°00'00"	0°00'00"	0°00'00"			
53			716°21'30"	536°22'00"	536°21'45"	89°23'37"	89°23'38"	89°23'37"
53		Down	0°00'00"	180°00'00"	0°00'00"			
			51'44"	231'44"				
48			310°21'30"	490°21'00"	310°21'15"	51°43'38"		
53		Up	179°39'30"	0°00'00"	359°39'45"			
48			490°21'30"	310°21'30"	310°21'30"	51°43'37"	51°43'35"	51°43'40"
48		Down	0°00'00"	180°00'00"	0°00'00"			
			218°53'	398°53'				
50			1313°16'00"	1493°15'30"	1313°15'45"	218°52'51"		
48		Up	180°00'30"	0°00'00"	0°00'15"			
50			1493°16'30"	1313°16'00"	1313°16'15"	218°52'40"	218°52'39"	218°52'43"
							359°39'46"	360°00'00" ck.

D. Hobbs, K. G. Whit. Notes. 2 hrs. August 10, 1915. Very Hot. Berger Transit No. 6. One Minute Vernier Estimated to 30". Six Repetitions.

SAMPLE NOTES 18.

ings and the total angle turned. (6) In a like manner, setting the *B* vernier at zero, measure the explement of the angle in a *counter-clockwise* direction with the *bubble down*, but read the horizontal circle as though the *angle itself* had been measured clockwise. (7) Go through the same process for all other angles about the point. (8) Compute the value of each of the angles for each direction turned and compare with the single measurement. (9) Find the mean of each of these sets of single angles. For a transit reading to single minutes the *total error should not exceed* $10''\sqrt{\text{number of angles}}$. (10) Adjust the angles so that their sum shall equal 360° by distributing the error equally among the mean values.

Hints and Precautions.—(1) Level the transit very carefully before each repetition, but do not disturb the leveling screws while a measurement is being made. (2) The mean of each set of single angles should furnish a value free from instrumental errors. The station adjustment is an attempt to distribute the accidental errors so that the condition, that there are 360° about a point, shall be fulfilled. (3) Do not become confused when calculating the total angle turned. Observe how the horizontal limb is graduated, and do not omit 360°. (4) The instrument should be handled very carefully. When turning on the lower motion, the hands should be in contact with the *lower* plate (not the alidade), and when making an exact setting on a point, the last movement of the tangent-screw should be *clockwise* or against the opposing spring. (5) After each repetition the instrument should be turned on its lower motion in a direction *opposite* to that of the measurement. (6) The single measurement is taken as a check on the number of repetitions. It should agree closely with the mean value.

Practical Applications.—This method is used in triangulation work. The number of sets of readings and the number of repetitions in each set observed depend upon the desired accuracy.

Problem 19. Laying off Angle by Repetition

Object.—To lay off a given horizontal angle more accurately than is possible with a single setting of the vernier.

Procedure.—(1) Drive and tack two stakes about 500 ft. apart. (2) Set the transit carefully over one end of the line.

Sight at the point at the other end, and lay off the angle. (3) Set a stake on the line of sight about 500 ft. from the instrument (distance by pacing), and carefully set a tack. (4) By repetition measure the angle laid off, as in the previous problem, making six repetitions in each direction. (5) Find the difference between the angle laid off and the required angle, and by trigonometry compute the linear distance that the tack must be moved perpendicular to the line of sight. (6) Set the tack accordingly.

Hints and Precautions.—(1) The angle laid off must be increased or decreased by the difference between it and the required angle. This difference is too small to lay off with the transit, but the length of its arc can be computed. For very small angles the length of this arc is equal to the radius times the sine of the angle of error.

Practical Applications.—This method is of use in laying out large buildings or valuable city lots. With a 01′ transit an error of 30″ in a single reading might easily occur; in 300 ft. this would amount to approximately ½ in.

Problem 20. Deflection-angle Traverse of Field with Transit

Object.—To determine by the deflection-angle method the necessary data for plotting traverse and obtaining the area inclosed. (The data may be used in Problems 73, 84, 85, and 86, pages 183, 206, 211, and 213.)

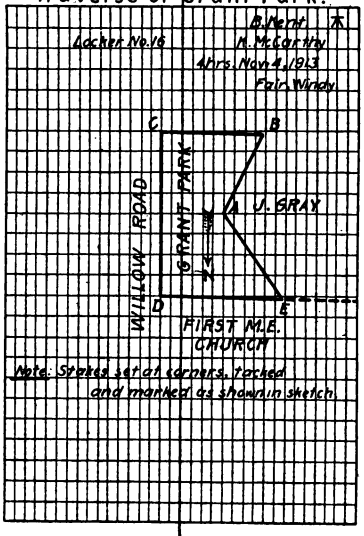
Procedure.—(1) Stake out an irregular field containing an area of an acre or more. (2) Measure the sides carefully with the steel tape and chaining pins (see Problems 2 and 3), recording the distances to the nearest 0.01 ft. (3) Set the transit over one station and with the *A* vernier set at 180°, sight upon the preceding station. (4) Read and record the magnetic bearing of this backsight. (5) Turn the transit in azimuth on the upper motion to sight the next forward point. The vernier reading will now indicate the amount of the deflection angle. Observe whether the deflection is right or left from the prolongation of the preceding line, and record *R* or *L* accordingly, after the value of the angle in the deflection-angle column of the notes. Also read and record the magnetic bearing of the forward line. (6) Before the transit is moved, calculate the

Prob. 20 Deflection Angle

Sigas, N.B.

Sta. At—To	Dist.	Def. Ang.	Mag. Bear.	Ang. of Bear.	Cal. Bear.
A E		57°54'R	N 28°W	58° R	
B	507.65		S 30° W		S 29°37'W
B A		113°36'L	N 29½°E	113½° L	
C	784.68		S 84° E		S 84°00'E
C B		98°15'L	N 84°W	98½° L	
D	994.60		N 2½°W		N 2°15'W
D C		88°19'L	S 2° E	88½° L	
E	739.72		S 89½°W		S 89°26'W
E D		117°43'L	N 69½°E		
A	526.00		S 20½°E	117½° L	S 26°17'E
		359°59'			

Traverse of Grant Park. 9.



SAMPLE NOTES 20.

deflection angle from the magnetic bearings taken at the station, and compare with the deflection angle indicated on the horizontal circle, as a rough check upon the amount and deflection of the angle. (7) Repeat the process at each station until the fieldwork is completed. (8) Add the deflection angles, assuming those to the right as positive and those to the left as negative. The algebraic sum should be 360° for any closed traverse. The *angular error of closure should not exceed* $30'' \sqrt{Number\ of\ Sides}$. (9) Select and assume as correct the magnetic bearing of some course whose back and forward bearings have the same angular value, and from this and the successive deflection angles, calculate the bearing of each of the other courses. See Sample Notes 20.

Hints and Precautions.—(1) Do not forget to read the magnetic bearing both forward and back from each station. Angles computed from these bearings, read from the same station, will contain no error due to local attraction (unless caused by passing disturbance), as the needle has the same declination no matter in which direction the transit is sighted. (2) When reading the deflection angle on the horizontal circle, be sure to read from the circle which is graduated from 0° in the direction of the deflection. (3) If the instrument is in good adjustment, the *A* vernier may be set on 0° and the backsight taken with the telescope in the reversed position (bubble up). When the telescope is again plunged, the line of sight will lie on the line of backsight prolonged, and the vernier will be in a position to be read without the observer moving about the transit. (4) Hubs should be driven well into the ground so that they will not be easily moved. (5) A witness stake is usually driven at one side and leaning over each hub. The stake should be marked with the station number on the side next to the hub. Points likely to be disturbed before they are needed should be tied to nearby permanent objects.

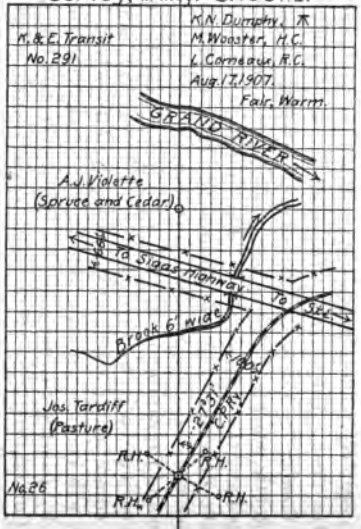
Practical Applications.—The deflection-angle method is one of the two most common methods of transit traversing either for closed or continuous traverses. It is particularly well adapted to railroad location and similar work. On such surveys stations are established every hundred feet and numbered consecutively from the beginning of the line, "Sta. 103" indicating that the point is 10,300 ft. from the point of beginning. Intermediate stations are indicated by plusses.

Prob. 20a Preliminary

Def. Ang. Traverse.

Sta.	Def. Ang.	Mag. B.	Cal. B.	Remarks.
12				
+82				Water Line
+17				" "
11				
10				
9				
08+36.3	23°14'L	N 35° W	N34°41'W	
8				
+99.8				
+68				& Due E to St.L.
+33.1				
7				
6+00				Ctr. of Brook.
5				
4				
3				
2				
+23.6				
1				
00+00		N 11° W	N11°33'W	on C.L. C.P.Ry.
				523'N. M'l. Post-

Survey, I.N.Ry. C. to St.L. 10.



SAMPLE NOTES 20(*a*).

Thus, "Sta. 103 + 57" represents a point 10,357 ft. from the point of beginning. Such notes are usually begun at the bottom of the page instead of the top, and the center line of the right-hand page represents the transit line. Sample Notes 20 (*a*) show a portion of the survey of this kind.

Problem 21. Azimuth Traverse with Transit

Object.—To collect data sufficient for plotting and determining the area of a field by the azimuth method. (The data may be used in Problems 73, 84, 85, and 86, pages 183, 206, 211 and 213.) See Sample Notes 21, page 70.

Procedure.—(1) Stake out the field and measure the sides as in Problem 20. (2) Set the transit over one station. Set the *A* vernier* at 0° and turn on the lower motion to sight along the meridian either north or south according as azimuths are to be reckoned from north or south. Clamp in this position. (3) Unclamp the upper motion, and turn the instrument to sight the next point forward. The angle turned off in a clockwise direction and read from the *A* vernier is the azimuth of the forward line. (4) Read and record the magnetic bearing. (5) From the azimuth, calculate the bearing of the line, and check with the magnetic bearing. Compute the back azimuth of the line by adding 180° to the forward azimuth. (6) Set the instrument on the next forward point with the vernier set on the back azimuth of the line, and backsight on the point previously occupied. Now turn upon the upper motion to sight the next point in advance; the azimuth of the forward line will be indicated by vernier *A*. (7) Proceed in this manner until all of the stations have been occupied. (8) Note the angular error of closure, which should not exceed $30'' \sqrt{Number\ of\ Sides}$.

Hints and Precautions.—(1) Since the traverse forms a closed field, a check will be obtained if the back azimuth is read from the first point occupied. This should be 180° more or less than the forward azimuth of the same line obtained from the last station occupied. (2) Each time an azimuth is determined, its calculated bearing should be com-

* Either vernier may be used, but with the telescope in the direct position (bubble down) it will be more convenient with most transits to read the *A* vernier.

Prob. 21. Azimuth Traverse,

Proposed Mill Pond, El. 741.36

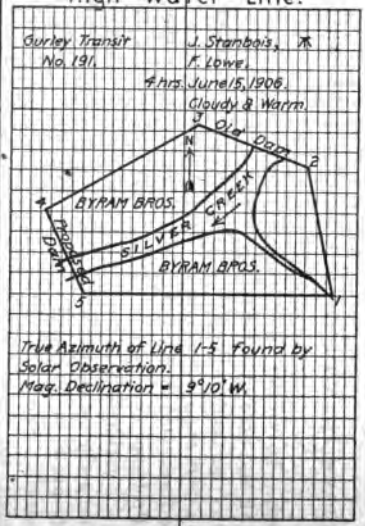
Silver Creek, Penn.

(for Land Damage Est.)

Sta.	Obj.	Dist.	Azimuth	Mag. B.	Cal. Bear.
1	5		270°28'	N80¼°W	N89°32'W
	2	689.32	350°30'	N	N 9°30'W
2	1		170°30'	S	S 9°30'E
	3	509.66	303°05'	N48°W	N56°55'W
3	2		123°05'	S48¼°E	S56°55'E
	4	678.68	236°13'	S65¼°W	S56°13'W
4	3		56°13'	N65¼°E	N56°13'E
	5	572.50	177°58'	S 7°W	S 2°02'E
5	4		357°58'	N 7¼°E	N 2°02'W
	1	1082.71	90°29'	S 80°E	S89°31'E
			Error = 01'		

12.

High Water Line.



True Azimuth of Line 1-5 found by Solar Observation.

Mag. Declination = 9°10' W.

SAMPLE NOTES 21.

pared with the magnetic bearing of the course. (3) If the azimuth of any line of the traverse is known, the transit may be set over one end of this line and the vernier at once set to read the proper azimuth. The transit will be oriented when its line of sight lies along the line, the azimuth of which is indicated by the vernier setting. The azimuth of the initial line may be arbitrarily assumed, but for facility in checking, the azimuth should be so chosen that the calculated and magnetic bearings will agree very closely. (4) The transitman may, instead of increasing the horizontal reading by 180° for each backsight, simply use the other vernier which, if there has been no slip between the plates in carrying the transit from one station to the next, will record the back azimuth of the line. This will necessitate plunging the telescope for each backsight. If the upper motion is left clamped after each foresight until the corresponding backsight has been taken in the manner just described and both foresight and backsight from any point are taken with the telescope in the same position (bubble up or down) and with the same vernier, neither compensating error due to imperfect vernier settings nor cumulative error due to the line of sight not being perpendicular to the horizontal axis will be introduced. Thus, this method has some advantage over the one described in the main part of the problem. If this method is used, the transitman should always read the vernier before a backsight is taken to make sure that no slip has occurred. As an insurance against using the wrong vernier, or having the telescope in the wrong position, the note keeper may mark the station numbers in the notes alternately A and B, or he may note that at odd numbered stations azimuths are read from vernier A with the bubble down and at even numbered stations from vernier B with the bubble up.

Practical Applications.—Perhaps more traverse lines are run by the azimuth method than by any other. Its advantage is that the direction of any line is indicated by the simple statement of one angle. This fact makes the method especially convenient for topographic surveying where many angles are taken about one point. The bearing of a line can be calculated at a glance, and thus a check between the magnetic and calculated bearings can be obtained much more readily than when deflection angles are used.

Problem 22. Altitude with Transit

Object.—To determine the height of a building above the water-table.

Method 1. Procedure.—(1) Set the instrument at *A* (Fig. 16), a distance from the building about equal to double its height. Level the telescope, and note the position of the horizontal hair on building; mark the point sighted, and measure the distance h_1 above or below the water-table. (2) Determine the index error of the vertical circle. Sight on the

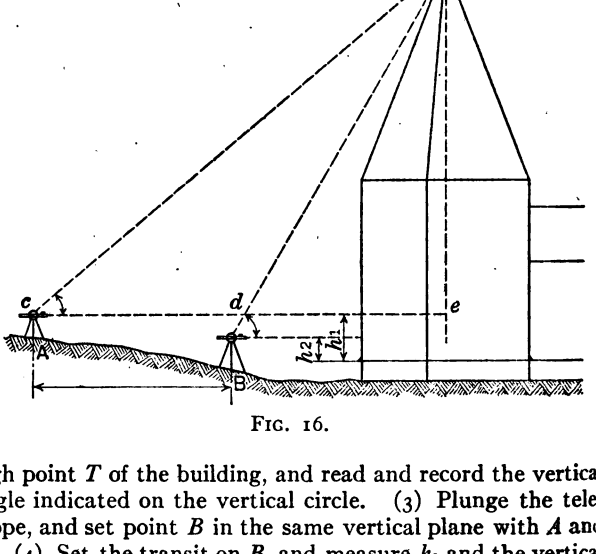


Fig. 16.

high point *T* of the building, and read and record the vertical angle indicated on the vertical circle. (3) Plunge the telescope, and set point *B* in the same vertical plane with *A* and *T*. (4) Set the transit on *B*, and measure h_2 and the vertical angle to point *T*, as at *A*. (5) Measure the horizontal distance *AB*. (6) Calculate the difference in elevation between *T* and the water-table by solving for the lengths of the lines *cd*, *dT*, and *Te*.

Method 2. Procedure.—(1) Set the transit at *A* (Fig. 17) in a convenient position and at a distance from the building roughly equal to its height. (2) Note the index error of the vertical circle. Set the horizontal vernier *A* at 0°, sight at

the high point T, and read the vertical angle. (3) Depress the telescope to a horizontal position, mark the point a, and measure the distance h above or below the water-table. (4) Now turn off 90° on the horizontal circle, and set point B on line 100 ft. from A. (5) Set the transit at B, and with horizontal vernier at 0°, sight at T. Turn on the upper hori-

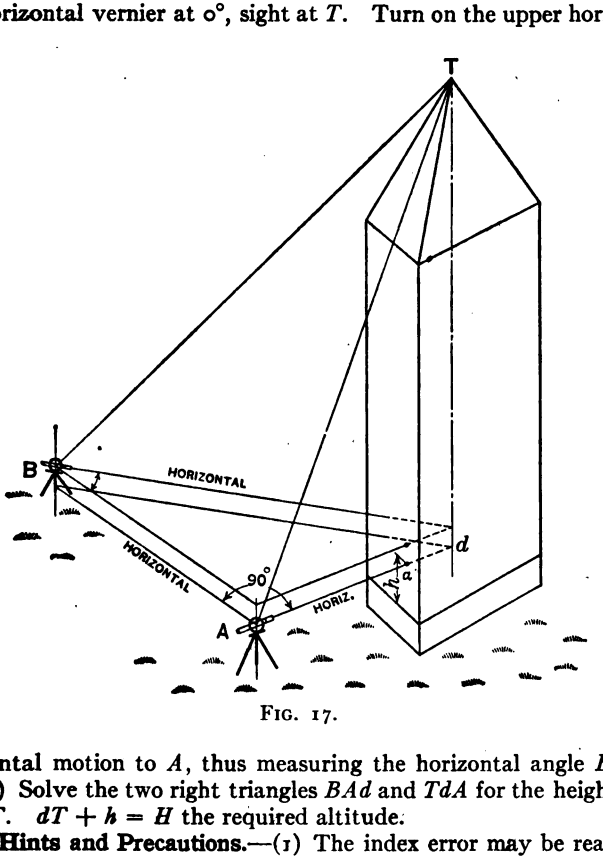


FIG. 17.

zontal motion to A, thus measuring the horizontal angle B. (6) Solve the two right triangles BAd and TdA for the height dT. $dT + h = H$ the required altitude.

Hints and Precautions.—(1) The index error may be read either before or after observing a vertical angle, but should be determined with the *line of sight in the same vertical plane as the point to which the vertical angle is taken, i.e.*, with the same horizontal vernier reading. (2) In case the transit used is provided with a movable vertical vernier controlled

by a tangent-screw, the index error should be eliminated by making the vernier read zero when the telescope is level.

Practical Applications.—Although a problem of this kind is not of frequent occurrence in the work of the engineer, it is sometimes desirable to obtain altitudes which can not be measured directly, as in a topographic survey of mountainous country.

Problem 23. Details with Transit and Tape

Object.—To obtain sufficient data for plotting a detailed map of a portion of the campus. (The data may be used in Problem 88, page 225.)

Procedure.—(1) Run a closed traverse by the azimuth method (Problem 21) through the area to be mapped, so locating the lines and corners of the traverse that linear and angular measurements necessary to fix the position of details with respect to the traverse lines may be taken with the least labor. (2) Obtain the location of buildings, streets, walks, hydrants, fences, etc., by the several methods for locating such details from a traverse line. (3) Sketch indefinite details such as trees, streams, shrubbery, etc., without measurements other than by pacing.

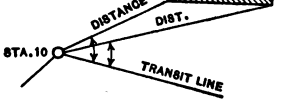


Fig. 18.

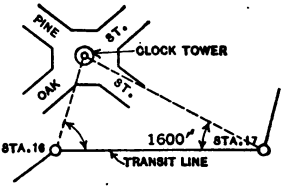


Fig. 19.

Hints and Precautions.—(1) Details may be taken as the traverse is run. (2) Note the angular error of closure of the traverse. If this exceeds $30''\sqrt{Number\ of\ Sides}$ re-run the traverse until the mistake is discovered. (3) Reference the hubs so that if discrepancies are later discovered the points may again be occupied by the transit. (4) Details may be located with the transit by an azimuth and distance from one point (Fig. 18), or by azimuths from two points (Fig. 19). Details may be located with the tape by distances from two

transit points (Fig. 20), or by perpendicular offsets from the transit line and distances along the line (Fig. 21). In addition to the foregoing, buildings may be readily located when not too far from the transit line by range ties, swing offsets, or simple ties (Fig. 22). No matter how many points of an

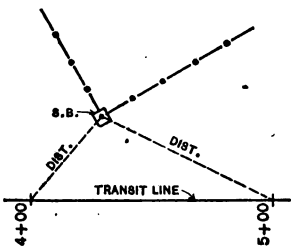


FIG. 20.

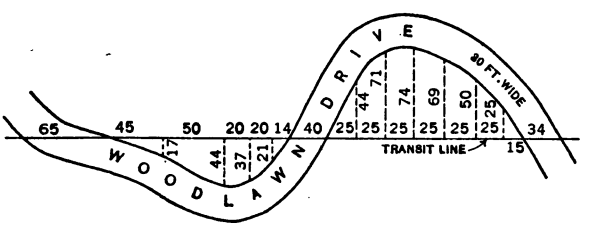


FIG. 21.

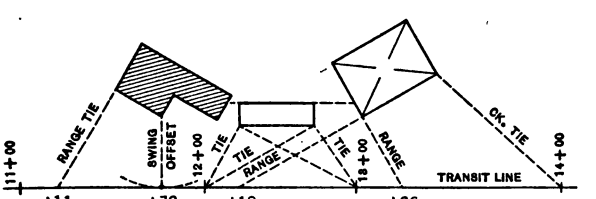


FIG. 22.

object have been located, its dimensions should be obtained by direct measurement where such is possible. This applies particularly to the length of land lines, streets, and sides of buildings. The location of important details should be checked, preferably by some other method than that used in

the first location. No set rules can be laid down as to how details shall be located, for conditions are too varied; but the surveyor should endeavor to choose that method which will give the required accuracy with the least time and labor. (5) Sketches should be made approximately to scale, and should not be so crowded with measurements as to become confusing. Many measurements can be tabulated on the left-hand page and referred to on the sketch by a single letter or number.

Practical Applications.—The methods of this problem are those in general use where a considerable degree of precision is required and the details are definite in character, as in city surveying or surveys for large-scale maps.

CHAPTER IV

LEVEL PROBLEMS

PROBLEM 24. ADJUSTMENTS OF THE WYE LEVEL*

(1) To Make the Horizontal Cross-hair Lie in a Plane Perpendicular to the Vertical Axis.

Test.—Sight at a clearly defined point about 150 ft. distant, and turn the instrument slowly about its vertical axis so that the point will appear to move from one end of the horizontal cross-hair to the other. If the point moves continually on the hair, it lies in a plane perpendicular to the vertical axis.

Correction.—If the point fails to follow the hair, loosen two adjacent reticule screws slightly, and turn the reticule in the barrel of the telescope until the point appears to traverse the entire length of the hair when the instrument is revolved. Tighten the *same* screws.

(2) To Make the Line of Sight Coincide with the Axis of the Wyes.

Test.—Unclamp the wyes, and sight the intersection of the cross-hairs upon a well-defined point 150 to 200 ft. from the instrument. Clamp the vertical axis. Rotate the telescope 180° in the wyes (bubble up). If in adjustment, the intersection of the cross-hairs will remain unchanged.

Correction.—If not, adjust the reticule until the intersection takes a position *midway* between the original point and the point sighted in the rotated position. Loosen only one screw at a time and tighten the opposite screw.

(3) To Make the Axis of the Bubble-tube Lie in the Same Plane with the Axis of the Wyes.

Test.—Bring the bubble to the center of the tube by means of the foot-screws. Rotate the telescope in the wyes through a small angle (perhaps 10°). If in adjustment, the bubble will remain in the center of the tube.

* In describing the adjustments of the wye level it is assumed that the optical axis coincides with the axis of the wyes and that the objective slide is permanently fixed as far as lateral motion is concerned. For level with adjustable objective slide see page 78. Also see Note, page 47.

Correction.—If the bubble moves, bring it back to the center of the tube by means of the azimuth screws.

(4) To Make the Axis of the Bubble-tube Parallel to the Axis of the Wyes.

Test.—Level the instrument, and clamp the telescope over one pair of foot-screws. Loosen the wye clips, and very carefully turn the telescope end for end in the wyes. If the bubble is in adjustment, its position will remain unchanged.

Correction.—If the bubble moves from the center of the tube, bring it *halfway* back with the altitude screws. Level up the instrument, and repeat the test to insure perfect adjustment.

(5) To Make the Axis of the Bubble-tube Perpendicular to the Vertical Axis.

Test.—Level the instrument, as well as its condition will permit, over both pairs of foot-screws; then bring the bubble carefully to the center over one pair. Revolve the level 180° about its vertical axis. If in adjustment, the bubble will retain its position.

Correction.—If the bubble moves away from the center of the tube, bring it back *one-half* of the displacement with the wye nuts. Level up, and repeat the test to insure perfect adjustment.

Level with Adjustable Objective Slide.—(See Transit with Adjustable Objective Slide, page 51). The adjustments of a level having an adjustable objective slide are the same as those just described except that after Adjustment (2) for a distant point, a near point is sighted and the telescope revolved in the wyes as in Adjustment (2). If the intersection of the cross-hairs moves off the point the adjustable objective ring is moved by its controlling screws until the cross-hairs appear to traverse over one-half the apparent error. The relation between this adjustment and the cross-hair adjustment is such that each must be repeated until both are found to need no further correction.

To Center Eyepiece Slide.—See Corresponding Adjustment of the Transit, page 52.

Hints and Precautions.—(1) The test of every adjustment should be repeated after the correction has been attempted, in order to insure perfection of adjustment. (2) Adjustment (4) fails if the telescope rings are worn or are not of equal size. In this case the wye level must be tested as a dumpy level

PLATE V.—ENGINEERS' WYE LEVEL (ERECTING EYEPIECE).
(From a photograph furnished by C. L. Berger & Sons.) (*Facing page* 78.)

and adjusted as such, by the peg adjustment. (See Adjustments of the Dumpy Level, Problem 25.) (3) The adjusting screws should be brought to a snug bearing, but care should be taken not to strain the metal. (4) The azimuth and altitude adjustments of the bubble depend on each other. For this reason each should be repeated after the other has been made, until no further adjustment of either is necessary.

Problem 25. Adjustments of the Dumpy Level

(1) To Make the Horizontal Cross-hair Lie in a Plane Perpendicular to the Vertical Axis.

The method of procedure is identical with that of Adjustment (1) of the wye level.

(2) To Make the Axis of the Bubble-tube Perpendicular to the Vertical Axis.

The method of procedure is identical with that of Adjustment (5) of the wye level, except that the correction is made with the altitude screws at the end of the bubble-tube instead of the wye-nuts.

(3) To Make the Line of Sight Parallel to the Axis of the Bubble-tube (Two-peg Method).

Test.—Set two stakes on approximately level ground about 300 ft. apart. Set the instrument in such position that the

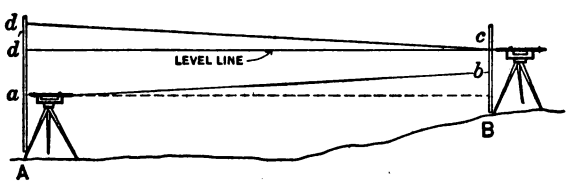


Fig. 23.—The Two-peg Test.

eyepiece is ½ in. or less in front of the rod held on one stake, as at *A* (Fig. 23). Take a rod reading *a* on *A*, sighting through the objective end of the telescope and centering the small circle of rod seen through the eyepiece. Take a rod reading *b* on *B*. Move the instrument to *B*, set up as before, and take rod readings *c* and *d*. The correct rod reading at *A* for a level line with the instrument in its last position at *B* will be

$d' = c + \frac{(a-b)+(d-c)}{2}$.* If the line of sight is in adjustment, d will equal d'. The mean of the two differences in elevation found from the rod readings taken from each position of the instrument will represent the true difference in elevation, since the line of sight will be inclined as much in one position of the instrument as in the other.

Correction.—If not in adjustment, move the cross-hair ring vertically until the line of sight cuts the rod at d', which is the rod reading for a level line.

Hints and Precautions.—(1) The screws controlling lateral movement of the reticule should not be moved during the peg test. After Adjustment (3) has been completed, the test for Adjustment (1) should be repeated. (2) Each test should be repeated after the correction has been attempted in order to insure perfect adjustment. (3) Undue strain should not be brought upon the adjusting screws. (4) If a tape is at hand or if the instrument is equipped with stadia-hairs, the true difference in elevation between the points A and B may be found by setting the instrument at the mid-point of the line AB. With the level in this position the error in rod reading, due to line of sight not being horizontal, will be the same at A as at B. The difference in elevation will, then, be the difference between rod readings. The correction is made by setting up the level at one peg and bringing the line of sight to the rod reading for a level line on the other peg as previously explained. (5) Readings should be observed to thousandths of feet.

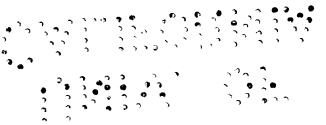
Problem 26. Differential Leveling with Engineers' Level and Self-reading Rod

Object.—To determine the elevations of points in an assigned level circuit. See Sample Notes 26, page 83.

Procedure.—(1) Set up the level at a convenient distance from a *bench-mark* whose elevation is known or assumed, and along the general direction of the course. The distance from the initial bench-mark to the instrument should not

* For exact adjustment this rod reading should be corrected by adding the effect of curvature and refraction. This correction in feet is approximately 0.02 M^2 in which M is the distance from instrument to rod in units of 1000 ft. For a 300-ft. sight the correction amounts to a little more than 0.002 ft., a negligible quantity in ordinary differential leveling but one which must be considered in the adjustment of precise levels.

PLATE VI.—ENGINEERS' DUMPY LEVEL (INVERTING EYEPIECE).
(From a photograph furnished by C. L. Berger & Sons.) (*Facing page* 80.)



be greater than 300 ft., and the position of the level should be such that the line of sight will not only strike the rod when held at the initial point but will also be favorable for a sight to a *turning-point* farther along the line. (2) With bubble centered read the rod (held on the initial bench-mark by the rodman) to 0.01 ft. and estimate to 0.001 ft. (3) Record the reading in the *backsight* (*B.S.*) column of the notes in line with the number and description of the bench-mark. (4) Center the bubble, and again read the rod as a check. (5) If the level is equipped with stadia-hairs, observe and record the intercept on the rod; if not, record the distance from bench-mark to level as paced by the rodman as he goes forward. (6) Calculate the *height of instrument* by *adding* the observed backsight to the elevation of the point sighted, and record in the *H.I.* column of the notes. (7) Take a *foresight* to a solid point (turning-point) chosen by the rodman, a convenient distance in advance of the instrument. Observe the same precautions as when reading the rod at the initial point. (8) The turning-point is designated in the station column of the notes by the initials *T.P.* On a new line record the number of the turning-point, its description if in the future it might be used as a bench-mark, the foresight reading, the elevation of the turning-point determined by subtracting the foresight reading from the height of instrument, and the distance from level to turning-point. (9) Move the instrument forward, and take a backsight in the same manner as at the first point occupied. Record the rod-reading and distance in their columns *in the same line with other data concerning the turning-point on which sight is taken.* (10) Continue in the same manner around the circuit, except at bench-marks where total lengths of backsights and foresights should be balanced.

Hints and Precautions.—(1) Select solid ground for instrument stations. Be sure to plant the tripod feet firmly in the soil. Spread the tripod well, especially if the wind is strong. (2) Remember, in leveling up, that the bubble will travel in the same direction as the left thumb. (3) Do not try to get the bubble in the center over one pair of leveling screws before bringing it approximately to the center over the other pair. It is a waste of time to center the bubble exactly before the rod has been sighted. (4) If one pair of leveling screws turn hard, loosen the other pair a little. Finish level-

ing up with all foot-screws bearing solidly but not so tightly as to spring the plate. (5) Test for parallax as follows: Slightly move the head from side to side while looking through the telescope with the instrument focused on the rod. If parallax exists, the cross-hairs will appear to move on the face of the rod. To remove this condition, first hold a white sheet of paper 2 or 3 in. in front of the instrument and focus the eyepiece until the cross-hairs appear sharp and distinct, then re-focus the objective carefully upon the rod. Once adjusted the focus of the eyepiece should remain perfect, but the objective focus must be changed for each different distance. (6) Look at the bubble just before taking each reading, and glance at it again just after, to be sure that it has not moved. The observer should stand in such a position that this will be possible without moving his feet. (7) Be sure the rod is held vertical while a sight is being taken; keep the foot of the rod free from dirt. (8) When the Philadelphia rod or similar rod is extended (long rod), it should be firmly clamped in the proper position; and between rod readings the rodman should examine it to see that no slip has occurred. Do not let the "long rod" down "on the run." (9) Slowly *wave the rod* toward and from the instrument when the "long rod" is used, and take the least reading on the rod. (10) Remember that the reading is recorded opposite the station number of the station on which the rod is held and has nothing to do with the instrument station. (11) Backsights are always taken upon points of *known* elevation; foresights, on points of *unknown* elevation. (12) Check all calculations by showing that the difference between the sum of the backsights and the sum of the foresights equals the difference in elevation between the initial and terminal stations. (13) Give a clear and concise description of each bench-mark and its location, on the right-hand page in line with numerical notes concerning that bench-mark. (14) Make the sum of backsight distances between bench-marks equal the sum of foresight distances as nearly as possible, to eliminate the effect of lack of adjustment of the instrument and curvature of the earth. (15) There should be a definite, well-understood system of signals between the instrumentman and rodman. (See Art. 14, page 11.) (16) The rodman should choose the position of the turning-point with an eye to simplicity of field operations. The turning-point may be an iron plug

13.

Prob. 26. Differential Levels.

Eng. Hall to Ag. Bldg & Return.

Self-Reading Rod.

Berger, Dumpy. J.M. Pratt, Jr.
No. 14, Locker No. 30. C.P. Kidd, Rod
2 hrs. Oct. 21, 1912.
Cold & Cloudy.

Sta.	B.S.	H.I.	F.S.	Elev.	Dist. B.S.	Dist. F.S.	
B.M. 1	6.102	419.212		413.11	97		U.S.G.S. B.M. Cor. Eng. Hall.
T.P. 1	9.842	425.858	3.196	416.016	96	108	+11 Top Hydrant Front S. N. House.
T.P. 2	11.276	432.950	4.184	421.674	85	100	+18 Plug.
T.P. 3	3.616	435.654	0.912	432.038	104	64	+8 −2 N.E. Cor. Lower Step Fellows Hall.
B.M. 2	2.256	431.796	6.114	429.540	84	108	+10 Concrete Monument S.W. Cor. Big Barn.
T.P. 4	3.387	423.535	11.648	420.148	44	96	Plug.
T.P. 5	1.110	414.958	9.687	413.848	115	76	+42 Plug.
B.M. 3	5.302	415.887	4.373	410.585	96	73	0 Copper Bolt in Sidewalk Front of Ag. Bldg.
T.P. 6	10.415	423.655	2.647	413.240	80	91	−2 S.E. Cor. Brick Pier at Observatory.
T.P. 7	6.788	428.330	2.113	421.542	93	80	−5 Plug.
T.P. 8	0.669	423.837	5.162	423.168	75	82	−16 Plug.
B.M. 1			10.768	413.069		93	+2 U.S.G.S. B.M.
	60.763		60.804	413.110	969	971	−3 ch.
			60.763	413.069			
			0.041 ≠	0.041	ck.		
	Error = 0.041		dist. in miles				

Distances Expressed in Paces.
37 Paces = 100 ft.

SAMPLE NOTES 26.

driven in the earth and removed by the rodman as soon as backsight and foresight have been taken, or it may be on any solid object such as a rock, curb, rail, or root of tree. Permanent turning points should be marked and described in the notes.

Practical Applications.—This method of differential leveling is employed on nearly all work in which the relative elevation of points is desired and extreme precision is not necessary. With careful observers and a good instrument the error should not exceed 0.05 *ft.* $\times \sqrt{\textit{distance in miles}}$. Such a degree of precision is maintained by the primary levels of the U. S. Geological Survey. If the allowable error is greater than this, readings to the nearest 0.01 ft. will be sufficiently exact and lengths of sights need not be observed.

Problem 27. Differential Leveling with Engineers' Level and Target Rod

Object.—To determine the elevation or difference in elevation of points assigned.

Procedure.—The procedure differs from Problem 26 only in that the rodman sets the target as directed by the instrumentman. Both men read the rod from the attached vernier to the nearest 0.001 ft. The rodman should record the distances and rod readings in a book for that purpose. The field notes will be kept by the leveler as explained in the preceding problem. Compare the results of this method with those of the preceding problem. Note the relative errors of closure of the circuits and the time required for each method per set-up of the instrument.

Hints and Precautions.—In addition to those for Problem 26, (1) The leveler should make his signals easily distinguishable, holding his hand well up to raise the target and well down to lower the target. It is not necessary to wave the hand. (2) The rodman should move the target rapidly at first until the opposite signal is given by the instrumentman; he should then move it slowly until the instrumentman indicates that it is in the proper position by the "all right" signal (extending arms horizontally). (3) After the target is clamped, the rod should be waved slowly toward and from the instrumentman, particularly when the "long rod" is used. If the horizontal line on the target appears above the horizontal

hair, the target should be lowered. (4) When using the "long rod" the target must be clamped to read exactly the reading of the vernier on the back of the rod when the rod is short.

Practical Applications.—The target rod is usually used on surveys where a considerable degree of precision is required. However, it is possible with proper instrumental equipment and care to run levels with the self-reading rod with about the same precision and at a less cost. Mistakes are less likely to occur with the target rod, and for this reason it may well be used on lines of levels that are not to be checked by running in both directions or by a complete circuit.

Problem 28. Profile Leveling for a Railroad

Object.—To determine the elevations necessary for plotting the profile of a line. (The data of this problem may be used in Problem 81, page 202.) See Sample Notes 28, page 86.

Procedure.—(It is assumed that the line has been laid out and stakes, marked with consecutive station numbers, driven every 100 ft.) (1) If no bench-mark is given, select some permanent point as a bench-mark, assuming an elevation such that no station will fall below the datum plane. (2) Set up the instrument near the line in such position that it will be possible to read the rod, not only on the bench-mark, but also at all stations and necessary intermediate points between the bench-mark and the instrument. (3) Obtain the height of instrument by backsighting upon the rod held upon the bench-mark, reading the rod to the nearest 0.01 ft. (4) Obtain elevations of stations and necessary plus points on the line by foresights, reading to the nearest 0.1 ft. *with the rod held on the ground.* (Plus distances may be obtained by pacing, or by measuring with the rod when the distance is short.) (5) In this manner determine ground elevations as far along the line as is convenient from the position of the instrument. (6) Establish a turning-point as described in Problem 26. Take a foresight upon this point to the nearest 0.01 ft., and move the instrument forward to another convenient point some distance in advance of the turning-point. (7) Take a backsight and determine the new height of instrument. Proceed with the determination of ground elevations as before. (8) Continue the process to the end of the line. (9)

20.

Prob. 28. Profile Levels

Cox Brook to Big Forks.

Sta.	B.S.	H.I.	I.F.S.	F.S.	Elev.
B.M. 28	1.56	565.87			564.31
605			0.7		565.2
606			2.9		563.0
607			3.5		562.4
608			6.7		559.2
609			11.9		554.0
T.P.	0.41	554.65		11.63	554.24
609+50			3.2		551.5
610			8.6		546.1
+40			9.0		545.7
+50			12.2		542.5
+65			9.3		545.4
611			8.4		546.3
612			7.1		547.6
613			8.4		546.3
614			7.0		547.7
615			4.1		550.6
T.P.	8.02	559.94		2.73	551.92
616			9.7		550.2
+40			6.3		553.6
	9.99		564.31	14.36	
			559.94	9.99	
			4.37	4.37	ck.

I.N.R. Location.

Cliff Dumpy Level. J.C. Brown, R. F. Graham, Rod. Sept. 16, 1907. Fair.

On Spruce Root 50 ft. Lt. Sta. 605.

On Stone

Bank Cox Brook.
Ctr. " "
Bank " "

Water 1.5 ft. deep.

On Plug.

Ctr. Highway to St. Leonards.

SAMPLE NOTES 28.

Here establish another bench-mark. (10) Run a line of check (differential) levels back to the initial bench-mark.

Hints and Precautions.—(1) Read the rod carefully to the nearest 0.01 ft. on bench-marks and turning-points and roughly and quickly to the nearest 0.1 ft. on the ground points. (2) To be consistent, ground elevations should be recorded only to the nearest 0.1 ft. (3) Rod readings should be taken at all full stations and such other points (plus stations) as are necessary to obtain a sufficiently accurate profile. These plus stations in general will be at points where the slope of the ground changes noticeably and at highways, railroads, and streams. (4) Bench-marks should be established every 1500 or 2000 ft. if the line is long. These should be placed at some distance to one side of the line, in such positions that they are not likely to be disturbed during construction. All bench-marks should be well described in the notes. (5) It is customary to mark the number and elevation on each bench-mark at the time it is established. It is important, therefore, that the computations for turning-points be performed as the work progresses, and each page of notes should be checked as soon as it is filled. The accompanying sample notes for a portion of a line of profile levels shows the computations for this check at the foot of the page.

Practical Applications.—Profile levels are run for railroads, sewers, pipe-lines, canals, and highways. Data for the elevations of contours on large scale topographic maps are often obtained by this method slightly modified.

Problem 29. Profile Leveling for a Pipe-line

Object.—To prepare a line for proposed sewer or water main for excavators. It is supposed that the line has already been run and that center stakes marked with station and plus have been set every 25 or 50 ft. (Data may be used in Problem 82, page 205.)

Procedure.—(1) Opposite each stake on the line and far enough from the line to insure its not being disturbed by the excavation, drive a short peg (or spike) flush with the ground, and beside this peg drive a stake marked with the station number of the center stake and the offset of peg from center stake. (2) Start from a bench-mark as in Problem 28, and take profile readings on the ground pegs to the nearest 0.01 ft.

Complete the level work as in Problem 28. (3) Roughly plot the profile, fix the grade of the bottom of the trench, and determine the amount of cut at each station. (4) Mark the cut in plain figures to the nearest ⅛ in. upon the side stakes, with waterproof kiel.

Hints and Precautions.—(1) Take rod readings upon the turning-points with greater care than upon the ground pegs. (2) The form of notes will be similar to Sample Notes 28, page 86, except that columns must be added for offsets of pegs from center line, grade elevations, and cuts. (3) All stakes should be marked to read down the stake. Center stakes should be driven with the marked side toward the beginning of the line. Side stakes should be driven side to the line in order that they will not be confused with center stakes. On the back or the side farthest from the line should appear the station number and offset; on the front or side nearest the line, the cut. (4) In paved streets or hard roads it is impossible to drive stakes or ground pegs, and 6-in. spikes are used instead, the spikes being driven flush with the road surface. In order that they may be found without difficulty, their position with respect to more prominent objects is carefully noted. (5) The purpose of the ground pegs is to furnish grade and line (after the center stakes are removed) to ditchers and pipers. (6) Observe the hints and precautions of Problem 28.

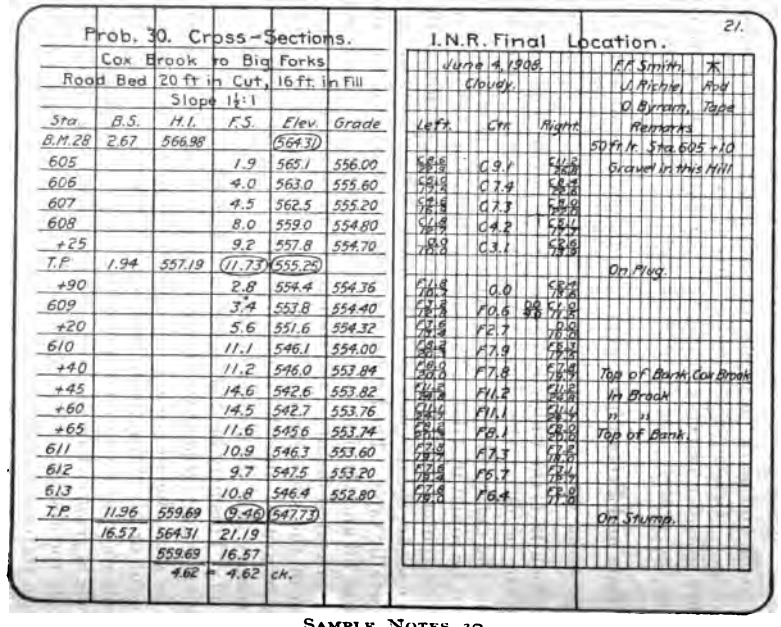
Problem 30. Setting Slope-stakes. Cross-sections

Object.—To prepare a proposed railroad for grading, and to obtain data for calculating earthwork. (The data may be used in Problem 80, page 199.) See Sample Notes 30, page 90.

Procedure.—(1) From the level notes of Problem 28 plot a profile and fix a grade (Problem 80) such that the amount of cut will approximately balance the amount of fill. (2) Drive short pegs flush with the ground beside the center stakes; run profile levels over the line, checking the elevations obtained with those of Problem 28; and mark on the back of each center stake the cut or fill at that point as *C* 3.9 or *F* 4.7. (3) Opposite each center stake at right angles to and on both sides of the line set slope-stakes at points where the side slope of the cut or fill will intersect the surface of the ground. These stakes should be driven side to the line, leaning toward the center line if in cut and away from the center line if in fill. They should

be marked on the side facing the center line with the cut or fill and distance from the center stake, as *C* 6.2/19.3; on the side farthest from the center line with the station number of the center stake and whether on the right or left, as *L* 17 + 00. The numbers should read down the stake. (4) Drive ground pegs at "grade points" or points where the grade passes from cut to fill, and mark the position of such pegs by stakes marked "grade." Grade pegs should be driven at the center and on either side at a distance of one-half the width of the roadbed from the center. (5) Record all data as shown in sample notes.

Hints and Precautions.—(1) The distance from center to slope-stake may be found by the expression $D = \frac{W}{2} + SC$, in which W is the width of roadbed, S the rate of slope, and C the cut or fill at the slope-stake. Except on level ground, since the cut or fill will not remain constant, the distance D must be found by trial. The rodman, knowing the cut or fill at the center, holds the rod in the estimated position of the slope-stake, the distance from the center being measured with a metallic tape. The leveler reads the rod, and calculates the cut or fill and the distance D for such cut or fill. If this is not the measured distance, the rodman again estimates the position of the slope-stake. Thus the process is repeated until the measured and the computed distances agree. (2) It is necessary that the leveler be able to calculate rapidly the cut or fill. This may be done as follows: Calculate the rod reading for grade (*grade-rod*) at the station to be cross-sectioned. If the elevation of grade is *less* than the height of instrument, the cut or fill at any point will be the *difference* between the grade-rod and the ground-rod at that point; if the elevation of grade is *greater* than the height of instrument, the *sum* of the grade-rod and ground-rod will give the fill at that point. That these statements are true may be easily shown by sketches. (3) The width of roadbed is somewhat greater in cut than in fill on account of side ditches. The roadbed of the average American single-track railway is perhaps 22 ft. wide in cut and 16 ft. wide in fill. (4) The slope of the cut or fill will depend upon the material. For earth a slope of 1½ (horizontal) to 1 (vertical) is commonly used in practice; for soft clay, 3 to 1; for solid rock, ¼ to 1 or less depending upon its structure. (5) As an example, suppose that a cross-section is to be taken



Prob. 30. Cross-Sections.

Cox Brook to Big Forks

Road Bed 20 ft in Cut, 16 ft in Fill

Slope 1½:1

Sta.	B.S.	H.I.	F.S.	Elev.	Grade
B.M.28	2.67	566.98		(564.31)	
605			1.9	565.1	556.00
606			4.0	563.0	555.60
607			4.5	562.5	555.20
608			8.0	559.0	554.80
+25			9.2	557.8	554.70
T.P.	1.94	557.19	(11.73)	(555.25)	
+90			2.8	554.4	554.36
609			3.4	553.8	554.40
+20			5.6	551.6	554.32
610			11.1	546.1	554.00
+40			11.2	546.0	553.84
+45			14.6	542.6	553.82
+60			14.5	542.7	553.76
+65			11.6	545.6	553.74
611			10.9	546.3	553.60
612			9.7	547.5	553.20
613			10.8	546.4	552.80
T.P.	11.96	559.69	(9.46)	(547.73)	
	16.57	564.31	21.19		
		559.69	16.57		
		4.62	4.62	ck.	

21.

I.N.R. Final Location.

June 4, 1908.
Cloudy.

A.A. Smith, K.
J. Richie, Rod
O. Byram, Tape

Left.	Ctr.	Right.	Remarks
			50 ft. lr. Sta. 605 +10
C6.6 / 56.6	C 9.1	C4.2 / 260	Gravel in this Hill
C5.0 / 59.2	C 7.4	C6.2 / 260	
C5.6 / 58.6	C 7.3	C5.0 / 259.0	
C1.8 / 62.4	C 4.2	C5.1 / 258.9	
0.0 / 70.0	C 3.1	C5.6 / 258.4	
			On Plug.
F1.8 / 70.7	0.0	C5.4 / 258.6	
F7.8 / 76.8	F0.6	00 / 95 F1.0 / 264.0	
F5.6 / 74.6	F2.7	D10 / 270.0	
F8.2 / 80.9	F7.9	F8.2 / 274.2	
F8.0 / 260.0	F7.8	F7.6 / 273.7	Top of Bank, Cox Brook
F4.5 / 76.8	F11.2	F1.2 / 268.0	In Brook
C4.4 / 268.0	F11.1	C4.4 / 268.4	" "
F8.6 / 272.6	F8.1	F8.0 / 272.0	Top of Bank.
F7.5 / 271.5	F7.3	F7.2 / 271.0	
F7.5 / 271.5	F5.7	F7.4 / 271.2	
F7.6 / 271.6	F6.4	F7.0 / 271.6	
			On Stump.

SAMPLE NOTES 30.

of an earth cut with 1½ to 1 slope. The roadbed is 20 ft. wide; the grade-rod is computed to be 12.0 ft. A slope-stake is to be set on the uphill side of the line. The rodman estimates the stake will be 19.0 ft. from the center. The leveler reads 5.4 ft. on the rod, which gives a cut of 12.0–5.4 equals 6.6 ft. and computes a distance out of $\frac{3}{2} \times 6.6 + 10.0$ equals 19.9 ft. Since the ground is rising it is evident that at a distance out of 19.9 ft. there would be a still greater cut. The rodman, realizing this, estimates the point at 22.0 ft. The rod reading at this point is 4.0 ft., the corresponding cut 8.0 ft., and the computed distance 22.0 ft. Thus the position of the slope-stake is determined. (6) In a party of two the tape may be held at the center by a knife stuck in the stake. (7) All hints and precautions of Problem 28, page 85, should be observed.

Practical Applications.—This method is essentially the same as that used on railroads, canals, and highways. For short sights, as when the cut or fill is small, the hand-level may well be used. The use of the Ward tape and tape-rod make greater speed possible. After setting the tape-rod to read the cut or fill at the center stake, other cuts or fills may be read directly from the rod. The Ward tape is a metallic tape graduated on one side in feet and tenths, and on the other side corresponding to the distance out, are amounts of cut or fill for a particular width of roadbed. The tapes are made for various slopes and widths of roadbed. Thus, with these accessories no computations are necessary.

Problem 31. Ballast Grades for Railroad

Object.—To set grade stakes for ballast at changes of grade and full stations. (It is assumed that the road is finished to sub-grade.)

Procedure.—(1) From the elevations of sub-grade calculate the elevations of base or top of rail for the specified thickness of ballast beneath the tie. (2) Run a line of levels from some point of known elevation, drive a stake to the calculated grade (nearest 0.01 ft.) opposite each station and change of grade, and far enough from the center (6 or 7 ft.) so that it will not be disturbed. Mark the top of each stake with kiel to show that it is a grade point. (3) Where there are no curves and no changes of grade, "shoot in" the grade for distances as great as the condition of the atmosphere and the magnifying

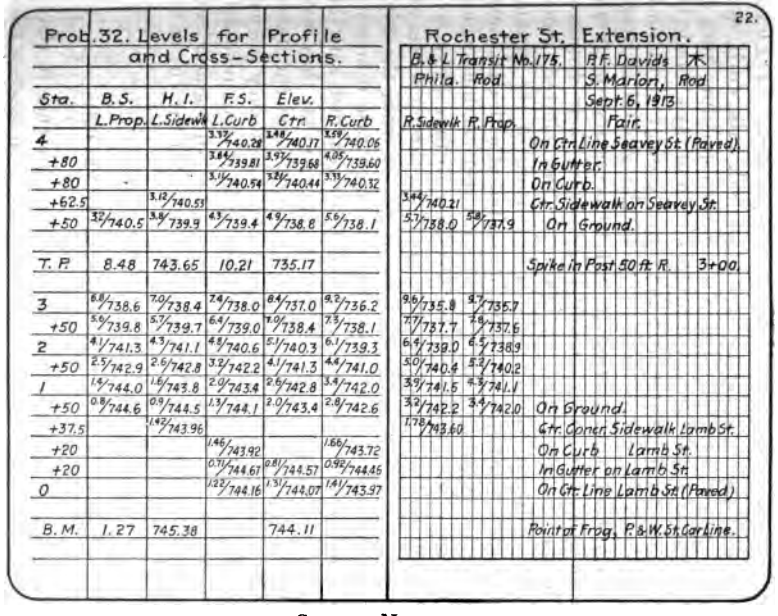
power of the telescope will allow. (4) Check the notes at each reading, and check the accuracy of the line of levels at every bench-mark.

Hints and Precautions.—(1) To "shoot in" a grade line proceed as follows: Drive a stake to grade at the end of the line near the instrument. Run a line of differential levels the desired distance, and set another stake to grade. With the instrument close to the stake last set, read the rod by looking through the objective end of the telescope and record the reading. With the rod held at the stake first set, elevate or depress the telescope until the horizontal hair cuts the rod at a similar reading. If possible, note the position of the horizontal hair on some prominent mark of the landscape. The line of sight is now a uniform distance above the grade line, and intermediate grade stakes may be set with the same rod reading. Check the line of sight frequently by observing the position of the horizontal hair on the reference mark. (2) If a target rod is used in "shooting" grade, it will be convenient to set the target at the constant reading. If the rod has no target, a card fastened with a rubber band will answer the purpose. (3) If a level is used in "shooting" grade, the telescope should be elevated or depressed by a single pair of foot-screws; in order that the horizontal hair may remain truly horizontal, this pair of screws should be brought in line with the distant stake. (4) Observe the precautions stated in Problem 28. (5) It frequently happens that stakes can not be driven to grade; it is customary to mark very plainly the position of grade on the sides of these stakes and to omit marking their tops.

Problem 32. Levels for Determining Quantities of Excavation for Grading Street

Object.—To determine the elevations of points necessary to plot the profile and cross-sections of a proposed city street. (The data may be used to Problem 83, page 205.)

Procedure.—(1) Drive stakes on a line 50 ft. apart to represent the center line of the street. Mark each with its proper station number beginning at 0 (50-ft. points are marked + 50). (2) Drive stakes opposite each of these on both sides of the center line at such distances from the center as 20, 35, and 40 ft. to mark right and left curb lines, right and left sidewalk lines, and right and left property lines. (3) Follow the



Prob. 32. Levels for Profile and Cross-Sections.

22.

Rochester St. Extension.

B. & L. Transit No. 175. Phila. Rod. H.F. Davids S. Marion, Rod. Sept. 6, 1913. Fair.

Sta.	B.S. L. Prop.	H.I. L. Sidewlk	F.S. L. Curb	Elev. Ctr.	R. Curb	
4			3.32/740.28	3.48/740.11	3.59/740.06	
+80			3.64/739.81	3.97/739.68	4.05/739.60	
+80			3.19/740.54	3.33/740.44	3.33/740.32	
+62.5		3.12/740.53				
+50	3.2/740.5	3.8/739.9	4.3/739.4	4.3/738.8	5.6/738.1	
T. P.	8.48	743.65	10.21	735.17		
3	6.8/738.6	7.0/738.4	7.4/738.0	6.4/737.0	9.2/736.2	
+50	5.6/739.8	5.7/739.7	6.4/739.0	7.0/738.4	7.3/738.1	
2	4.1/741.3	4.3/741.1	4.5/740.6	5.1/740.3	6.1/739.3	
+50	2.5/742.9	2.6/742.8	3.3/742.2	4.1/741.3	4.4/741.0	
1	1.4/744.0	1.5/743.8	2.0/743.4	2.6/742.8	3.4/742.0	
+50	0.8/744.6	0.9/744.5	1.3/744.1	2.0/743.4	2.8/742.6	
+37.5		1.42/743.96				
+20			1.46/743.92		1.66/743.72	
+20			0.71/744.67	0.81/744.57	0.92/744.46	
0			1.22/744.16	1.31/744.07	1.41/743.97	
B. M.	1.27	745.38		744.11		

9.6/735.8	9.7/735.7		
7.7/737.7	7.8/737.6		
6.4/739.0	6.5/738.9		
5.0/740.4	5.2/740.2		
3.9/741.5	4.3/741.1		
3.2/742.2	3.4/742.0	On Ground.	
1.78/743.60		Cor. Corner Sidewalk Lamb St.	
		On Curb Lamb St.	
		In Gutter on Lamb St.	
		On Ctr. Line Lamb St. (Paved)	
		Point of Frog, P. & W. St. Car Line.	

SAMPLE NOTES 32.

same method of procedure as in Problem 28. Keep the notes as shown in the accompanying sample notes, page 93. The figures above the short diagonal lines are ground-rod readings; those below are the elevations of the points.

Hints and Precautions.—(1) All hints and precautions mentioned in Problem 28, p. 87, apply to this problem. (2) Elevations of the points of intersection of the sidewalk, curb, and center lines of the proposed street with corresponding lines of existing paved streets or with bridges and railways should be determined with greater accuracy than ground elevations, as shown in the sample notes. (3) If the ground is rough or irregular at any section it will be necessary to obtain the elevations of extra ground points with their distances from the center stake, for otherwise the plotted cross-sections will not represent the true contour of the ground and cross-sectional areas can not be computed with exactness. If there are appreciable irregularities between sections where stakes have been set, it will be necessary to take extra sections, but no stakes need be set. (4) If the survey is over an existing street the ground may be so hard as to make the driving of stakes well-nigh impossible. In such case 6-in. wire nails may well be used. They should be driven flush with the ground, and through a piece of tough paper so that they may be found readily. (5) It is common practice to set the stakes for sidewalk and curb on offset lines so that a re-survey will not be necessary after excavation. When set in this manner each stake is marked with its offset distance. (6) After grade lines have been determined (Problem 83, page 205) the cut or fill is marked on each stake.

Problem 33. Reciprocal Leveling

Object.—To determine accurately the difference in elevation between two points (*B.M. a* and *B.M. b*) on opposite sides of a wide stream or ravine. See Sample Notes 33.

Procedure.—(1) Set up the level in such a position that rod readings may be taken upon each bench-mark. (This, of course, necessitates the instrument being much closer to one point than to the other.) Carefully take a series of five consecutive readings upon *B.M. a*. The mean of these is to be used as a backsight. (2) Take ten careful readings upon *B.M. b*, the distant point. The mean of these readings is to

23.

Prob. 33. Reciprocal Leveling Across Sun River at Lemo.

F.A. Charles K
N.L. Christy, Rod
R.M. Evans.
June 13, 1913.
Fair.

Survey 20" Wye
2-Philo Rods.

Obs. No.	B.S. on a	B.S. on b	F.S. on a	F.S. on b	Elev.
1	4.689	4.166			739.215
2	4.687	4.168			
3	4.686	4.169			
4	4.688	4.167			
5	4.690	4.166			
Mean	4.688	4.167			
1			1.030	7.822	
2			1.025	7.833	
3			1.018	7.825	
4			1.036	7.819	
5			1.022	7.826	
6			1.017	7.837	
7			1.025	7.822	
8			1.023	7.830	
9			1.031	7.818	
10			1.017	7.815	
Mean			1.024	7.825	
	7.825	4.167			
	4.688	1.024			
	3.137	3.143	Diff. El.	+3.140	736.075

B.M.a. Iron Plug, U.S.R.S.

Note: B.S. distances to B.M.a & B.M.b were 100 ft.

B.M.b. Point on ledge marked U.S.R.S. B.M.

SAMPLE NOTES 33.

be used as a foresight. (3) Now set up the level on the opposite side of the stream in such position that the distances from the instrument to *a* and *b* are respectively the same as the distances to *b* and *a* from the former position of the instrument. Take a series of readings on the near and distant points as before. (4) The difference between the mean of the backsight readings on *b* and the mean of the foresight readings on *a* from this series will also give a difference in elevation between the two points. (5) The mean of the two differences in elevation secured from the two settings of the instrument should be the true difference. (6) If the stream or ravine is imaginary, run a line of differential levels between the two points and note the discrepancy.

Hints and Precautions.—(1) Be sure that the bubble is exactly in the center at the time of each reading. The effect of bubble displacement will be particularly great on long distance readings. For distant sights the bubble should be moved and reset after each observation. (2) If the instrument can be set up near the bench-mark used as a backsight, only one observation need be taken to that point. (3) If the rod has no target, a white card may be used as such for long sights. The target should be moved and reset for each observation.

Practical Applications.—The method of reciprocal leveling is used to transfer accurately a line of levels across a river or deep ravine. On account of the curvature of the earth and lack of perfect adjustment in the instrument a considerable error is likely to be introduced if the ordinary method of differential leveling is used. This is due to the difference in length between backsight and foresight.

Problem 34. Test of Accuracy of Setting Level Target

Object.—To determine the probable error of setting the level target at distances of 100, 300, and 600 ft. from the instrument, and to determine what length of sight will give best results in running a line of levels.

Procedure.—(1) Set the level in position to permit a 600-ft. sight. Drive stakes solidly at 100, 300, and 600 ft. from the instrument (distances by pacing). (2) Take a series of ten consecutive rod readings upon each stake, reading the target

Prob. 34. Accuracy of Setting Level Target at 100, 300 & 600 ft.

24.

Gurley 20" Wye No. 69. J. Craig,
Philadelphia Rod. L. Jacobs, Rod.
Nov. 11, 1913.
Cloudy, Moderate.
No Wind.

	At 100 ft.		At 300 ft.		
Rod	V	V^2	Rod	V	V^2
4.675	0.000	0.000000	3.178	0.003	0.000009
4.675	0	0	3.175	0	0
4.676	1	1	3.174	1	1
4.674	1	1	3.176	1	1
4.675	0	0	3.176	1	1
4.676	1	1	3.173	2	4
4.676	1	1	3.174	1	1
4.673	2	4	3.177	2	4
4.676	1	1	3.172	3	9
4.674	1	1	3.175	0	0
4.675	ΣV^2 = 0.000010		3.175	ΣV^2 = 0.000030	
	At 600 ft.				
2.467	0.002	0.000004	E_s (100 ft.) = ±0.00071		
2.460	5	25	E_s (300 ft.) = ±0.00123		
2.469	4	16	E_s (600 ft.) = ±0.00262		
2.465	0	0	E_m (100 ft.) = ±0.00023		
2.471	6	36	E_m (300 ft.) = ±0.00039		
2.461	4	16	E_m (600 ft.) = ±0.00083		
2.463	2	4			
2.466	1	1			
2.460	5	25			
2.468	3	9			
2.465	ΣV^2 = 0.000136				

Probable Error in Running 600 ft. with
Single Obs. at 100 ft. Sights = $\pm\frac{1}{6}\times 0.00071 = \pm 0.00174$
" " at 300 ft. " = $\pm\frac{1}{2}\times 0.00123 = \pm 0.00174$
" " at 600 ft. " = ± 0.00262

SAMPLE NOTES 34.

vernier to the nearest 0.001 ft. Center the bubble, and reset the target at each observation. (3) Compute the mean rod reading for each distance. (4) Record in the column for residuals (Sample Notes 34, page 97) the difference between each rod reading and the mean of all the rod readings for each distance. (5) Compute the probable errors of each set of observations by the following formulas:

Probable error of a single observation, $E_s = \pm 0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$

Probable error of the mean of all the observations, $E_m = \pm \frac{E_s}{\sqrt{n}}$

in which Σv^2 is the sum of the squares of all residuals and n is the number of observations. (6) From the probable error of a single observation at 100, 300, and 600 ft., compute the probable error in running a line of levels of any given length when the sights are in one case all 100 ft. long, in a second case all 300 ft. long, and in a third case all 600 ft. long.

Hints and Precautions.—(1) Be sure that the bubble is in the center of the tube for each reading even though you may think that the level has not moved. (2) The rodman should move the target several inches between each setting and reset it without prejudice, as directed by the instrumentman. (3) Note carefully the effect of distances or length of sight upon the precision of rod readings. In considering the effect of distance upon the accuracy of a line of levels, it must be remembered that three times as many 100-ft. sights are necessary as 300-ft. sights, and that the probable error of a line of levels varies as the square root of the number of set-ups.

Practical Applications.—This problem should give the student not only an idea of his own reliability at target setting with the particular instrument he is using, but also the effect of length of sight upon the probable accuracy of target setting. The elements affecting the possible accuracy are magnifying power of telescope, sensitiveness of bubble, size of cross-hairs, power of observation of the leveler, and condition of the atmosphere.

Problem 35. Magnifying Power of Telescope

Object.—To determine the number of diameters an object viewed through the telescope is magnified.

Procedure.—(1) Sight at the rod held erect about 15 ft.

in front of the instrument. (2) With both eyes open turn the instrument until the images, as seen by the naked eye and as seen through the telescope, appear to fall one upon the other. (3) Compare 0.1 ft. on the rod as seen through the telescope with a space as seen with the naked eye. The number of tenths apparently covered on the unmagnified image is the magnifying power of the telescope.

Hints and Precautions.—(1) Some practice will probably be necessary before the student will be able to see both images at the same time. First sight the image through the telescope; then still keeping the image in sight, look at the rod with the other eye. After a little practice both images will appear distinct. Turning the level slightly, if necessary, will cause the unmagnified image to fall upon the magnified image. For observation select the tenth on the magnified image which is located wholly in the field of vision. Observe the reading of the upper line of this tenth on the unmagnified image, and then observe the lower. The difference of these readings in tenths of feet will be the magnifying power.

Practical Applications.—This is a convenient, quick, and sufficiently accurate method of determining the magnifying power of any telescope for the purpose of appraising or comparing instruments. For the engineers' transit the magnifying power should be from 20 to 30 diam. depending upon the least count of the vernier, and the sensitiveness of the bubble-tube; for the level, 30 to 40 diam.

Problem 36. Radius of Curvature of Bubble-tube

Object.—To determine, in the field, without the use of special apparatus, the radius of the curvature of the bubble-tube of transit or level.

Procedure.—(1) Hold the rod on a solid point 300 ft. from the instrument. With one end of the bubble at a division near the end of the tube, take a careful rod reading to the nearest 0.001 ft. Note the exact position of each end of the bubble. (2) Adjust the foot-screws until the other end of the bubble falls near the other end of the tube. Take another rod reading, and measure the exact distance traversed by each end of the bubble. (3) Determine the bubble movement (this should be expressed to the nearest 0.001 ft.) and the difference between the two target readings or target movement.

(4) In this manner obtain a series of five bubble movements and their corresponding target movements. (5) Compute the radius of curvature by the following formula: $R = \frac{b}{t} D$ in which R is the radius of curvature, D is the distance from the instrument to the rod, b is the mean of the five bubble movements, and t is the mean of the five target movements. (6) Compute the value of one division of the bubble-tube in seconds of arc.

Hints and Precautions.—(1) Observe the bubble and target movements very carefully, as they are very small in comparison to R and D. (2) One division on the bubble tube of most instruments is equal to $\frac{1}{10}$ in.

Practical Applications.—This is a quick and sufficiently accurate means of testing the delicacy of the bubble of a transit or level, as the delicacy is directly proportional to the radius of curvature, or inversely proportional to the value of one division expressed in arc. Thus one is enabled to judge the reliability of an instrument in so far as it is affected by the sensitiveness of the bubble.

In a well-balanced instrument the relation between the sensitiveness of the bubble and the magnifying power should be such that for the slightest perceptible movement of the bubble there will be a perceptible movement of the line of sight. Neglecting the effect of size of cross-hairs and assuming that the image is well illuminated, this condition is approximately approached when $M = 400/S$ in which M is the magnifying power of the telescope and S the value in seconds of arc of one division ($\frac{1}{10}$ in.) of the bubble-tube.

Problem 37. Differential Levels—Double-rodded Line

Object.—To run a line of levels with two sets of turning-points in such manner as to furnish two independent heights of instrument for each set-up of the level.

Procedure.—The method of procedure is similar to that of Problem 26 or 27 (p. 80–85), except that at each set-up of the level backsights and foresights are taken on two separate lines of turning-points, the order of observing being backsight, foresight, foresight, backsight. Two separate sets of level notes are kept as shown in Sample Notes 37. Usually

26.

Prob. 37. Levels, Double Rodded Line Along P. & R. R. Ry.

Sta.	B.S.	H.I.	F.S.	Elev.
B.M.$_1$	5.241	532.871		527.63
B.M.$_1$	5.239	532.869		
TP$_1$ H	6.943	535.896	3.916	528.955
TP$_1$ L	7.897	535.893	4.873	527.996
TP$_2$ H	8.337	541.804	2.431	533.467
TP$_2$ L	9.746	541.797	3.842	532.051
TP$_3$ H	5.173	541.508	5.469	536.335
TP$_3$ L	7.549	541.504	7.842	733.955
TP$_4$ H	3.411	536.731	8.188	533.320
BM$_2$ L	4.963	536.725	9.742	531.762
TP$_5$ H	2.344	531.837	7.238	529.493
TP$_5$ L	5.729	531.830	10.624	526.101
BM$_3$ H	7.004	731.043	7.798	724.039
TP$_6$ L	8.021	731.039	8.812	723.018
	87.597		80.775	

Dixfield to Peru.

Gurley Wye Level, 2- Philo. Rods.

F.R. Burton, J.J. Mantel, Recorder, Lowe & Smith, Rods.

July 9, 1908.

U.S.G.S. in Culvert 800 ft. S. of Mile Post 13.

Fair & Warm.

731.043 731.039
727.63 727.63
87.597 3.413 3.409
80.775 3.409
6.822 = 6.822 ch.

Spk. in Tel. Pole at Road to Abbotts Mills.

d.h. in Culvert at Alder Brook.

SAMPLE NOTES 37.

two rodmen are employed, each rodman remaining at the two corresponding turning-points until all sights to these points have been taken.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 26 and 27. (2) Upon the conclusion of observations at any set-up the instrumentman should *immediately* calculate the height of instrument for each line. If there has been no mistake in observing, these heights should agree very closely at the beginning of the line, but following the law of compensating errors, may differ by a considerable amount before the end of the line is reached. In ordinary leveling where lengths of sights are less than 300 ft., reasonable care is taken to prevent unequal heating of the level, and the rod is read to thousandths of feet, the difference between any two corresponding heights of instrument should not vary from the difference between the two preceding heights of instrument by more than 0.004 ft. In case the notes indicate a greater variation than is allowable, without moving the level, a new set of readings should be taken and the first set should be discarded. (3) The reason for taking sights in the order stated above is to partially produce conditions likely to be met in running two lines of levels in opposite directions. If both backsights were taken before the foresights or if each backsight were taken before its corresponding foresight, constant errors, such as settling of the instrument, might go undetected since they would affect the heights of instrument in one line as much and in the same direction as those in the other line. It will be seen that the method here used is such that, while the error may be the same in each line, it will in one case be positive and in the other case negative, and for this reason the heights of instrument will tend to diverge. It should be noted, however, that such a course of procedure will not eliminate errors due to unequal backsight and foresight distances. (4) It is best to have the turning-points of one line a foot or more above the corresponding turning-points of the other line, for this will lessen the possibility of mistakes of 1 ft. being made in two corresponding rod readings. It is evident that such mistakes would still make the heights of instrument check. The turning-points of one line may be distinguished from those of the other by the subscripts H and L (meaning high and low) as shown in the sample notes. (5) Backsight and foresight

distances should balance between bench-marks, but very frequently where the line passes through smooth country or along a railroad these distances are not recorded in the notes because approximately constant lengths of sight are maintained throughout the length of the line. All bench-marks should be used as turning-points.

Practical Applications.—The double-rodded line is used principally on courses that do not form a complete circuit or do not close upon some point of known elevation. It is more economical than two lines of ordinary differential levels; it can not be said, however, that in point of accuracy it is the equal of two single-rodded lines run in opposite directions over the same course. Its principal advantage over a single line lies in the greatly increased probability of the resulting elevations being free from large errors or mistakes. The U. S. Geological Survey has used it extensively on short branch lines for the purpose of furnishing vertical control to topographers, and formerly it was used extensively on lines of precise levels.

CHAPTER V

PLANE-TABLE AND SEXTANT

PROBLEM 38. ADJUSTMENTS OF THE PLANE-TABLE

Note.—There are two quite distinct types of plane-table alidades in use. The telescope tube of one type (see Plate VII) is rigidly attached to or is an integral part of the horizontal axis, as with the engineers' transit. The telescope tube of the other type is fastened into a cylindrical sleeve which is an integral part of the horizontal axis; the telescope may be turned about its axis in the sleeve much as may the telescope of the wye level in its wyes, and on the telescope tube are turned two shoulders, perhaps 5 in. apart, upon which rests a striding level which may be removed at will. The telescope of the former type may be equipped with striding level as just described or may have an attached bubble-tube, like the transit telescope.

With either type the vertical vernier may be fixed or movable or there may be an auxiliary bubble-tube attached to the vernier arm, as with the transit; and the levels for controlling the table may be mounted on the base-plate or straight-edge of the alidade or they may be mounted with the compass on a smaller plate.

Adjustments.—The relations that should exist in a *perfectly* adjusted instrument are: (1) that the vertical cross-hair lie in a plane perpendicular to the horizontal axis, (2) that the axes of the control bubble-tubes be parallel to the plane of the plate, (3) that the line of sight revolve in a plane, (4) that the horizontal axis lie in a plane parallel to the plane of the plate, (5) that the line of sight coincide with the optical axis, (6) that the axis of the striding level or bubble-tube attached to the telescope be parallel to the line of sight; further, for convenience in measuring vertical angles, (7) if the vernier is fixed, that it should read zero when both straight-edge and line of sight are horizontal, or (8) if the vernier has an attached bubble-tube, that its axis should be

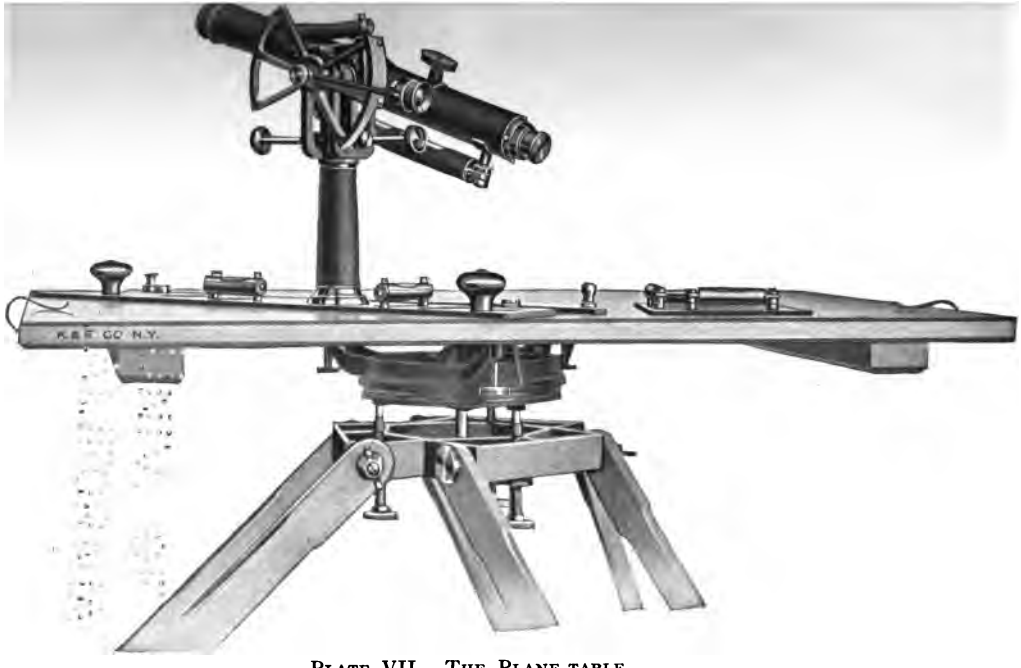


PLATE VII.—THE PLANE-TABLE.
(From a half-tone furnished by Keuffel & Esser Co.)

(*Facing page* 104.)

parallel to the axis of the telescope bubble-tube for a zero reading of the vernier.

In principle the methods of performing the adjustments are identical with those of corresponding adjustments of the transit (Problem 11, page 47) or of the wye level (Problem 24, page 77), but in general the adjustments do not need to be made with such precision.

Adjustment (1) is performed exactly as in the transit.

Adjustment (2) is like the plate-bubble adjustment of the transit, except that the plate on which the bubble-tubes are mounted is turned end for end after its position has been marked on the plane-table board. It should be noted that the plane-table *is not* turned 180° about its axis.

Adjustments (3) and (4) rarely, if ever, need attention. Many alidades have no provision for Adjustment (4).

Adjustment (5) in the alidade with telescope in axial sleeve is performed by sighting at a point and turning the telescope 180° about *its* axis as in Adjustment (2) of the wye level. This also takes care of Adjustment (3). In the alidade with telescope rigidly attached to horizontal axis the adjustment may be performed as explained under Hints and Precautions (6), page 53.

Adjustment (6), if the level is attached to the telescope, is performed by the peg method as with the transit; if it is a striding level, like Adjustment (4) of the wye level except that the striding level is lifted from its supporting shoulders and turned end for end, instead of lifting the telescope from the wyes and reversing it.

Adjustments (7) and (8) are performed exactly as are Adjustments (7) and (8) of the transit.

Hints and Precautions.—(1) If Problems 11 and 24 have not been performed, study them carefully. (2) In plane-table work it is essential that vertical angles be measured accurately. For this reason Adjustments (5), (6), and (8) should be performed with considerable care.

Problem 39. Survey of Field with Plane-table by Radiation Method

Object.—To make a survey of the assigned field by the method of radiation.

Procedure.—(1) Choose some point in the field from which

all corners are visible. Set up and clamp the plane-table, and choose some point *o* on the plane-table sheet such that no corner to be plotted will fall off the paper. This point on the paper represents the point occupied on the ground. (2) With compass or declinator draw the meridian. (3) Move the alidade with the straight-edge cutting this point until the line of sight strikes some corner such as *A* (Fig. 24).

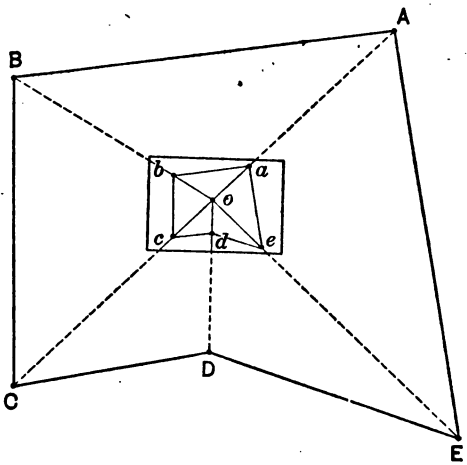


FIG. 24.—RADIATION WITH PLANE-TABLE.

(4) Draw a line of indefinite length along the straight-edge, and read the stadia intercept on the rod held by the rodman at *A*. (5) Move the alidade to one side, and scale the distance *oA* corresponding to the stadia reading, thus locating the point sighted. Letter the point so that it will not be confused with the points plotted later. (6) In like manner locate other corners *B*, *C*, *D*, etc. (7) Make a neat title containing the following information: name of survey, method used, members of party, date, and scale of the drawing.

Hints and Precautions.—(1) Set the plane-table low enough to permit taking sights without leaning against or placing the elbows on the board. (2) Be sure that the table is securely clamped. To guard against the possibility of undetected movement of the board sight at some prominent feature of the landscape as soon as the plane-table is in position, and

draw a line. Before leaving the station, with the straightedge along this line note whether or not the line of sight passes through the same point. (3) The stadia intercept can be calculated with greater facility if the lower stadia hair is made to intersect the stadia rod at some footmark. To avoid mistaking the middle hair for one of the stadia hairs, always note the half interval. (4) In most instruments the stadia interval factor is 100; the stadia interval may be tested as explained in Problem 58, page 151. (5) If the stadia intercept is greater than the length of the rod, note the interval cut by the lower and middle hairs, then plunging the telescope slightly, note the interval between the middle and upper hairs. Obviously the sum of these two readings is the stadia reading for that distance. (6) A stadia constant of $1 \pm$ ft. is added to all distances determined from rod readings. The amount of this constant is usually determined by the maker and marked in the instrument box. It will be unnecessary to consider this constant on small scale work. (7) Keep the telescope as nearly level as is practicable. On large scale work vertical angles up to 3° may be used without any appreciable error in the horizontal distance, but when the vertical angle is greater than 3° a horizontal correction must be applied. If the scale is small no horizontal correction need be made unless the country is very rough. (8) Too much care can not be taken to keep the pencil well sharpened, and a soft pencil should never be used. Make fine lines and no more of them than are absolutely necessary. (9) Considerable time can be saved by sticking a pin in the paper at the point from which all sights radiate.

Practical Applications.—The plane-table is not intended for precise work, nor can it be used where the numerical values of angles are required; hence it is generally made use of in such classes of surveys as map making, preliminary surveys, and topographical work. The plane-table is especially useful in plotting indefinite details such as the banks of rivers, the shores of lakes, roads, woods, and parks. It is advantageous over transit surveying in that more work can be done at a less cost and in less time. Since the map is made in the field, it dispenses with field notes; consequently no useless data are collected, lengthy computations are avoided, and omissions in the map are prevented. On the other hand, it is disadvantageous because angles are not recorded, hence is of little value

in calculating areas; because the table is cumbersome and awkward to handle, and the accessories are numerous; and because in wet weather good results can not be obtained. The problem as here given is of little practical value except as a means of acquainting the student with fundamental methods.

Problem 40. Plane-table Survey by Intersection

Object.—To make a plane-table survey of the same field assigned in the previous problem by method of intersection.

Procedure.—(1) Measure with a steel tape or by stadia a base-line *MN* (Fig. 25) 150 or 200 ft. long within or without

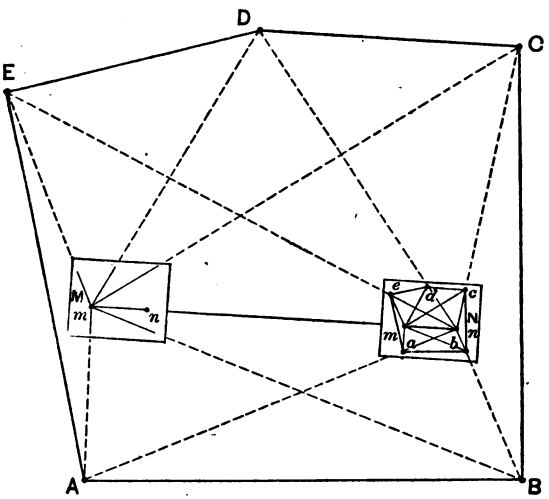


Fig. 25.—Intersection with Plane-table.

the field, so that when produced it will not pass near any corner of the field. (2) Plot the base-line *mn* on the plane-table sheet to the desired scale in position such that when the field is plotted all corners of the field will fall on the paper. (3) Set up over one end of the base-line *M*, and make the line *mn* on the paper coincide with the base-line *MN* on the ground (orient) by moving the board about the vertical axis of the plane-table until, with the straight-edge of the alidade along the line *mn*, the line of sight cuts *N*. (4) Clamp

the table in this position, and move the alidade about *m* as a center, drawing radial lines of indefinite length toward *A*, *B*, *C*, etc. Letter these lines so that there may be no doubt as to their identity. (5) By means of the compass or declinator draw a meridian, correcting for declination if desired. (6) Move the table to *N* and orient as before, making the line *nm* on the paper lie in the same vertical plane with *NM* on the ground. (7) With the edge of the alidade passing through *n*, draw radial lines toward *A*, *B*, *C*, etc., to intersect the corresponding lines drawn from *m*. Each point of intersection is (from similar triangles) the plotted position of the corner toward which the line of sight was directed when the radial lines were constructed. (8) Check the meridian line previously drawn. (9) Mentally compare the method with that of the preceding problem. Make a neat title.

Hints and Precautions.—(1) Use the same care in setting up the plane-table as in the previous problem. When orienting the board, if the scale of the drawing is large, make sure that the point on the paper is approximately over the corresponding point on the ground. (2) To make sure that intersections will be definite, the angle between intersecting lines should not be less than 30° nor more than 150°; when laying out the base-line this should be borne in mind. If it is impossible to obtain a good intersection from the points occupied on the base-line, a third point previously located by intersection may be occupied for that purpose, and the work may be carried on as before. (3) Make the radial lines as light as can be easily distinguished and only long enough to insure intersections. (4) In small-scale work it is not essential that the point on the paper should be directly over the point on the ground because the error is negligible, but on large-scale work it is quite necessary.

Practical Applications.—The method of intersection is generally used in connection with other methods, principally in the location of inaccessible objects or as a check. It is very convenient at times to choose some well-defined landmark such as a church steeple, which can be seen from several different points to be occupied by the plane-table. If the point is plotted at the intersection of lines directed toward the object from any two stations, a line drawn in the same manner from every other station occupied should pass through this same point. Any failure indicates either that the board has

not been properly oriented or that the point occupied has not been plotted in its proper position on the plane-table sheet.

On small or intermediate scale work this method is often used to locate points that are likely to be used as plane-table stations.

Problem 41. Survey with Plane-table by Method of Progression or Traversing

Object.—To make a survey of the same field used in the previous problem by method of traversing or progression.

Procedure.—In this method, as the name implies, the plane-table is moved from one station to the next in order of location,

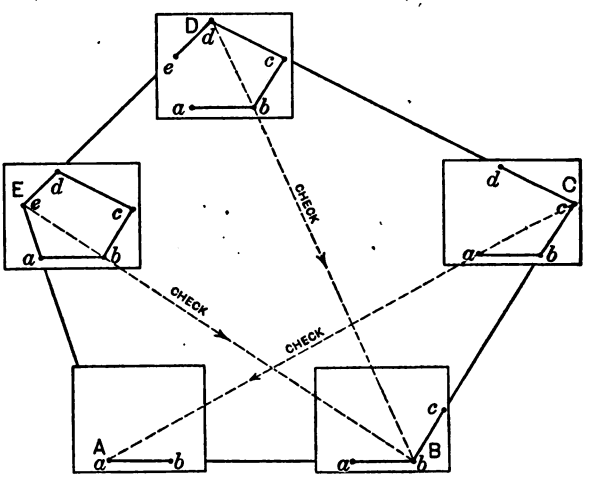


Fig. 26.—Transverse with Plane-table.

around the area to be mapped. The board is oriented at each point by sighting at the station previously occupied with the edge of the alidade passing through the plotted positions of the station occupied and the station sighted. (1) Set up over some corner *A* (Fig. 26). Sight to *B* and note the stadia distance. (2) Plot *b* as explained in Problem 39. (3) Draw a meridian with the compass. (4) Move the plane-table to *B*, and after orienting the board by sighting on *A*, plot the

point c in the same manner as b. With the compass, check the meridian drawn while the first station was occupied. (5) Continue in the same manner around the entire field. (6) Calculate the error of closure. (7) Make a comparison of the three methods. Make a title.

Hints and Precautions.—(1) Choose the first point on the paper with due regard for the limits of the plane-table sheet. (2) If from A the two adjacent corners are plotted, there will be no necessity of occupying the last station of the traverse except as a check. (3) Observe the hints and precautions of Problem 39 that are applicable to this problem.

Practical Applications.—The method of progression is especially adapted to the survey of roads and streams and is used in combination with the methods taken up in the two previous problems. In topographic work, traverses are usually run between two points some distance apart which have been established by a more accurate method of surveying. The details are located from the various traverse points by the radiation method, and a check on the work may frequently be furnished by the method of intersection. Thus, except in very small surveys, one rarely has occasion to use one method without using them all.

Problem 42. Plane-table Survey of Campus

Object.—Make a plane-table survey of a portion of the campus, locating all buildings, walks, trees, etc., by a combination of the methods of radiation, progression, and intersection.

Procedure.—(1) Set up at some point from which a considerable number of objects to be located may be seen. Locate these points by method of radiation (Problem 39, p. 105). (2) By progression (Problem 41) establish some other point from which more of the details to be mapped can be seen. (3) As a check, sight on some distant object which can be located by intersection (Problem 40, p. 108) from the next station occupied. (4) Set up over the traverse point located and continue as before. (5) In this manner make a closed traverse, locating from the traverse points all the points necessary to construct a complete map. If at any time there is any marked error made, as will be shown by the intersection of the various lines drawn to the distant point, the previous station should be re-occupied or the

orientation of the plane-table tested. (6) Finish by making a neat title as explained in Problem 39.

Problem 43. Location of Plane-table by Resection

Object.—To locate plane-table station from a base-line only one end of which can be conveniently occupied. The base-line may be the line between two triangulation stations. It is supposed that it has already been plotted on the plane-table sheet.

Procedure.—(1) Set up the plane-table at one end of the base-line, orient the board by bringing the plotted base *ba* (Fig. 27) over the actual base *BA*. (2) Sight to the point to

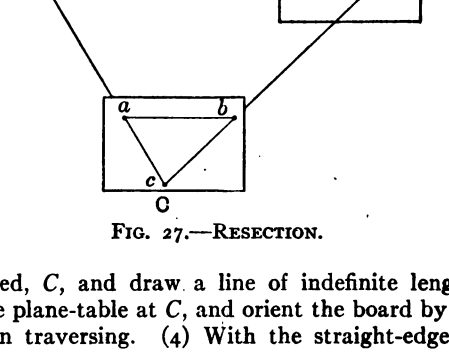


Fig. 27.—Resection.

be occupied, *C*, and draw a line of indefinite length. (3) Set up the plane-table at *C*, and orient the board by sighting at *B* as in traversing. (4) With the straight-edge passing through *a*, sight to *A* and draw a line to intersect *bc*. The point of intersection is the plotted position of the station occupied.

Hints and Precautions.—(1) It is evident that the strength of the determination of the point is greatest when the intersecting lines make an angle of 90° and decreases as the angle increases or decreases from 90°. The solution is indeterminate for all points on the base-line or its prolongation. (2) On large-scale maps the position of the point *c* must be carefully estimated before orienting at *C*, and the estimated position of the point should be over or nearly over the actual point on

the ground. On small-scale maps the plane-table may be roughly set over the station and the plotted point allowed to come where it will. The resulting error will be negligible.

Practical Applications.—This method of resection is used but little for actually locating stations. It may sometimes happen for one reason or another that it is impractical to locate the point by intersection or progression. The method is more often used as a check on other methods. As an example, suppose the station had been located by progression or traversing. A check would have been performed if on resecting through a to A the straight-edge passed through the point c. It follows that, if the point occupied is correctly plotted and the table is oriented, resecting lines from plotted points to their corresponding actual points will intersect at the plotted position of the station occupied. Fig. 26, page 110, illustrates the above method as a check on the traverse method.

Problem 44. The Three-point Problem with Plane-table (Mechanical Solution)

Object.—To plot the position of the station occupied by resection on three known points whose positions have already been located on the plane-table sheet.

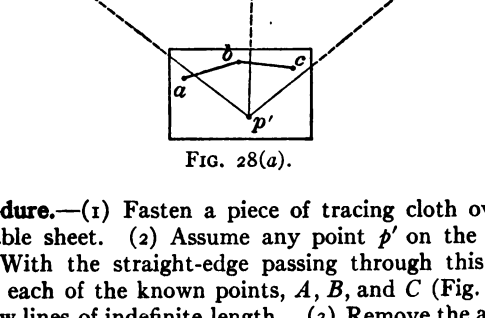


Fig. 28(a).

Procedure.—(1) Fasten a piece of tracing cloth over the plane-table sheet. (2) Assume any point p' on the tracing cloth. With the straight-edge passing through this point, sight at each of the known points, A, B, and C (Fig. 28(a)), and draw lines of indefinite length. (3) Remove the alidade,

and shift the tracing cloth until each of the lines drawn to the three actual known points passes through the corresponding plotted points, *a*, *b*, and *c* (Fig. 28(*b*)). The new position of the intersection of the radial lines on the tracing cloth is the point sought or the plotted position of the station occupied.

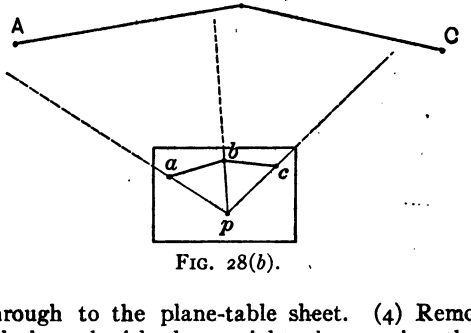


FIG. 28(*b*).

Prick through to the plane-table sheet. (4) Remove the tracing cloth, and with the straight-edge passing through *p* and one of the plotted points, turn the board until the line of sight strikes the corresponding actual point (Fig. 28(*c*)).

Hints and Precautions.—(1) Observe the hints and precautions of Problems 39, 40, and 41, that are applicable to this

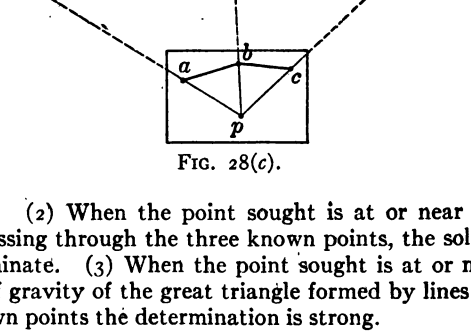


FIG. 28(*c*).

problem. (2) When the point sought is at or near a great circle passing through the three known points, the solution is indeterminate. (3) When the point sought is at or near the center of gravity of the great triangle formed by lines joining the known points the determination is strong.

Practical Applications.—On large surveys where the horizontal control is furnished by a system of triangulation and by accurate transit traverses, the three-point problem is particularly applicable because of the readiness with which known points can be plotted on the plane-table sheet and located in the field. By this method the position of each plane-table station is located independently; thus the error in the position of one station has no effect upon other stations. This is a decided advantage over the method of progression where the error in the position of one station must necessarily affect others. The tracing-cloth solution is not in general favor with experienced topographers; it has, however, the advantage of extreme simplicity over other methods.

Problem 45. The Three-point Problem with Plane-table (Coast Survey Solution)

Object.—To plot the position of the station occupied by resection on three known points whose positions have already been located on the plane-table sheet.

Procedure.—(1) By means of the compass or declinator orient the table as nearly as possible. Resect through each of the plotted points from the corresponding actual points, and draw lines of indefinite length on the plane-table sheet. If the lines intersect at a common point, the table is correctly oriented and the plotted position of the station occupied is at that point of intersection. (2) If the table is not correctly oriented, the three lines will form a triangle, termed the "triangle of error" as in Illustration 29, p. 117. The position of the "point sought" p will depend upon the size and shape of the triangle of error and its position relative to the great circle passing through the three fixed points. (3) Estimate the position of the point sought as explained in the following article, and with the straight-edge through this point and one of the fixed points, turn the table until the corresponding actual point is sighted. The table should now be nearer perfect orientation. (4) Again resect from the three known points, and determine a new triangle of error. (5) Estimate the position of the point sought, and perform the trial orientation. (6) Continue in this manner until the resection lines meet at a point. The table is, of course, to be clamped after each trial orientation.

Position of Point Sought.—The following extract taken from *A Plane-table Manual*,* by Mr. D. B. WAINWRIGHT, simplifies the estimation of the position of the point sought.

"*The relative positions of the three fixed points with reference to the new station* have an important bearing on the strength of its determination.

"In the following statement in regard to the different groupings of points met in practice, for the sake of brevity, the term "fixed points" will be understood to mean points already determined and plotted on the sheet; the "great triangle" referred to is one formed by the three fixed points; and the "great circle" is the circle passing through them.

"*When the new station is outside the great circle*, the strength for determination of a position will be weak when the middle point as seen from the new station is the farthest of the three and the angles are small. (See Illustration 29, Fig. 3.) If the new station is located outside the great circle, and some distance below it, the angles are small and the determination correspondingly weak.

"The determination increases in strength for given angles as the middle point approaches the new station (Fig. 1).

"When one angle is small or 0° (points in range); the determination will be strong, provided the two points making the small angle or range are not too near each other when compared to the distances to the new station and to the third point; provided also the angle to the third point is not too small (Fig. 2).

"*When the new station lies on or near the great circle*, its position is indeterminate (Fig. 3).

"*When the new station is within the great circle*, the strength of its determination increases as it approaches the center of gravity of the great triangle (Figs. 3, 4, and 5).

"There are a number of graphic solutions, but all save three are better suited to the drafting-room with its appliances than to the conditions which exist in the field.

"*Lehmann's method* of solution is the simplest and most direct, and applies under all circumstances. The directions are stated in the form of rules.

"The term "*point sought*" will be understood to mean the true position on the sheet of the projected point of the station

* Department of Commerce and Labor, U. S. Coast and Geodetic Survey. Report of 1905.

occupied. The surveyor is assumed to be facing the signals, and the directions right and left are given accordingly.

"*Rule* 1.—The point sought is always distant from each of the three lines drawn from the three fixed points in proportion to the distances of the corresponding actual points from the

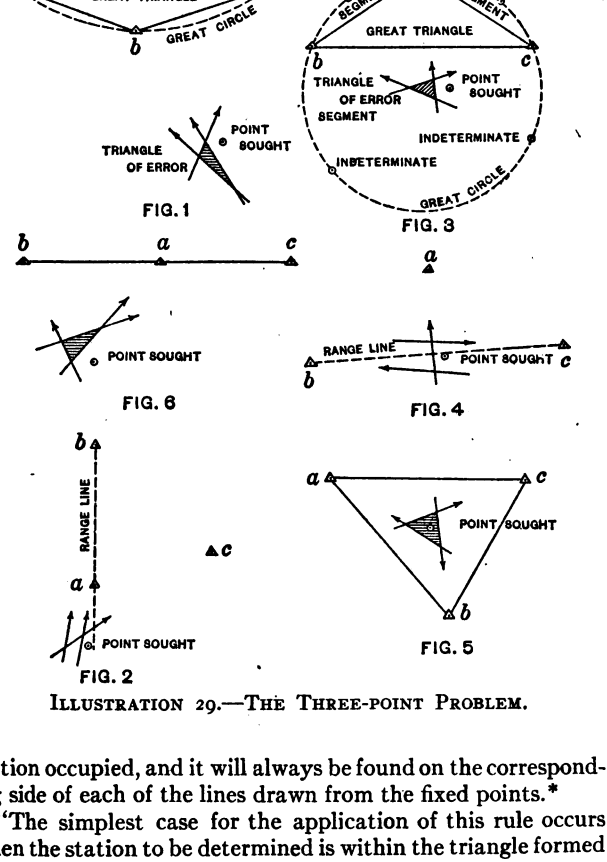


ILLUSTRATION 29.—THE THREE-POINT PROBLEM.

station occupied, and it will always be found on the corresponding side of each of the lines drawn from the fixed points.*

"The simplest case for the application of this rule occurs when the station to be determined is within the triangle formed

* That is, if it is on the right side of one line it will be on the right of the other two.

by the three fixed points; the point sought must then be within the triangle of error to satisfy the conditions (Fig. 5, p. 117).

"Although Rule 1 is sufficient in itself for the solution of the problem, there are two subordinate rules which materially assist the topographer in reaching a decision as to the proper location of the point sought with reference to the lines from the fixed points.

"*Rule* 2.—When the point sought is without the great circle, it is always on the same side of the line from the most distant point as the intersection of the other two lines (Fig. 1, p. 117).

"*Rule* 3.—When the point sought falls within either of the three segments of the great circle formed by the sides of the great triangle, the line drawn from the middle point lies between the point sought and the intersection of the other two lines (Figs. 3, 4, and 6, p. 117).

"Application of Rules.—In practice the topographer first decides the relation of the new station with reference to the fixed points, whether it is within the great triangle or in one of the segments or outside the great circle. He then determines the position of the point sought with reference to one line (if within one of the segments or without the great circle by Rule 2 or 3); it then follows from Rule 1 that it must be on the corresponding side of the other two lines. Finally, he estimates the relative distances of the three actual points from him and marks the position of the point sought a proportionate distance from each of the three lines."

Practical Applications.—The Coast Survey method of solving the three-point problem is extensively used by topographers of the U. S. Coast and Geodetic Survey and the U. S. Geological Survey. The length of time necessary to locate the station occupied is much less than one might suppose, and a solution is possible for all positions of the table (except on or near the great circle when the problem is indeterminate). Some of the other graphical methods depend upon construction lines which, for certain positions of the plane-table relative to the known points, will fall off the paper. The Coast Survey solution is certainly preferable to the mechanical method of the previous problem; for in damp weather the tracing cloth will soon become unfit for use, and in windy weather the topographer will experience some difficulty in keeping the cloth flat as it is shifted about the board.

Problem 46. The Two-point Problem with Plane-table (Mechanical Solution)

Object.—To locate the position of an unknown point P by resection on two known points A and B, and to orient the plane-table.

Procedure.—(1) Plot the two known points on the plane-table sheet. (2) Set up the plane-table over a fourth point C from which the other three points can be seen and which will give intersection angles from P and C to A and B between 30° and 150°. Fasten a piece of tracing cloth to the board. Assume a point c' on the tracing cloth corresponding to C on the ground; and, with the straight-edge of the alidade passing through this point, take sights to A, B, and P, and draw lines of indefinite length, Fig. 30(a). (3) Estimate the distance from C to P, and plot p' (the estimated position of P) on the line drawn toward P. (4) Set up the plane-table at P, and orient the board with respect to the line PC.* With p' as the center take sights to A and B, and draw lines until they intersect the corresponding lines from c', at the points a' and b'. (5) Remove the alidade, and shift the tracing cloth until the line $a'b'$ falls on the line ab, Fig. 30(b), page 120. Fasten the cloth in this position. (6) Replace the alidade on the tracing cloth and

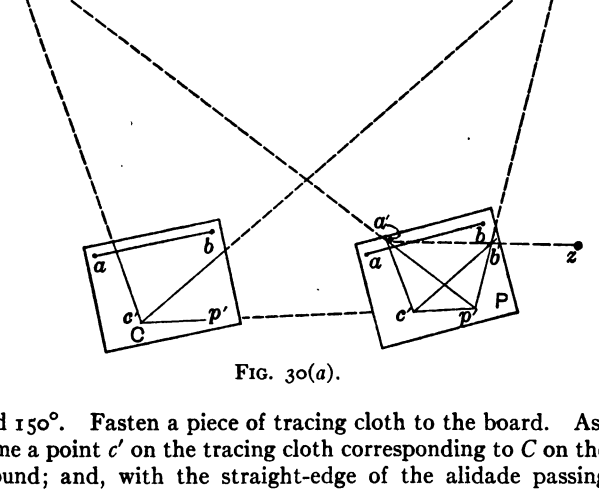


Fig. 30(a).

* That is, bring the line $p'c'$ into the same vertical plane with PC.

with the straight-edge along one of the lines $p'a'$, $p'b'$, or $p'c'$, revolve the board until the corresponding actual point A, B, or C is sighted, Fig. 30(*c*). The plane-table is now oriented.

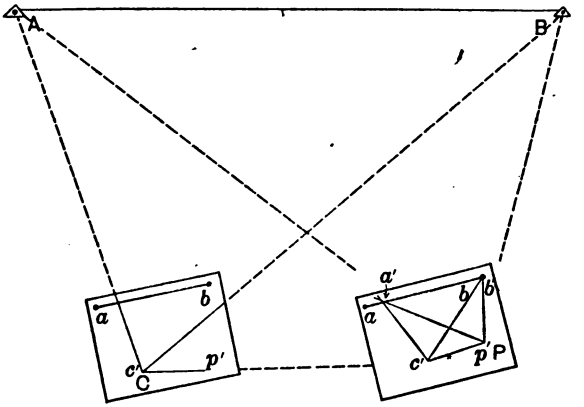


FIG. 30(*b*).

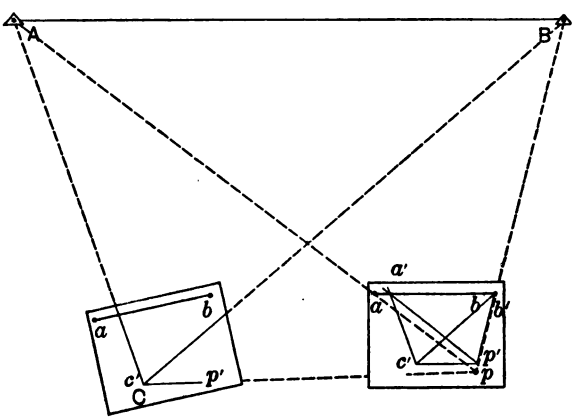


FIG. 30(*c*).

(7) Remove the tracing cloth. Resect through a from A, and through b from B, and draw lines on the plane-table sheet. The intersection of these two lines, shown as dotted lines in the figure, will be the position of p, the point sought.

Hints and Precautions.—(1) If the fourth point C is chosen in range with the known points A and B, the plane-table can be oriented at the start. This makes the problem comparatively simple, for when P is occupied, the table can be correctly oriented by the line drawn from c toward P. It is then only necessary to resect through a and b. (2) If the true length of the line CP be known, the tracing cloth may be shifted so that a' will coincide with a, and b' with b. The correct position of the point sought will then be at p'. (3) Intersecting lines should form angles between 30° and 150°. The position of the point C should be chosen with this in mind.

Practical Applications.—Owing to the length of the problem it is not practical except where other methods fail. If the known points are accessible and are near the station to be occupied, it will be found much simpler to use the method explained in Problem 43, page 112, or to run a traverse to the unknown point. If, however, the known points are at a distance or are inaccessible, as would be the case with church spires or chimneys, the two-point problem must be solved.

Problem 47. The Two-point Problem with Plane-table (Graphical Solution)

Object.—To locate the position of an unknown point P by resection on two known points A and B, and to orient the plane-table.

Procedure.—(1) Plot the two known points on the plane-table sheet. (2) Set up the plane-table over a fourth point C from which the other three points can be seen and which will give intersection angles from P and C to A and B between 30° and 150°. Orient the board as nearly as possible with the compass. Assume a point c', Fig. 30(a), page 119, on the plane-table sheet corresponding to C on the ground; and, with the straight-edge of the alidade passing through this point, take sights to A, B, and P and draw lines of indefinite length. (3) Estimate the distance from C to P and plot p' (the estimated position of P) on the line drawn toward P. (4) Set up at P, and orient the board with respect to the line PC. With p' as a center, take sights to A and B, and draw lines until they intersect corresponding lines from c' at the points a', b'. The line $a'b'$ will be parallel to a line between the corresponding actual points. (5) With the straight-edge of the alidade along the line $a'b'$,

note the position of the line of sight on some prominent object of the landscape or set a point z on line at some distance from the table. Move the alidade to the line ab, and turn the table until the same point is sighted. The plane-table is now oriented. (6) By resection through a and b, locate the point sought.

Hints and Precautions.—Observe the hints and precautions of Problem 46 that apply to this problem.

Practical Applications.—This method of solving the two-point problem is in favor with many topographers of the U. S. Coast and Geodetic Survey and U. S. Geological Survey. The fact that no extra equipment is needed makes it preferable to the solution of the previous problem.

Problem 48. Adjustments of the Sextant

(1) **To Make the Index Mirror Perpendicular to the Plane of the Sextant.**

Test.—Set the vernier to read about 30°. With the arc held away from the body and the eye slightly above the plane of the sextant examine the arc and its reflected image in the index mirror. If the mirror is in adjustment, the arc and its reflection should form a continuous curve.

Correction.—If there is a break in the continuity of the curve, change the inclination of the mirror by the screws provided for that purpose or by inserting slips of paper under its base. If the reflected image appears above the arc, the top of the mirror should be moved backward; if below, forward.

(2) **To Make the Horizon Glass Perpendicular to the Plane of the Sextant.**

Test.—Sight at some well-defined point a mile or more away (a star or the sun may be used). Move the index arm back and forth until both direct and reflected images are brought as near as possible to coincidence. If the images may be brought to exact coincidence, the desired relation exists.

Correction.—If the glass is not in adjustment, move the screws at the back or foot of the horizon glass until coincidence occurs. This adjustment should be made after the adjustment of the index mirror.

(3) **To Make the Horizon Glass Parallel to the Index Mirror for a Zero Reading of the Vernier.**

PLATE VIII.—THE SEXTANT.
(From a half-tone furnished by Keuffel & Esser Co.)
(Facing page 122.)

Test.—Set the vernier to read zero and sight at a distant object as in Adjustment (2). If coincidence of images occurs the mirrors are parallel.

Correction.—If not, move the horizon glass about an axis perpendicular to the plane of the sextant. Many sextants have no provision for such adjustment. With such instrument the index error is the reading of the vernier at the conclusion of Adjustment (2).

4. To Make the Line of Sight of the Telescope Parallel to the Plane of the Sextant.

Test.—Place the sextant on some solid object about 20 ft. from a wall with the plane of the graduated arc in a horizontal position. Make two wooden blocks of length equal to the height of the axis of the telescope above the plane of the graduated arc. Set the blocks on the graduated arc, and sighting over them, mark a point on the wall. Sight in the same direction with the telescope.

Correction.—If the line of sight (center of square) strikes above or below the point, the desired relation does not exist. Correct by tipping the collar into which the telescope is screwed. If the lines of sight come within an inch of coinciding at 20 ft., no correction need be made. Some sextants have no provision for this adjustment.

Problem 49. Angles of a Triangle with Sextant

Object.—To measure each of the angles of a triangle with the sextant, both with the handle below the telescope and with the handle above. The sum of the angles should be within 05′ of 180°.

Procedure.—(1) Sight at some clearly defined distant object, and move the vernier arm until the image of the same object reflected from the index mirror nearly coincides with that one directly viewed through the telescope. (2) Clamp the vernier arm in this position, and bring the images into exact coincidence by use of the tangent-screw. Read the vernier. The reading thus obtained is the index error and should be added to or subtracted from each observed angle according as the zero of the vernier lies to the right or to the left of the zero on the arc. (3) Place flag poles at two vertices of the triangle, and standing on the third point, with the telescope directed toward one of the flag poles, move the vernier until the

images of the two flag poles nearly coincide. Clamp the arm in this position, and bring the images exactly together with the tangent-screw. Read the vernier and record the angle. (4) Repeat with the instrument inverted. (5) From the mean of these two readings add or subtract the index correction. (6) Repeat the process for the other two angles of the triangle.

Hints and Precautions.—(1) Beginners usually find considerable difficulty in getting both images in the field of the telescope. The reflected image will usually be the dimmer of the two. Estimate the angle and set it off on the arc. Sight at the flag pole which is to furnish the direct image, and move the sextant slightly about the axis of the telescope. If two images fail to come into the field of view, move the vernier arm back and forth, at the same time rocking the sextant slightly about the axis of the telescope until they do appear. If the observer finds that he is unable to distinguish two images in this manner, he may remove the telescope and place his eye in the center of the ring. In this way he can distinguish the images without difficulty, and having brought them approximately together, can bring them into exact coincidence after replacing the telescope. (2) Use the eyepiece intended for terrestrial work. (3) The colored shades are intended for solar observations. (4) Notice that the angles measured are horizontal angles only when the eye of the observer and the two objects are in a horizontal plane. (5) The possible accuracy of angle determination with the sextant depends upon the size of the angle and the length of sight. The angle actually measured has its vertex not at the eye, but at the point where the ray coming directly to the eye and the ray striking the movable mirror would intersect were it not for the intercepting mirror. It is evident that the distance to this intersection will increase as the angle decreases and that it will be large for very small angles. For this reason the sextant is not an instrument of precision for distances less than 1000 ft. and angles less than 15°.

Practical Applications.—The advantage of the sextant over other surveying instruments is that sights may be taken to two objects simultaneously. While other instruments must be kept stationary between backsight and foresight, the sextant may be held in the hand. For this reason, in surveying it is used principally on hydrographic surveys for locating the

position of soundings. Two angles are taken from the boat to three known points on shore at the instant of taking the sounding. This may be accomplished by one observer with two sextants or by two observers each having a sextant. The sextant is also used in astronomical observations.

CHAPTER VI

FIELD ASTRONOMY

PROBLEM 50. LATITUDE BY OBSERVATION ON THE SUN AT NOON

Object.—To determine the latitude of the place by an observation on the sun at local apparent noon with the engineers' transit.

Procedure.—(1) Set up the transit some time before the sun is at the highest point in the heavens, with the telescope pointing in the general direction of south, and determine the index error of the vertical circle. (2) With the horizontal cross-hair on the lower limb of the sun, follow it along until a maximum altitude has been reached and the sun begins its apparent descent. Record the vertical angle and watch time. (3) Correct the watch time by comparing it with a clock keeping standard time, and determine the Greenwich mean time (*G.M.T.*) of the observation by adding to the corrected time the longitude (in hours) of the meridian from which standard time is reckoned. (4) In the Nautical Almanac* (page II for month) or other solar ephemeris giving values for Greenwich mean noon, look up the apparent declination δ_0 for mean noon, and the hourly change of declination on the day of the observation, noting that the declination is *positive* when *north* and *negative* when *south*. (5) Calculate the change in declination $\Delta\delta$ between Greenwich mean noon and the Greenwich mean time of observation.

$$\Delta\delta = (\textit{hourly change} \times G.M.T.)$$

(6) Compute the apparent declination for the Greenwich mean time of the observation.

$$\delta = \delta_0 \pm \Delta\delta$$

(7) Compute the true altitude h of the sun's center by correcting the observed vertical angle h' for the index error, re-

**American Nautical Almanac*, published for each year by Nautical Almanac Office, U. S. Naval Observatory, Washington, D. C.

fraction and parallax, and the sun's semi-diameter. The sun's semi-diameter may be taken as 16′. (For a closer value consult the Nautical Almanac.) The correction for parallax and refraction depends upon the altitude. It may be found in Table I, page 243. Calculate the latitude by the equation

$$\phi = 90^\circ - h + \delta$$

Follow the accompanying form, page 128.

(8) Some of the pocket solar ephemerides give declinations for *Greenwich apparent noon*. If the ephemeris used is of this kind, after step (3) change Greenwich mean time to Greenwich apparent time by equation

$$G.A.T. = G.M.T. \pm E$$

in which E is the equation of time or difference between mean solar time and apparent solar time at the Greenwich apparent time of observation. The value of E is found by interpolating. The values given in the table headed "*Equation of Time*" are for Greenwich apparent noon. The sign of the equation of time as expressed at the head of the column is for changing *apparent* time to *mean* time; since here the reverse operation is desired, the opposite sign must be used. The operation of finding the declination will now be similar to that explained in steps (4), (5), and (6), except that Greenwich apparent time will be used. Since the maximum rate of change of declination is less than 01′ per hour and since the equation of time is always less than 17^m, the changing of mean time to apparent time may often be neglected.

Hints and Precautions.—(1) Particular care should be taken to apply corrections with their proper sign. The parallax and refraction correction will *always be negative;* the semi-diameter correction will be positive when the lower limb of the sun is observed; and the index error will depend upon the instrument used. (2) If the longitude of the place of observation is known, the time of observation need not be determined. Thus, if the hourly change in declination be denoted by c, the apparent declination is

$$\delta = \delta_0 + c\lambda$$

in which λ is the longitude *in hours*. (3) Prismatic eyepieces with colored shades are usually furnished with instruments for use in solar observations. In case none is to be had,

14.

Prob. 50. Latitude of Town Hall, Gorham, Me.

Observation on Sun at Apparent Noon.

Field Work.

Circle	Obj.	Time	V. Circle	Index E.	
L	"A"		+2°58'30"		h_1
R	"A"		+2°55'30"		h_2
				-0°01'30"	
L	☉	11ʰ34ᵐ0ˢ	48°05'00"		h'

Computations.

Watch	Time		11ʰ 34ᵐ 01ˢ
"	Slow		29
G.M.T.	(E.S.T. + 5ʰ)	4ʰ	34 30
Eq. of Time	(from Ephemeris) +		04 40
G.A.T.		4ʰ	39ᵐ 10ˢ
δ_o	(Declin. at G.A. Noon)	3°	09' 59.3"
$\Delta\delta$	(57.7" × 4.65)	−	04' 18.5"
δ		3°	05' 40.8"
Obs.	h' on ☉	48°	05.'0
Index-error		−	01.'5
Ref. & Parallax (Table I)		−	00.'7
Sun's Semi-Diameter		+	15.'9
h	(Corrected Altitude)	48°	18.'7
δ		3°	05.'7
ϕ	$= 90° - h + \delta$	44°	47.'0

Wm. Bolton, H. L. Brown } Observers.
Sept. 15, 1913.
Remarks. Fair, Warm, Calm.
"A" is Mark on Barn
400 ft. South.
B. & B. Transit No. 142.
See Note.
Waltham Watch.
By Watch.
29ˢ slow E.S.Time.

NOTE:- Index-Error found by Reversal on Point "A"

Index E. $= -\frac{(2°58'30") + (2°55'30")}{2}$

" " $= -0°01'30"$ with Circle Left.

Ephemeris gave Values for G.A. Noon

Latitude of Town Hall.

SAMPLE NOTES 50.

and the sun is not at a high altitude, very good observations can be made by allowing the sun's image to come to focus on a white card held several inches from the eyepiece. A pointing on the sun should be made as nearly as possible by sighting over the telescope; the eyepiece should then be drawn out and the objective focused until the sun's image and the cross-hairs are clearly seen on the card. If the eyepiece of the telescope is erecting, the image on the card will be inverted; if the eyepiece is inverting, the image will be erect. (4) In using the card with an instrument equipped with stadia hairs, the observer should make sure that the *middle* cross-hair is brought tangent to the sun; this can be ascertained by revolving the telescope a few minutes in altitude until all the cross-hairs have been seen. (5) With a transit having a full vertical circle the index error may be most accurately found by noting the vertical circle readings to some well-defined point with the telescope in both positions (circle right and circle left). This will not only take into account the errors due to vertical axis not being truly vertical and zero of vernier's being displaced, but will also dispose of the error due to line of sight's not being parallel to axis of telescope bubble-tube. The determination of the index error by this method is shown in Sample Notes 50, h_1 and h_2 being the observed vertical angles to the mark. The reason for the sign of the correction should be ascertained. (6) Some engineering instrument companies issue pocket ephemerides for use with the solar attachment. There is on the page for each month a table of refraction corrections with argument expressed in hours before or after noon. It should here be noted that *these refraction corrections are to be applied to apparent declinations (when using the solar attachment), not to observed altitudes.* (7) In the United States the watches and clocks usually keep standard time. Eastern, central, mountain, and western standard times are respectively five, six, seven, and eight hours slower than Greenwich mean time. Thus when it is 12 o'clock central standard time it is 6 o'clock Greenwich mean time.

Practical Applications.—The above method provides a rough value of the latitude for use in determining the meridian by astronomical observations. Under favorable conditions the error need not exceed 02′, and the mean of a series of observations will render a closer determination.

Problem 51. Azimuth by Direct Solar Observation

Object.—To determine the true azimuth of a line by an observation of the sun with the engineers' transit.

Procedure.—(1) Set up the transit over one end of the line and level very carefully. If the vertical vernier is not adjustable, note the index error of the vertical circle with the telescope level and pointing in the general direction of the sun. Set the A vernier to read zero on the horizontal circle, and note the reading of the B vernier. Sight along the line, and clamp the lower motion. (2) Loosen the upper motion, and bring the vertical and horizontal cross-hairs tangent to the sun in the upper left-hand quadrant if the observation is in the forenoon, in the lower left-hand quadrant if in the afternoon. Record the time of tangency and horizontal and vertical circle readings, reading all verniers. (3) Loosen the upper motion, plunge the telescope, and bring the vertical and horizontal cross-hairs tangent to the sun in the lower right-hand quadrant if in the forenoon; in the upper right-hand quadrant if in the afternoon. Read and record the horizontal and vertical angles, and note the time of observation as before. (4) Loosen the upper motion, and again sight along the line with the instrument in the plunged position. Record the horizontal circle readings. (5) Compare the watch used with a time-piece keeping correct standard time, and note the correction. (6) Compute the Greenwich mean time of observation by adding five, six, seven, or eight hours (according to the time belt in which the place of observation is located) to the corrected mean of the times of observation. (7) From the Nautical Almanac or Solar Ephemeris, determine the apparent declination δ of the sun for the Greenwich mean time of observation as explained in the previous problem. (8) If the ephemeris gives values of the declination for Greenwich apparent noon, follow Procedure (8) of the previous problem. (9) Correct the mean of the observed altitudes for index error, and refraction and parallax. The latter correction may be found in Table I, page 243, and is applied as in the previous problem. (10) Compute the azimuth of the line (using five-place logarithmic tables) by the formula

$$\text{Cos } A = \frac{\sin \delta}{\cos h \cos \phi} - \tan h \tan \phi$$

in which ϕ is the latitude of the place, h is the corrected mean altitude to the sun's center, and A is the angle east or west of south according as the observation is made in the morning or afternoon. (11) Calculate the azimuth of the line by algebraically subtracting the mean of the observed horizontal angles from the computed azimuth of the sun, angles taken in a clockwise direction from reference line to sun being considered as positive. (12) Make several similar observations. Determine the most probable value of the azimuth and the probable error of a single observation and of the mean (Procedure (5), page 98). See Sample Notes 51, page 132.

Hints and Precautions.—(1) The time between sights should be short, but the observer should not read the vernier carelessly in an endeavor to hurry. An interval of five minutes between sights will not cause an appreciable error in the final result. (2) The reason for taking one pointing with the instrument direct and the other with the instrument inverted is to eliminate instrumental errors. As an additional precaution, eccentricity of circles is eliminated by reading both verniers. If the transit has two vertical verniers, they should both be read and a mean value taken. (3) On account of the size of the sun, a sight can not be taken to its center with any degree of precision; but by taking the mean of the observations as already described, the position of the sun's center is determined with accuracy. If for any reason, however, it is desired to obtain the azimuth of a line by a single pointing, the sun may be brought tangent in any of the quadrants. The altitude correction for the sun's center will then be the amount of the sun's semi-diameter or approximately 16′;* the horizontal angle correction will be equal to $16' \times$ *secant of the altitude.* (4) Great care should be taken in leveling the instrument before making the observation; first level with the plate bubbles and then, as a final test, see that the telescope bubble remains in the center of the tube when the instrument is revolved about its vertical axis. (5) If no solar eyepiece is to be had, a card held in the rear of the eyepiece may be used as explained in the previous problem. If the eyepiece of the telescope is erecting, the image on the card will be inverted; thus the image of the sun will appear in the upper right-hand quadrant when it is in the lower left-

* For a closer determination of the sun's semi-diameter see Nautical Almanac.

15.

Prob. 51. Azimuth of Line Δ 46 – Δ 63

Circle	Object	Direct Time	Solar Vert. Cir.	Observation Horiz. Vert A	Horiz. Circle Vert B
				0°00'00"	180°00'00"
R	Δ 63				
R	☉	8h42m46s	34°46'	331°37'40"	191°38'20"
L	☉	8h47m23s	34°55'	152°05'40"	332°05'20"
L	Δ 63			179°59'40"	359°59'20"
Watch	Time	8h45m05s		– LOGS –	
"	Slow +	33		sin δ	8.73595
C.S.	Time	8h49m38s		cos h	9.91431
G.M.T.		2h45m38s		cos φ	9.85112
Eq.	Time +	04 42.1			8.97052
G.A.T.		2h50m20s		tan h	9.84235
δ_n		3°09'39.3"		tan φ	9.99672
Δ δ	–	02'43.7"			9.83907
δ		3°07'15.6"	– NUMBERS –		
h'		34°50.5'	Diff. δ / cos h cos φ		0.09344
Ref. & Par.	–	01.2	– tan h tan φ		0.69034
h		34°49.3'	cos A = –		0.59690
			A =		126°39'
			H =		28°08'
					154°47'

Duren & Co. Transit. Watch 33s Slow. C.S. Time.

J.N. Mc Dougal. W.W. Wilson. Sept. 15, 1913. A.M. Hot, Windy.

Hor. Angs. on Azimuth Circle.

Note: Ephemeris gave Values for G.A. Noon.

Azimuth of Sun from North.
Angle from Line to Sun.
Azimuth of Line (from North)

SAMPLE NOTES 51.

hand quadrant. If the eyepiece is inverting, the image will appear in its true position. (6) For good results, observations should be taken between the hours from 8 to 10 a.m. and 2 to 4 p.m.; for, if taken when the sun is near the horizon, the refraction correction is large and uncertain, and if taken near the hour of noon, the resulting astronomical triangle will be poorly proportioned. Under favorable conditions of atmosphere and time, the azimuth of a line may be determined within 01′ by a single observation. A series of observations will render a much closer determination.

Practical Applications.—The method of determining the meridian by a single altitude of the sun is much used in surveys where a high degree of accuracy is not necessary. Although more accurate results may be obtained by an observation on Polaris, this necessitates work at or near night, which is not always desirable. The solar attachment makes it possible to turn to the meridian at the time of the observation and eliminates the solution of the spherical triangle; but the direct method possesses the advantage that no additional instrumental equipment is needed besides being the more accurate method. At the present time the direct method is looked upon by surveyors with increasing favor.

Problem 52. Azimuth with Saegmuller Solar Attachment

Object.—To determine the true azimuth of a line by a single observation with the Saegmuller solar attachment.

Procedure.—(1) Look up the sun's declination for the Greenwich mean time of the proposed observation as explained in Problem 50, and correct the result for refraction by Tables III and III(*a*), pages 245 and 247, *adding* the correction if the declination is *positive* and *subtracting* if the declination is *negative*. (2) Set up the transit, with solar telescope attached, over one end of the line the azimuth of which is to be determined, carefully leveling the instrument. Note the index error of the vertical circle. (3) Sight the transit telescope at some well-defined point, and bring the lines of sight of both telescopes into the same vertical plane by a similar pointing with the solar telescope. (4) Set the horizontal vernier to read zero. (5) Incline the transit telescope an amount equal to the *corrected declination* (allowing for index

error if such exists), depressing the objective end if the declination is north and elevating it if the declination is south. Level the solar telescope without disturbing the transit. Set off

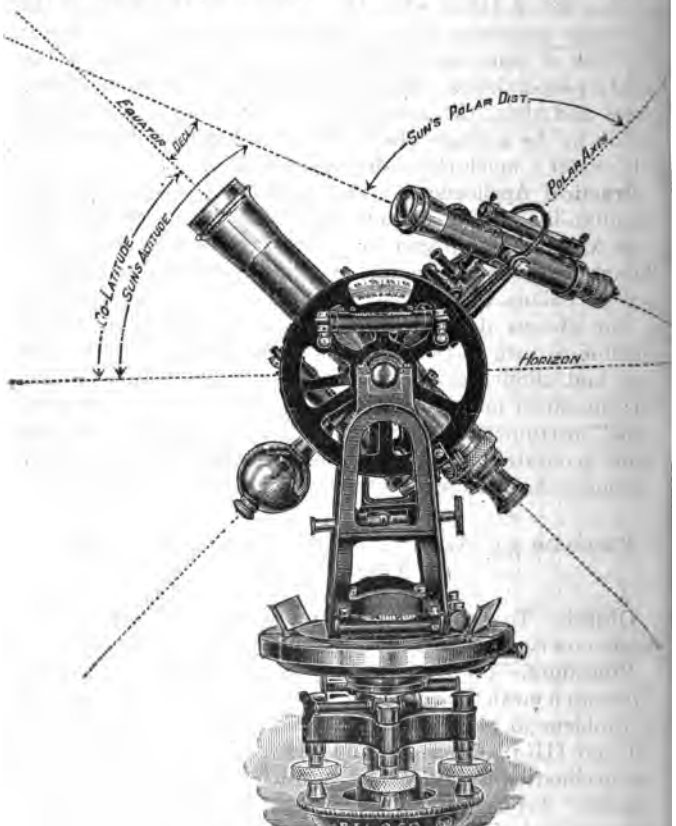


PLATE IX.—SOLAR ATTACHMENT MOUNTED ON TRANSIT.
(In position for determining the meridian.)
(From an electrotype loaned by Bausch & Lomb Optical Co.)

a positive angle on the vertical circle equal to the co-latitude of the place (allowing for index error). (6) Point the transit telescope as near south as can be determined with the mag-

netic needle. With the lower motion unclamped, and with the solar telescope free to revolve about the polar axis, bring the sun into the center of the lined square of the solar telescope near the time for which the declination was computed. Clamp the lower motion and the polar axis. Make an exact setting by moving simultaneously the tangent-screw controlling the lower motion of the transit and the tangent-screw controlling the movement of the solar telescope about the polar axis. If no mistake has been made in laying off the co-latitude or declination, the line of sight of the large telescope will lie in the plane of the meridian and the polar axis will point toward the pole. If this condition is realized, since the declination changes very slowly, it will be possible to keep the sun in the center of the square for some time by simply revolving the solar telescope about the polar axis. *This should be done as a check.* Plate IX shows the solar attachment mounted on transit with the line of sight of the transit pointing south in the plane of the equator and the solar telescope pointing at the sun. (7) Loosen the upper motion, and turn to the line. Read the horizontal angle, and record the azimuth of the line. (8) Repeat the observation for other times until several satisfactory determinations have been made. (9) Compute the most probable value of the azimuth, and probable error of a single observation and of the mean (Problem 34, Procedure 5, page 98). If conditions are similar, compare the results with those of the previous problem.

Hints and Precautions.—(1) Many of the suggestions of the previous problem apply to observations with the solar transit. (2) The solar attachment should be in perfect adjustment, and vertical angles should be set off very carefully, for an error in the declination or co-latitude will be increased in the meridian determination. (3) The refraction corrections to be applied to declinations for latitude 40° with arguments in hours before and after noon may be found in Table III, p. 245. For latitudes other than 40° the refraction may be found by multiplying the value for 40° by the coefficient corresponding to the latitude of the place of observation, Table III(*a*), p. 247. These tables are particularly adapted to observations with the solar transit, and it should be noted that *these corrections are for declinations, not altitudes.* (4) To the beginner, the operation of getting the sun into the center of the lined square of the solar telescope presents some difficulties. This can best be done as

follows: Point the solar telescope (as near as can be judged) along a vertical plane through the sun. Hold it in that position with one hand, and with the other hand revolve the transit about its vertical axis. Since the solar telescope is free to revolve about the polar axis, the effect of this movement will be to elevate or depress the line of sight. The sun will soon appear in the field, and a final setting may be made as previously explained. (5) Where observations are likely to extend over some time, a table of declination settings should be made out before going into the field. Declinations, corrected for refraction, should be computed for intervals of time depending upon the rate of change of declination and the least reading of the vertical circle. Thus, if the vertical vernier reads to 20″ and the hourly change in declination is 58″, declinations should be calculated at intervals of say 15^m. (6) The method of procedure with other attachments is essentially the same as with the Saegmuller solar attachment. On the Burt solar attachment, the declination is laid off on a declination arc, and the sun's image is brought to the center of a ruled square on a silver disc.

Practical Applications.—As a quick method of determining the meridian in the field the solar attachment possesses a decided advantage over the ordinary transit. Most engineers, however, agree that its cost and the bother of adjusting it more than offset the advantage gained, unless it is to be used very frequently, as is sometimes the case in mining and city surveying. Under the most favorable circumstances its accuracy is less than that to be obtained by the direct method of observation. For this reason most engineers do not view the use of the solar attachment with favor.

Problem 53. Time by Observation on the Sun at Noon

Object.—To determine the correction of a time-piece by observing the sun as it crosses the meridian. It is assumed that the longitude of the place of observation has already been determined either by astronomical observations or from a map; and that a meridian has been established.

Procedure.—(1) Set up the transit over the north end of the established meridian, carefully level the instrument, and sight along the meridian. Elevate the line of sight to in-

Prob. 53. Time By Observation
Meridian at Observatory, Urbana.

Circle	Object	Time
L	☉ 20	
L	☌	$12^h07^m32^s$
R	☉ 20	
R	☽	$12^h09^m49^s$

Time of Passing of Suns Ctr.	$12^h08^m40.5^s$	
Eq. of Time at G.A. Noon	−	15 40.8
Longit. x Hourly Ch. (5.87 x 0.34)	+	02.0
Watch Time of L.M. Noon	$11^h53^m01.7^s$	
Δ T_m	+	06^m58^s
Δ T_s	−	08^s

16.

On Sun at Noon, Nov. 13, 1913.

Young & Sons Transit. J.C. Moore.
Watch. E. Williams.
Fair, Cold.

So End of Meridian.

" " " "

Note - Longitude of Observatory $5^h52^m54.1^s$

From Ephemeris

" "

Correction to Local Mean Time

" " Central Std. "

SAMPLE NOTES 53.

tercept the path of the sun. (2) At the instant of tangency between the western limb of the sun and the vertical cross-hair, note the time. (3) Quickly plunge the telescope, reverse in azimuth, and again sight along the meridian. Elevate the telescope, and note the time when the vertical cross-hair is tangent to the eastern limb of the sun. The mean of the two times will be the time of transit of the center of the apparent sun over the meridian. (4) If the longitude of the place of observation is expressed in degrees, change to hours by dividing by 15. Since the sun transits at apparent noon, and longitude is reckoned from Greenwich, the longitude in hours is the Greenwich apparent time of observation. (5) From the Nautical Almanac or Solar Ephemeris determine the equation of time for the Greenwich apparent time of observation. Then if T be the observed time, E the equation of time, and ΔT_m the correction of the time-piece to local mean time,

$$\Delta T_m = 12^h - (T \pm E)$$

The sign of the equation of time will be positive or negative according as mean solar time is slower or faster than apparent solar time. (6) If the correction to give standard time is desired, it may be found by the equation

$$\Delta T_s = \Delta T_m \pm \Delta\lambda$$

in which $\Delta\lambda$ is the difference in longitude in hours between the place of observation and the meridian at which standard time and local mean time agree. If the place of observation is east of this meridian, the sign of $\Delta\lambda$ will be negative; if west, positive.

Hints and Precautions.—(1) The length of time taken by the sun in crossing the meridian will depend somewhat upon its declination and semi-diameter but will not vary much from two and one-third minutes. The observer should, therefore, lose no time while reversing the transit for a second sight along the meridian and the sun's eastern limb. (2) If for any reason it is impracticable to observe but one limb, the time in seconds earlier or later that the sun's center passes the meridian is approximately $\frac{4\,S}{\cos\,\delta}$, in which S is the sun's semi-diameter in minutes of arc (approximately 16′) and δ is the declination of the sun. It is best, however, to take the time of passage of both limbs with a reversal of the instrument be-

tween sights, for by so doing, instrumental errors are eliminated. (3) If the transit is not equipped with eyepiece for astronomical observations and the sun is not too high, a card held back of the eyepiece may be used as explained in Problem 50, p. 127. (4) If the ephemeris used gives values for Greenwich mean noon, an exact determination of the equation of time will necessitate changing the Greenwich apparent time of observation (Procedure (4)) to Greenwich mean time by means of an approximate equation of time. The true equation of time may then be found by interpolating between values given for the preceding and succeeding mean noons. Since the equation of time is always less than 17^m and since its rate of change is relatively small, for ordinary observations for the purpose of determining time, the change from apparent to mean time need not be made. (5) The problem may be readily reversed; if the time is accurately known, the longitude may be computed.

Problem 54. Azimuth and Longitude by Solar Observation

Object.—To determine the azimuth of a line and longitude of the place of observation by an observation on the sun with the repeating theodolite. It is presumed that the theodolite used is a high-grade instrument, carefully adjusted, and that the time is taken with an accurate time-piece keeping standard time.

Procedure.—(Fieldwork similar to that of Problem 51, p. 130.) (1) Between the hours of 8 and 10 a.m. set up the instrument at one end of the line whose azimuth is to be determined. Level the instrument very carefully. (2) With upper motion clamped, sight along the line, clamp the lower motion, and record the horizontal circle readings (circle right). (3) Loosen the upper motion, reverse the telescope, and again sight along the line. Record the horizontal circle readings as before (circle left). (4) Loosen the upper motion, and turn to the sun, bringing the sun tangent to the cross-hairs in the upper left-hand quadrant (circle left). Record the time at the instant of tangency and the horizontal and vertical circle readings. (5) Loosen the upper motion, and again sight on the sun, repeating step (4) (circle left). (6) Loosen the upper motion, reverse the telescope, and bring the sun into the lower right-

15.

Prob. 54. Azimuth and Longitude at White Rock, Ky.

Observation on Sun.

Field Work

A.M.

Circle	Obj.	Time	Horizontal Ver. A	Horizontal Ver. B	Circle Mean
R	B		352°37'40"	38 00	352°37'50"
L	B		172 37 20	37 00	37'10"
Mean					352°37'30"
L	⊙	9h35m15s	291 40 00	40 20	111 40 10
L	⊙	36 10	291 49 00	49 00	111 49 00
R	☉	37 40	112 45 20	45 40	112 45 30
R	☉	38 47	112 56 20	56 40	112 56 30
Means		9 36 58 0			112°17'50"
Mean Hor. Ang. Line to Sun.					119°40'20"
R	☉	9 39 46	113 06 10	06 20	113 06 15
R	☉	40 49	113 17 00	17 00	113 17 00
L	⊙	42 20	112 49 00	49 00	112 49 00
L	⊙	43 24	112 59 10	59 20	112 59 15
Means		9 41 34 8			113°02'50"
Mean Hor. Ang. Line to Sun.					120°24'30"
L	B		172 38 00	37 30	352 37 45
R	B		352°38'50"	39 10	39 00
Mean					352°38'20"

Lat. 37°03'35"

J.M. Kirk
F.C. Camp.
July 2, 1901.
Tamp. 97°F.
Berger Theod. No. 43.
Chronometer 06'8 – Fast.
E.S. Time.

Vertical Circle Ver. A	Ver. B	Mean
44°17'30"	17'30"	44°17'30"
44 29 00	29 00	44 29 00
45 11 00	11 30	45 11 15
45 24 00	24 30	45 24 15
		44°50'50
Pan & Ref. –		0'76
45 36 00	36 00	45 36 00
45 48 30	49 00	45 48 45
45 41 30	42 00	45 41 45
45 54 30	54 30	45 54 30
		45°45'26
Pan & Ref. –		0.76

Note: Hor. Angs. are Clockwise from Line A+B to ⊙

SAMPLE NOTES 54.

and quadrant (circle right). Record the time and readings of circles. (7) Repeat step (6) (circle right). The four readings taken to the sun thus far constitute a set which for convenience will be called Set 1. (8) Without again turning to the line, sight on the sun taking the first two observations with circle right and the last two, with circle left. (9) Again turn to the line (circle left) and record the horizontal circle readings. (10) Reverse the telescope and repeat (9) (circle right). (11) Record the temperature. This completes Set 2. (12) Between the hours of 2 and 4 p.m. take two more sets in a similar manner except that the sun may best be observed in the upper right-hand and lower left-hand quadrants. (13) Compute the mean values of horizontal and vertical angles and time, for each set. (14) Correct each mean altitude for parallax and refraction (Table I, page 243) and look up the apparent declination for the Greenwich mean time of each set. (15) Calculate the azimuth of the sun for the mean time of each set by the equation

$$\text{Cot}^2 \tfrac{1}{2} A = \frac{\sin (s - \phi) \sin (s - h)}{\cos s \cos (s - p)}$$

in which A is the angle to the sun east of south in the morning, west of south in the afternoon; ϕ is the latitude of the place of observation; h is the corrected altitude; p is the polar distance of the sun or $(90° - \delta)$; and $s = \tfrac{1}{2}(\phi + h + p)$. Compute the mean azimuth of the line. (16) Determine the azimuth of the line by combining these results with the mean of the corresponding horizontal angles to the sun as shown in the sample computation. (17) Calculate the hour angle of the sun at the mean time of each set by the equation

$$\tan \tfrac{1}{2} t = \frac{\sin (s - h)}{\cot \tfrac{1}{2} A \cos (s - p)}$$

in which t is the hour angle expressed in degrees of arc. This expressed in hours is the local apparent time. (18) Convert into local mean time by applying the equation of time, the latter being added or subtracted according as mean time is faster or slower than apparent time. (19) Calculate the longitude of the place of observation. The difference between this local mean time of each set and the correct standard time of each set is the difference in longitude expressed in hours between the place of observation and the meridian at which

Prob. 54. (Cont'd.) Azimuth and Computations.

	Set	1	2	3	4
	h	44°45'.7	45°44'.5	44°05'.3	41°28'.5
	φ	37 03.6	37 03.6	37 03.6	37 03.6
	p	66 55.5	66 55.5	66 56.7	66 56.8
	2 s	148°48.8	149°43.6	148°05.6	145°28.9
	s	74 24.4	74 51.8	74 02.8	72 44.4
	s-p	7 28.9	7 56.3	7 06.1	5 47.6
	s-h	29 34.7	29 07.3	29 57.5	31 15.9
	s-φ	37 20.8	37 48.2	36 59.2	35 40.8
Check	2 s	148°48.8	149°43.6	148°05.6	145°28.7
colog	cos s	0.57056	0.56316	0.56090	0.52767
colog	cos(s-p)	0.00371	0.00418	0.00334	0.00222
log	sin(s-h)	9.69339	9.68723	9.69842	9.71516
log	sin(s-φ)	9.78293	9.78743	9.77933	9.76586
log	cot²½A	0.05059	0.06200	0.04199	0.01091
log	cot ½A	0.02530	0.03100	0.02100	0.00546
Az. of	⊙ from N	93°20'.2	94°05'.2	267°13'.8	269°16'.8
Ang. ⊙ to	Line	240 19.7	239 35.5	66 27.1	64 23.4
Az. of	A-B	333°39'.9	333°40'.7	333°40'.9	333°40'.2
Mean	Az	333°40'.4			
Cal.	Bear.	N 26°19'.6 W			

17.

Longitude at White Rock, Ky.

For Field Notes See P 15 & 16.
Computed by J.C. Kirk.
Checked by F.C. Camp.
July 2, 1901.

	Set	1	2	3	4
colog cos(s-p)		0.00371	0.00418	0.00334	0.00222
log sin(s-h)		9.69339	9.68723	9.69842	9.71516
colog cot½A		9.97470	9.96900	9.97900	9.99458
log tan²t		9.67180	9.66041	9.68076	9.71192
t°		50°19'.00"	49°10'.12"	51°13'.51"	54°10'.32"
t^h		-3^h 21^m 16^s.0	-3^h 16^m 40^s.8	-3^h 24^m 55^s.8	-3^h 36^m 01^s.3
E		3 40.2	3 40.2	3 43.4	3 43.5
Local M.T.		8 42 24.2	8 46 59.4	8 28 39.2	8 41 45.6
Obs. Time		9 36 58.0	9 41 34.8	9 23 10.8	9 36 27.8
Δ T_m		- 54 33.8	- 54 35.4	- 54 31.6	- 54 37.2
Δ T_s		- 06.8	- 06.8	- 06.9	- 06.9
Δ λ		54^m 27.0	54^m 28.6	54^m 24.7	54^m 30.3
Mean		54^m 27.6 = 13°56.9			
λ	88°36'.9				

standard time is also local mean time. If local mean time is ahead of standard time, the place of observation is to the east of the standard meridian, and the longitude is, therefore, less; if behind, to the west, and the longitude is greater. Record notes and computations as shown in Sample Notes 54 and Sample Computation 54, pages 140 and 142.

Hints and Precautions.—(1) Observe the hints and precautions of Problem 51, page 130. (2) Notice that Sample Notes 54 has not included the afternoon observations for which computations are shown.

Practical Applications.—The above method is employed by the U. S. Coast and Geodetic Survey in connection with magnetic observations. For longitude determination a reliable time-piece is of the utmost importance, since error in time will make a corresponding error in longitude. By this method, under favorable conditions the error in azimuth determination need not exceed 10″, and in longitude determination, 30″.

Problem 55. Latitude by Observation on Polaris at Culmination

Object.—To determine the latitude of the place by observing Polaris at upper or lower culmination.

Procedure.—(1) Determine the approximate local mean time of culmination of Polaris by Table IV, page 248. Change to standard time. Look up the star's polar distance $p = 90° - \delta$ in Table VIII, page 261. The polar distance may be determined more accurately by looking up the value of the declination δ in the Nautical Almanac (Table I). (2) A few minutes before culmination set up the transit leveling it very carefully (as a final test the telescope bubble should remain centered for all pointings) and note the index error of the vertical circle or adjust vernier to read zero. (3) Locate the star with the naked eye, and if the latitude is roughly known, set off its value, plus or minus the star's polar distance, on the vertical circle. Sight along the barrel of the telescope to range in the star; sight through the telescope and move the instrument about the vertical axis until the star is near the center of the field. (4) Illuminate the cross-hairs, and follow the star until it no longer appears to move off the horizontal hair but moves along it. Note the time, and record the vertical angle as quickly as possible. (5) Plunge the telescope, and

again sight on the star. Record the time and vertical angle as before. (6) Correct the mean of the observed vertical angles for refraction (Table II, page 244) and index error if such exists; and compute the latitude, $\phi = h \pm p$, in which the sign of p is positive or negative according as the star is at lower or upper culmination.

Hints and Precautions.—(1) The mean of the times of observation should agree closely with the computed time of culmination; this furnishes a check upon the star sighted. (2) Observations are taken with the telescope in both positions to eliminate instrumental errors. This would, of course, be impossible with a transit not equipped with a full vertical circle. (3) Knowing the local time of culmination, to find the standard time of culmination, add or subtract the difference in longitude expressed in hours between the place of observation and the standard meridian, *i.e.*, the place in the time belt of the observation where standard and local mean time agree, as the 75th meridian for eastern standard time, or the 90th meridian for central standard time. The difference in longitude will be added to local mean time of culmination if the place of observation is to the west of the standard meridian; and subtracted, if it is to the east. The longitude of the place of observation may be determined with sufficient accuracy for the problem from any good map. (4) Some transits are furnished with reflectors with which to illuminate the cross-hairs at night. The illumination is accomplished by holding a light to one side of the telescope. With the ordinary transit not so equipped the cross-hairs can be illuminated sufficiently by holding the light a few inches in front of the objective and a little to one side of the telescope barrel, thus causing the rays of light to enter the telescope diagonally. If held so that the rays strike the eye of the observer directly, the light will be so strong as to cause the star to disappear. A drop of candle grease in the center of the objective lens is also often used. The grease should be shaved to a very thin layer. (5) It should be noted that, while a double sight eliminates error due to line of sight not being parallel to axis of bubble, it will not eliminate error in the vertical circle reading due to the vertical axis not being truly vertical, and for this reason the instrument should be leveled with great care. (6) The position of Polaris in the sky may be easily determined by means of the two neighboring constellations, Cassiopeia and Ursa Major

(Fig. 31). The seven most brilliant stars of the latter constellation are known as the Great Dipper; the two stars forming that part of the bowl farthest from the handle are called the "*pointers*" because a line through these stars points very nearly at the Celestial Pole. Since Polaris is only a little more than a degree distant from the Pole, and since there are no other bright stars in its vicinity, the correct star can be readily

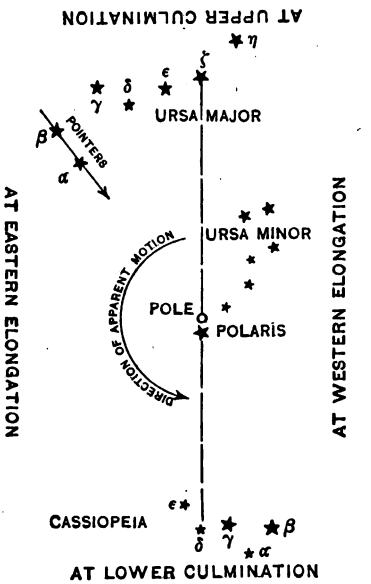


FIG. 31.—POSITIONS OF CONSTELLATIONS NEAR THE NORTH POLE WHEN POLARIS IS AT CULMINATION AND ELONGATION.

picked out with the eye. (7) If tables for determining the time of culmination of Polaris are not at hand, the position of the star relative to the Pole may be roughly estimated by noting the position of the stars δ Cassiopeia and ζ Ursa Major. A line through these stars passes very near Polaris.

When δ Cassiopeia and ζ Ursa Major (Fig. 31) are in the same vertical plane, Polaris is at upper or lower culmination, depending on whether δ Cassiopeia is above or below the Pole. (8) Observations may be taken on Polaris some time before it is dark enough to discern the star with the naked eye, provid-

ing the objective is properly focused and the air is clear. Government surveyors who find it necessary to make frequent observations, often mark on the barrel of the telescope the position of the objective slide for this focus. When the air is very clear, as in high altitudes, it is possible in this way to observe the star when the sun is still above the horizon. A method of focusing the objective often used when it is desired to observe near dusk is to get a clear definition of the evening star which can be clearly seen with the naked eye before sunset. The objective should be moved toward or from the cross-hairs until the star appears as a fine point.

Practical Applications.—Same as for Problem 50, p. 126. The method will, however, give more reliable results, and should be used when an error greater than 01′ is undesirable.

Problem 56. Azimuth by Observation on Polaris at Elongation

Object.—To determine the azimuth of a line by observation on Polaris at eastern or western elongation.

Procedure.—(1) Look up the local mean time of elongation in Table IV, p. 248, and change to standard time as explained in Problem 55. (2) A few minutes before the star is at elongation, set up the transit very carefully over one end of the line the azimuth of which is to be determined. (3) With the objective focused so that the star appears as a fine point, and the cross-hairs illuminated, follow the star until it no longer appears to travel horizontally but moves along the vertical hair. Depress the telescope, and carefully set a point at least 300 ft. distant. (4) Reverse the instrument, and again sight on Polaris. Depress the telescope, and set a point beside the first. If the instrument is in good adjustment, these two points may coincide; if they do not coincide, set a point midway between them. (5) Look up the azimuth of Polaris in Table V, p. 250. (6) By daylight determine by repetition (Problem 18, page 62) the angle between the line established by setting points beneath the star and the line of which the azimuth is desired. Calculate the azimuth of the line.

Hints and Precautions.—(1) The hints and precautions of Problem 55 should be noted. Fig. 31, p. 145, shows the relative positions of the neighboring constellations, Ursa Major and Cassiopeia, when Polaris is at elongation. (2) It is not essential that the exact latitude be known. It will be seen from Table

V that an error of 1° of latitude will make a relatively small error in the deduced azimuth. It will therefore, for ordinary observations, be sufficient, if the latitude of the place of observation has not been previously determined, to estimate it from a reliable map; or having no map, to assume that it is the vertical angle to the star at time of elongation. In the latter contingency it will, of course, be necessary to slightly alter the course of procedure outlined. (3) The azimuths given in Table V are deduced from mean values of the declination for each year. If very accurate results are desired, the apparent declination of Polaris for the date of observation should be determined from the Nautical Almanac; careful observations for latitude should be made (Problem 55); and the azimuth of the star should be computed by the formula $\sin A = \frac{\cos \delta}{\cos \phi}$ in which δ is the apparent declination and ϕ is the latitude. If the latitude is accurately known, the resulting azimuth should be in error less than 05″. Six place logarithms should be used. (4) If an error of 01′ in the calculated azimuth of the line is allowable, points need not be set beneath the star. Instead, the horizontal angle between the line of reference and the line of sight to the star may be measured with circle right and circle left as in direct solar observations (Problem 51, p. 130). (5) A very satisfactory target for night work is a strongly illuminated card upon which a line has been drawn of sufficient weight to be clearly seen through the telescope from the observer's station. The source of illumination should be shielded from the observer, for otherwise the target will appear indistinct. (6) The time between sights to the star should be as short as possible, but there is no necessity for extreme haste in setting the ground points, since the azimuth of the star is changing very slowly. At latitude 40° this change amounts to about 01.5″ when the star is 05^m from elongation, and about 06″ when the star is 10^m from elongation. (7) If it is necessary to observe Polaris when not at elongation but *near* that position, the correction to be applied to the azimuth at elongation is expressed approximately by $C = 0.06n^2$ (for observations south of latitude 50°) in which n is the time in minutes before or after elongation and C is in seconds of arc.*

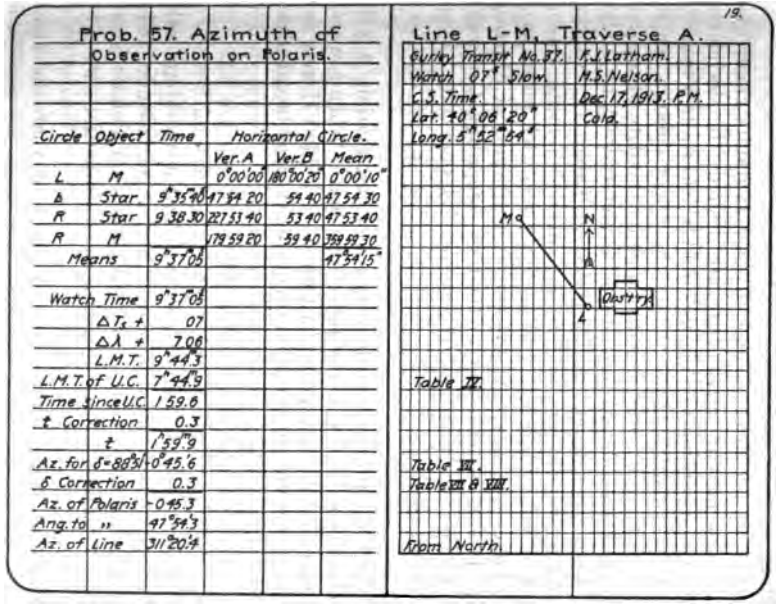
* More exactly the correction is $0.052n^2$ for latitude 30°, $0.058n^2$ for latitude 40°, and $0.069n^2$ for latitude 50°.

Practical Applications.—Because azimuth determination by observation on Polaris does not depend upon a time-piece and because the horizontal movement of the star is comparatively slow near the time of elongation, this method is superior to all others in point of precision. With modifications it has been extensively used on the U. S. Coast and Geodetic Survey, the Great Lakes Survey, the U. S. Geological Survey, and many other surveys of less magnitude. On some surveys this method is used to the exclusion of all others. Its chief disadvantage is that under the most favorable conditions observations can be taken but twice daily, and it often necessitates work at inconvenient hours.

Problem 57. Azimuth by Observation on Polaris at any Time

Object.—To determine the true azimuth of a line by observation on Polaris at any hour when the star can be seen, the latitude and longitude of the place of observation being known. See Sample Notes 57.

Procedure.—(1) Measure the horizontal angle between line and star with circle right and circle left, and note the time of the passing of the star across the vertical cross-hair at each setting as in direct solar observation (Problem 51, page 130). (2) Correct the mean of the two times of observation by comparing the time-piece used with some clock keeping correct standard time. (3) Change this correct standard time to local mean time by adding or subtracting the difference in longitude expressed in hours between the place of observation and the standard meridian (meridian where standard time is local mean time), adding, if the place of observation is east of the meridian; subtracting, if west. (4) From Table IV, page 248, determine the local mean time of the upper culmination of Polaris on the date of observation. (5) Compute the interval in hours before or after upper culmination. If Polaris were moving at the same angular velocity about the pole as the sun, this time interval would be the hour angle before or after upper culmination. In reality, Polaris is gaining on the sun at the rate of about 3.95^m per day or $10'$ per hour. Add this gain to the time interval and the result is the hour angle of the star before or after upper culmination. (6) By Table VII, page 253, determine the azimuth of the star for declination $88°51'$, the above hour angle, and the



Prob. 57. Azimuth of Line L-M, Traverse A.
Observation on Polaris.

Circle	Object	Time	Horizontal Circle. Ver. A	Ver. B	Mean
L	M		0°00'00"	180°00'20"	0°00'10"
L	Star	9h35m40s	47 54 20	54 40	47 54 30
R	Star	9 38 30	227 53 40	53 40	47 53 40
R	M		179 59 20	59 40	359 59 30
Means		9h37m05s			47°54'15"
Watch Time		9h37m05s			
	ΔT_c +	07			
	$\Delta\lambda$ +	7.06			
	L.M.T.	9h44s3			
L.M.T. of U.C.		7h44s9			
Time since U.C.		1 59.6			
τ Correction		0.3			
	τ	1h59s9			
Az. for δ=88°31'		0°45'.6			
δ Correction		0.3			
Az. of Polaris		045.3			
Ang. to "		47°54'3			
Az. of Line		311°20'.4			

19.

Gurley Transit No. 37. F.J. Latham.
Watch 07s Slow. M.S. Nelson.
C.S. Time. Dec. 17, 1913. P.M.
Lat. 40°06'20" Cold.
Long. 5h52m54s

Mo
N
A
Distance
L

Table IV.

Table VII.
Table VIII & VIII.

From North.

SAMPLE NOTES 57.

latitude of the place of observation, interpolating if necessary. By Table VIII, page 261, determine the mean declination for the time of observation, and by last columns of Table VII correct the azimuth for change in declination. (7) Compute the azimuth of the line.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 55 and 56. (2) If results closer than the major part of one minute are desired, the declination and right ascension of Polaris should be ascertained from the Nautical Almanac (Table I) for the upper culmination for the 90th meridian west of Greenwich on the date of observation; and the sidereal time should be determined, by adding the local mean time, the correction of Table III, Nautical Almanac, whose argument is the Greenwich mean time of observation, and the sidereal time of Greenwich mean noon. The latter quantity will be found in the Nautical Almanac, page II for month. For method of computing Greenwich mean time see Hints and Precautions (7), page 129. It can be shown that

$$t = \theta - \alpha.$$

in which t is the hour angle after upper culmination, θ is the sidereal time, and α is the right ascension. When t exceeds 12 hours it must be subtracted from 24. Table VII may now be used as before, or the azimuth of the star may be calculated by the equation

$$\tan A = \frac{\sin t}{\cos \phi \sin \delta - \sin \phi \cos t.}$$

in which A is the angle east or west of the meridian, t the hour angle expressed in degrees of arc, ϕ the latitude, and δ the declination. (3) It should be noted that Polaris is east of the meridian for 12^h before upper culmination; west, for 12^h after.

Practical Applications.—It will be seen that the above method theoretically is not dependent upon the position of the star. From inspection of the preceding formula or by Table VII it will be noted that when the star is near upper or lower culmination any small variation in the hour angle will produce a relatively large variation in the azimuth of the star, the angular rate of change of the star being about 01′ of arc for $2\frac{1}{2}^m$ of time (for latitude 40°). For this reason the method should not be expected to give accurate results when the star is far from elongation unless a trustworthy time-piece is used.

CHAPTER VII

TOPOGRAPHIC SURVEYING

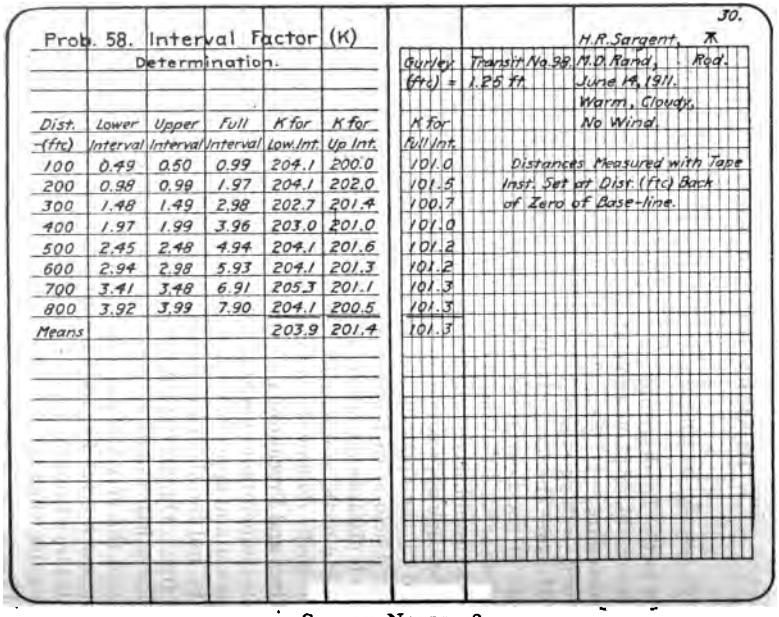
Problem 58. Stadia Interval Factor (Fixed Hairs)

Object.—To determine the stadia interval factor (K) of transit, level, or plane-table alidade.

Procedure.—(1) Establish an approximately level line 800 ft. long, setting stakes by tape measurement every 100 ft. (2) Measure the focal distance (f) and the distance from the center of the instrument to the plane of the objective (c). Set a stake at a distance ($f + c$) back of the zero end of the line just established. (3) Set up over this last point, and with the telescope approximately level, determine the spaces intercepted by the upper and middle, and lower and middle hairs on a rod held at each of the base-line hubs. (4) Calculate K for each distance for upper, lower, and full intervals, and take the mean of the computed values as the factor for the instrument used. Follow the accompanying form, page 152.

Hints and Precautions.—(1) Before laying out the base-line the instrumentman should make certain that the projections of the lower and upper hairs, when the telescope is level, will fall on the rod for all distances up to the length of the proposed base-line. On fair days it is essential that the line of sight of the lower hair should be some distance above the ground because of the very unequal refraction in the layers of air close to the earth. (2) The focal length f is approximately equal to the distance from the plane of the objective to the plane of the cross-hairs when the objective is focused for a distant object. (3) Placing the instrument at a distance ($f + c$) in the rear of the zero end of the base-line eliminates the constants ($f + c$) and therefore simplifies the computations. With the instrument in this position the stadia interval factor whether for upper, lower, or full interval may be determined by the formula

$$K = \frac{D}{S}$$



Prob. 58. Interval Factor (K) Determination.

Dist. (ftc)	Lower Interval	Upper Interval	Full Interval	K for Low. Int.	K for Up Int.
100	0.49	0.50	0.99	204.1	200.0
200	0.98	0.99	1.97	204.1	202.0
300	1.48	1.49	2.98	202.7	201.4
400	1.97	1.99	3.96	203.0	201.0
500	2.45	2.48	4.94	204.1	201.6
600	2.94	2.98	5.93	204.1	201.3
700	3.41	3.48	6.91	205.3	201.1
800	3.92	3.99	7.90	204.1	200.5
Means				203.9	201.4

30.

H.R. Sargent, K

Gurley Transit No. 98 M.D. Rand, Rod.
(ftc) = 1.25 ft. June 14, 1911.
Warm, Cloudy,
No Wind.

K for Full Int.
101.0
101.5
100.7
101.0
101.2
101.2
101.3
101.3
101.3

Distances Measured with Tape Inst. Set at Dist. (ftc) Back of Zero of Base-line.

SAMPLE NOTES 58.

in which D is the distance from zero of base-line to rod and S is the rod interval at the above distance. It will probably be found that values of K will differ somewhat for the various distances. This will be due partly to errors of observation and partly to unequal atmospheric refraction. (4) It will facilitate the calculation of the rod interval if the lower hair is set on a footmark, for then the subtrahend will be a whole number of feet. This may be done without appreciable error if the footmark nearest the apparent position of the hair is chosen. That is, if with the instrument leveled the lower hair cuts the rod at 1.57 ft. throw the telescope out of level until the hair falls on 2.00 ft.

Practical Applications.—The stadia interval factor should be determined before any sort of a stadia survey is attempted, and to secure a good average value a series of tests should be made under a variety of weather conditions and at various hours of the day. On an extensive survey, since radical changes in weather conditions may alter the value of the interval factor, occasional tests should be made. For most instruments the value of this factor is so nearly 100 that for side shots or short traverses it may be considered as such without introducing an appreciable error, but on long traverses its true value should always be used.

Problem 59. Topographic Survey by Transit-Stadia Method. (Elevations Carried Forward by Direct Leveling)

Object.—To collect sufficient data for making an accurate topographic map of an assigned tract. (The data may be used in Problem 90, page 229.) See Sample Notes 59(*a*) and 59(*b*).

Procedure.—(1) Make a rapid reconnaissance of the tract, picking out the most advantageous points for instrument stations. (2) Run a closed azimuth traverse through these points, observing the stadia intervals and vertical angles (where the latter are so great as to appreciably change the stadia distance when reducing to the horizontal). Note the angular error of closure. *Allowable error in minutes* $= \sqrt{\textit{number of sides}}$. (3) Calculate the horizontal distances, latitudes and departures (Problem 73, page 183), and linear error of closure of the traverse. *Allowable error of closure* $= \frac{1}{500}$. (4) Run a level circuit from some point of known elevation for the

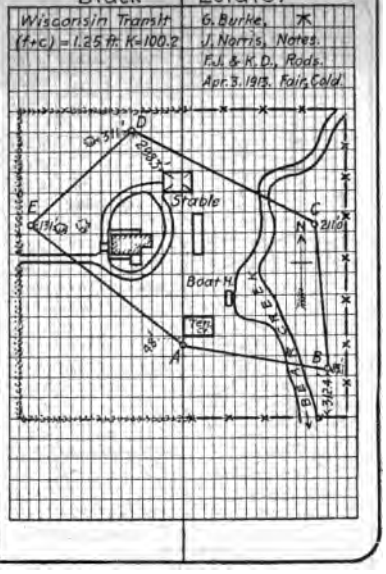
purpose of determining the elevations of the traverse stations and such other points as will facilitate the taking of topography. *Allowable error in feet* $= 0.05 \times \sqrt{}$*distance in miles.* (5) Occupy each of the traverse stations, and with the instrument correctly oriented, observe the azimuth, stadia distance, and vertical angle to all changes in ground-slope and to other natural and artificial features which are to appear on the finished map and which are within range of the instrument. (6) With the stadia slide-rule,* reduction diagram,* or by Table IX* compute the horizontal distance, difference in elevation, and elevation of each side shot.

Hints and Precautions.—(1) The line of zero azimuth may be either an assumed or true meridian, but in case an assumed meridian is used, for convenience in checking with the magnetic needle, it will be found advantageous to have the assumed and magnetic meridian closely related. In any case the azimuths of the traverse lines should ultimately be referred to the true meridian. (2) The hubs of the traverse lines should be referenced to permanent objects in such manner as to be easily re-located in case of loss, and a careful description of the location of the traverse stations should appear in the notes. Suitable witness stakes marked with the number of the line and station should be driven near the hubs. (3) In running the traverse the magnetic bearings of all lines should be recorded and immediately compared with the calculated bearing (deduced from the azimuth of the line). It is obvious that, if this method is adhered to, there will be no possibility of large angular errors. If the magnetic and calculated bearings are not referred to the same meridian, that is, if they do not agree, there should always be the same difference, and this difference is the check. (4) Vertical angles and stadia intervals should be observed from both ends of each traverse line; if they agree closely, their numerical mean is used in the later computations. Vertical angles are usually determined by sighting at a point as far from the foot of the rod as the center of the horizontal axis is above the station occupied; if for any reason this is impracticable, some other point is sighted upon (usually a whole

* Based on formulas $d = Kr \cos^2 v + (f + c) \cos v$ and $h = \frac{1}{2} Kr \sin 2v + (f + c) \sin v$ in which h and d are vertical and horizontal distances, v the vertical angle, r the rod interval, K the interval factor, and $(f + c)$ the stadia constant. Though the derivation of these formulas should be understood, the time and labor consumed in computing makes their direct use prohibitive.

Prob. 59a. Stadia Traverse for Horizontal Control.

Sta.	Obj.	Az.	Mag. B.	Rod Int.	Ver. Ang.	Hor. Dist.
B	A	279°00'	N 81°W	9.09	+2°41'	908
	C	358°14'	N 4° W	8.94	+0°31'	
C	B	176°14'	S 3°30'E	8.98	−0°32'	899
	D	296°58'	N 63°W	13.45	−1°17'	
D	C	116°56'	S 63°E	13.50	+1°17'	1350
	E	221°49'	S 42°W	8.49	+4°42'	
E	D	41°49'	N 41°E	8.47	−4°41'	845
	A	127°57'	S 52°E	11.90	−1°27'	
A	E	307°57'	N 52°W	12.00	+1°27'	1198
	B	98°58'	S 81°E	9.05	−2°40'	



SAMPLE NOTES 59(*a*).

32.

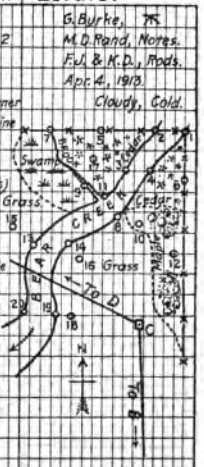
Prob. 59 b. Topographic Details, Black Estate.

	Inst.	at C.	El. 423.9, H.I. = 4.4		
Obj.	Az.	Rod Int.	Ver. Ang.	Hor. Dist.	Diff. El.
B	176°14′				
1	10°21′	7.23	−3°11′	723	−40.2
2	3°14′	7.02	−3°17′	702	−40.2
3	352°45′	5.64	−4°11′	563	−40.9
4	7°18′	5.76	−4°04′	575	−40.9
5	349°10′	(7.14 x 4)	+3°3	714	−31.9
6	16°55′	5.50	−2°50′	551	−27.3
7	315°20′	(7.86 x 5)	−2.5	786	−36.8
8	349°15′	4.13	−5°46′	410	−41.4
9	339°30′	5.40	−4°22′	539	−41.1
10	0°05′	3.71	−4°12′	371	−27.2
11	344°40′	4.85	−4°54′	484	−41.4
12	25°00′	2.86		288	+ 1.2
13	307°45′	4.88	−4°56′	487	−42.0
14	319°10′	4.02	−5°56′	400	−41.6
15	309°45′	5.80	−3°00′	581	−30.7
16	318°25′	3.27	−4°36′	327	−26.3
B	176°15′	ck.			
17	340°00′	6.34	−3°08′	635	−34.7
18	278°35′	2.51	−5°43′	250	−25.0
19	276°20′	3.07	−7°55′	303	−42.3
20	277°40′	4.24	−5°40′	422	−41.9

Wisconsin Transit
(F + C) = 1.25, K = 100.2
Elev.

383.7 Water's Edge - Corner
383.7 " " - On Line
383.0 " "
383.0 " "
392.0 Line (Intervals)
396.6 "
387.1 " (")
382.5 Water's Edge
382.8 Bank Brook 6' wide
396.7 " " 15' "
382.5 " " 15' "
425.1 Direct Levels.
381.9 Water's Edge.
382.3 " "
395.2
397.6
389.2
398.9
381.6 Water's Edge.
382.0 " "

G. Burke, Jr. M. D. Rand, Notes. F. J. & K. D., Rods. Apr. 4, 1918. Cloudy, Cold.


Swamp
Grass
Cedar
Cedar
Grass
Marsh
Brook
Brook
Brook

number of feet above or below the point for true vertical angle) and a note is made to that effect. For ordinary stadia work, inclined distances with vertical angles of less than 3° may be considered as horizontal without appreciable error. (5) Rod intervals can be most easily determined by sighting the lower hair on a footmark. (6) When the distance between stations is too great to allow a single reading to be taken the stadia distance may be found by adding readings between the lower and middle hair, and between the middle and upper hair; or by adding the readings taken from a point approximately midway between the two stations. The objection to the last method is that it allows no check. (7) The stadia constant $(f + c)$ should not be omitted from the calculations; with the ordinary transit and in stadia work of the usual precision this may be assumed to be 1 ft. (8) When a station is to be occupied for some time (as when side shots are being taken), it is well to record the azimuth to some prominent object as soon as the transit has been oriented. Any motion of the lower plate can then be detected by again sighting upon the object before leaving the station. (9) The azimuths of topographic details such as roads, streams, and changes of slope need be read only to the nearest 05′, without the use of the vernier; the rod intercept should be read to the nearest 0.01 ft. unless the scale of the prospective map be small; vertical angles should be observed to single minutes. (10) Many shots can be taken by reading the rod with the telescope leveled as in direct leveling. This should be done when possible on account of the labor saved in later computations. (11) The location of points where elevations would be useless can often be advantageously determined by azimuths from two or more traverse stations. (12) The middle hair should not be mistaken for one of the stadia hairs. (13) Because of the lack of checks on side shots, the instrumentman and noteman should form the habit of judging distances and angles with the eye. This precaution together with the method of sketching the approximate position of all shots on the right-hand page of the note-book will often prevent absurd mistakes. (14) Care should be taken to make clear, complete notes; the sketches should be made approximately to scale, and except for contour lines, should show all features that are to appear upon the finished map. The accompanying pages of notes are of a convenient form for traverse and side shots.

(15) Many topographers believe that time can be saved when vertical angles are not large by using the "*interval*" or "*stepping*" method for determining the difference in elevation, instead of reading the vertical angle and computing this difference with the stadia slide-rule. To illustrate this method suppose it is desired to determine the elevation of the ground at *a* (Fig. 32). The transit is sighted on the rod held at *a*, and the rod interval is noted as 4.55 ft. The telescope is then lowered in the same vertical plane until level, and the position of the *horizontal* line of sight on some prominent

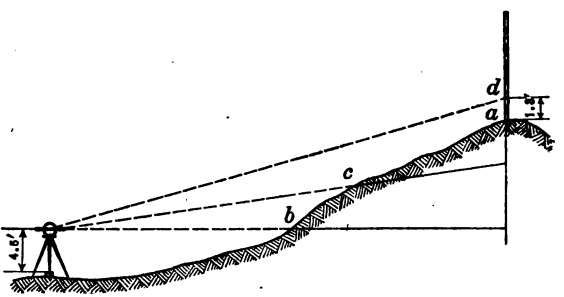


FIG. 32.—THE STEPPING METHOD.

mark of the landscape is noted at *b*. The telescope is raised until the lower hair rests on *b*, and the position of the upper hair on the landscape is noted at *c*. The lower hair is brought up to *c*, and the upper hair rests at *d* which is 1.3 ft. on the rod. The difference in elevation between the known point at the instrument and *a*, assuming that no error is introduced by reading the rod with line of sight inclined, is $4.5 - 1.3 + 2 \times 4.55 = 12.3$ ft. It is evident that for vertical angles of sufficient magnitude to cause an appreciable difference in the rod interval or to necessitate horizontal corrections this method would not be applicable, and it seems doubtful if, even at long distances, it possesses any advantage over the vertical angle method for angles greater than two or three intervals. (16) Where a high degree of precision is desired, as will generally be the case with large scale and small contour interval, the engineers' level is often used in conjunc-

tion with the transit. With the level a number of points are found on each contour; and with the transit, the horizontal position of these points is determined by angle and stadia distance. (17) It is sometimes advantageous, particularly if there is a large number of details, to plot the map in the field as the work progresses, and thus not only portray the topography with greater veracity but avoid taking voluminous notes as well. The usual method is to divide the area into a number of parts, plotting the control points of each subdivision on a sketch-sheet large enough for a complete map of that area and of a convenient size for field sketching. With this sheet mounted on a small drawing board, the draftsman or sketcher plots the map as fast as data is obtained. When all the field-sheets have been completed, they are assembled, any irregularities adjusted, and a map of the whole area made by tracing the assembled sheets.

Practical Applications.—The transit-stadia method of topographic surveying is in general use on railroad preliminary, hydraulic, municipal, highway, and many other surveys where the contour interval is small and moderate precision is required. The method of procedure may be somewhat altered from that above, but the principles are the same. For a more accurate horizontal control of surveys covering a large area, triangulation systems are established and traverses are run with transit and tape; for a less accurate vertical control, or on surveys where government or other bench-marks can be tied onto from time to time, direct leveling may be dispensed with and elevations may be carried forward by stadia and vertical angles. The transit-stadia method is advantageous over the plane-table in that less time is required in the field, a higher precision can be obtained, and progress is not hindered by showers or very cold weather; but more time is consumed in office work. It is advantageous over location of contours by direct leveling in that less time is consumed in the fieldwork and a smaller party is required; but the precision is lower, and since the position of contours must be interpolated, the resulting map is more generalized. In heavily wooded country the method is not practicable, but on cleared or partially cleared land it is, for the accuracy obtainable, one of the most economical methods in use.

Problem 60. Survey of Tract for Deed Description and Topographic Map

Object.—To obtain sufficient data for a proper legal description and an accurate topographic map by a combination of tape and stadia measurements. (The data may be used in Problem 90, page 229. For method of computing area within traverse see Problem 73, page 183.)

Procedure.—(1) Run an azimuth traverse referred to the true meridian about the field, measuring the sides with a steel tape, and setting hubs at the corners. Determine the angular error of closure, as in Problem 59. *Allowable angular error of closure in minutes* $= \frac{1}{2} \times \sqrt{\text{number of sides}}$. (2) Calculate the latitudes and departures, and linear error of closure; balance the survey, and compute the area of the tract by the double meridian distance method. *Allowable linear error of closure* $= \frac{1}{5000}$. (3) If it is impossible to obtain sufficient topography from the corner stations, establish other stadia stations at advantageous points, and tie these stations to the main traverse by azimuths and stadia distances. (4) Conduct the remainder of the survey as in Problem 59.

Hints and Precautions.—The same as for Problem 59.

Problem 61. Preliminary Survey for Railroad by Transit-stadia Method

Object.—To obtain data for plotting a topographic map of a proposed railroad route between two governing grade-points (the elevations of which are known) approximately 2 miles apart. (The data may be used in Problem 90, page 229.)

Procedure.—(1) Run a stadia azimuth traverse between the two points establishing stadia stations at advantageous points near where it appears that the line will eventually be placed. (2) Determine the distance between adjacent stations by reading the stadia distance on both forward and back sights. (3) Observe the vertical angles between instrument stations on both forward and back azimuths. (4) While running the traverse, take side shots 300 to 800 ft. on either side of the traverse line according to methods of Problem 59, p. 153. (5) Determine azimuth at the beginning and end of traverse by astronomical observations (Problem 51

or 56, pages 130 and 146). (6) Note the quality of the land, as to whether it is clay, rock, sand, or gravel. Note the condition of the land as to whether it is cleared land, pasture land, or woods, and if woods the kind of trees and density of growth.

Hints and Precautions.—(1) In determining the differences of elevation, and horizontal distances between traverse points, use the mean of the two vertical angles and mean of the two stadia readings taken along the line joining these points. (2) The width of the tract over which topography must extend will depend to a large extent upon the character of the country. If in a valley or on a ridge and the ground slopes are steep, it will be necessary to map only a narrow strip, but if the country is nearly level or rolling, side shots must be taken over a considerable width. (3) Before taking side shots about the station occupied, set the next stadia station in advance. (4) For other precautions see Problem 59, page 153.

Practical Applications.—Methods made use of in this problem are applicable to preliminary surveys for railroads, canals, and highways where the route is through open or partially cleared country. The accuracy obtainable is well within the allowable error of such surveys. The contour interval is usually 5 ft., but may be greater in rough country.

Problem 62. Railroad Preliminary Survey. (Topography with Hand-level)

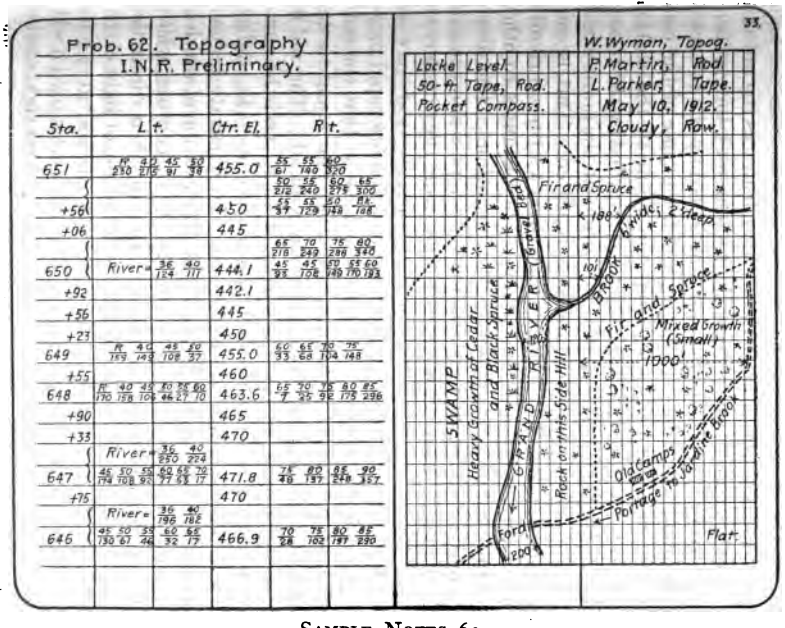
Object.—To obtain data for a topographic map along the route of a proposed railroad by locating the contours directly by hand-level and tape measurements.

The staff on a survey of this kind is made up of three parties, led by a chief who directs the course of line. The transit party, made up of the transitman, chainmen, flagman, and axemen, establish the horizontal control for the survey. Sample Notes 20(*a*), page 68, are of the form commonly used. The level party, made up of the leveler and rodman, establish vertical control for the survey by the methods of Problem 28, page 85. The topographer's party, made up of topographer, rodman, and tapeman, locate the details for a topographic map of a narrow strip of country on both sides of the transit line. It is with the latter operation that this problem is

chiefly concerned, though the work of the transit and leve parties is incidentally mentioned.

Procedure.—(1) Run a deflection angle or azimuth travers along the apparent general route of the proposed railroad Set stakes every 100 ft. (by tape measurements) along th line. Mark the station number so that it will read down th stake; when the stake is driven the number should face to ward the zero end of the line. Establish permanent hubs a all instrument stations, and mark the station and plus of th hub on a witness stake. Also reference the hubs so that the may be replaced in case of loss. (2) By direct levelin determine the elevations of all stations and such other point as are needed to plot an accurate profile of the line. Establis permanent bench-marks every 3000 ft. (3) With hand-leve metallic tape, and topographers' rod, determine the positio of 5-ft. contours at each station, and at such plusses as a necessary over a distance of 300 to 800 ft. on either side of th line; and the chainage of points between stations at which 5-ft. contours cross the traverse. (4) Measure distance rom the traverse line to other topographic features such as land lines, fences, streams, roads, lakes, and buildings; and determine the chainage of intersections of other lines with the traverse line. (5) Note the quality of the land, as to whether it is clay, rock, sand, or gravel. Note the condition of the land, as to whether it is cleared land, pasture land, or woods, and if woods, the kind of trees and density of growth. (6) Tabulate the numerical data on the left-hand page of the note-book, keeping the notes up the page. On the right-hand page make a neat sketch showing the approximate position of contours, roads, streams; and all other details. Follow the accompanying form.

Hints and Precautions.—(1) To avoid large angular errors in the traverse, the magnetic bearing of each line should be taken, and immediately compared with the calculated bearing; to avoid errors of 100 ft. in chaining, the rear chain-man should call out the station number at which he is holding the tape, and the head chainman or stakeman after marking and setting the forward stake should see that these numbers increase consecutively. As a further check the rear chainman as he leaves each station may enter its number in a small note-book. (2) The bench-marks should be placed on roots of trees, stumps, or large rocks if possible and should be marked



SAMPLE NOTES 62.

in such a manner as to be easily recognized. The station and plus of all bench-marks, their offset from the traverse line, and the name of objects on which they are placed should appear in the level notes. (3) At each station contours are located with the hand-level as follows: The topographer, having already obtained the elevations of the traverse stations from the level notes, stands over the station from which distances to contours are to be measured, and directs the rodman on a line perpendicular to the traverse until a point, indicated by the rod reading, is found which has an elevation of a multiple of the contour interval. The distance from this point to the station occupied is measured with the tape and recorded by the topographer. He then takes up a new position at the point where the rod was held, and the process is repeated. This is continued until topography has been taken over a strip sufficiently wide for the purposes of the survey. The distances to other important features are taken in the meantime. The directions may be determined with the pocket compass or estimated with the eye. Points a remote distance from the line may be located by pacing. (4) The ordinary leveling rod may be used, but a special topographers' rod makes more rapid progress possible. This has an adjustable base so that the zero point may be fixed at a distance from the foot equal to the height of the observer's eye above the ground. The rod is graduated in half-feet over a length of 10 ft. and the numbers increase from the zero point toward the ends. In reading the rod, tenths of feet are estimated. To illustrate its use, suppose the observer to be at a station the elevation of which is 481.7, and that he wishes to locate the 485-ft. contour, which is 3.3 ft. higher than the station. He will then direct the rodman on the high side of the line until the line of sight of the hand-level as it is held to the eye cuts the rod at 3.7 ft. below the zero point. (5) If the Locke hand-level is used it will be found after a little practice that a much clearer sight can be obtained with both eyes open. (6) In flat country the engineers' level is often used in place of the hand-level because of the great horizontal distance between contours and the fact that the survey usually extends over a much wider belt.

Practical Applications.—This method was formerly much used on railroad preliminary and other surveys of a like nature, but at the present time it is used chiefly in wooded country where stadia work would be impractical. It is

advantageous over the transit-stadia method in that the contours are located on the ground, but with an experienced topographer the increased accuracy is slight and is more than offset by the extra cost.

Problem 63. Topographic Survey of Park for Landscape Architect's Study. (Hand-level and Sketch-sheets)

Object.—To make accurate large scale field-sketches of the topography of a given area showing such details as the position, kind, and size of trees, walks, drives, buildings, and fences; the location of streams, hydrants, manholes, etc.; and the representation of the relief by contours located directly with the hand-level, the individual sketches being so constructed that together they will form a complete map of the area surveyed.

Procedure.—(1) With transit and tape establish a base-line along the middle of the area to be mapped, setting stations every 500 ft. (2) Through each station of the base-line run laterals (with transit and tape) at right angles to the base, setting stakes every 100 ft. to the boundary of the area. (3) By direct leveling determine the ground elevations at stakes on base-line and laterals. (4) Plot the base-line and laterals to the scale of the proposed map, showing the ground elevations at all stakes. Cut this sheet into sketch-sheets that may be fastened to a drawing board of convenient size for field use. (5) The topographer locates the position of contours crossing the laterals and crossing lines parallel to the base-line between corresponding stations on the laterals, using the hand-level as explained in the previous problem. (6) The sketcher plots contours on the sketch-sheet as fast as their positions are determined, taking note of the shape of the ground between contour crossings and sketching the contours accordingly. (7) The topographer locates the other details by offsets and distances along line. (8) The sketcher plots these details using conventional signs (Figs. 44–47, pages 218–222) when it is possible, but showing everything that should appear on the finished map. (9) When all sketch-sheets have been completed, assemble the drawing and paste to a backing. Make a tracing of the whole map.

Hints and Precautions.—(1) Observe the hints and precautions of Problem 62 which apply to this problem. (2) The

topographer in locating contours should call out the number of the contour and its distance from the station away from which he is moving. The elevation of this station should be given by the sketcher to the topographer on his arrival. (3) The topographer should check his work by comparing distances between stations with corresponding distances of the transit survey, and elevations should be compared in the same manner. The allowable discrepancy between these corresponding measurements will depend on the contour interval, the scale of the map, and the character of the country, but should not be of a magnitude sufficient to appreciably alter the position of contour or other object on the sketch-sheet. Small discrepancies may be assumed to have gradually accumulated and the topography should be adjusted accordingly. A large discrepancy indicates a mistake, and the work should be retraced until the error is discovered. (4) Usually it will be sufficiently accurate to locate many of the minor details by pacing or by estimating distances with the eye.

Practical Applications.—The preceding method is particularly adapted to large-scale topographic surveys where the details are many and varied in character and the country is rolling. It is simple, chances of error are reduced to a minimum by checks, and the entire ground must be covered, hence it may be used to good advantage by surveyors having little or no experience in topographic work; for field sketching the absence of computations make it preferable to the transit-stadia method; the map is made in the field, therefore field notes, which must necessarily be very copious for a survey of this kind, are unnecessary; and since the map is made in the field, much can be sketched with the eye that would otherwise have to be located by field measurements. Essentially the same methods have been used in numerous small surveys* throughout the country. It seems probable, however, that in the hands of an experienced topographer the plane-table would become a more efficient instrument for constructing the detail of the map, being sufficiently accurate providing horizontal and vertical control points are established with transit and tape and level, and requiring a much smaller number of these control points than the method of this problem.

* For description of such a survey see "Topographic Surveying Methods Used on the Vanderbilt Estate, Biltmore, N. C.," *Jour. Assoc. Eng. Soc.*, April, 1897.

Problem 64. Survey for Contour Map by "Square" Method (Transit and Level)

Object.—To obtain sufficient data for an accurate topographic map of large scale and small contour interval. The area to be mapped is small, very regular, and possesses few details. (The data may be used in Problems 89 and 90, pages 227 and 229.)

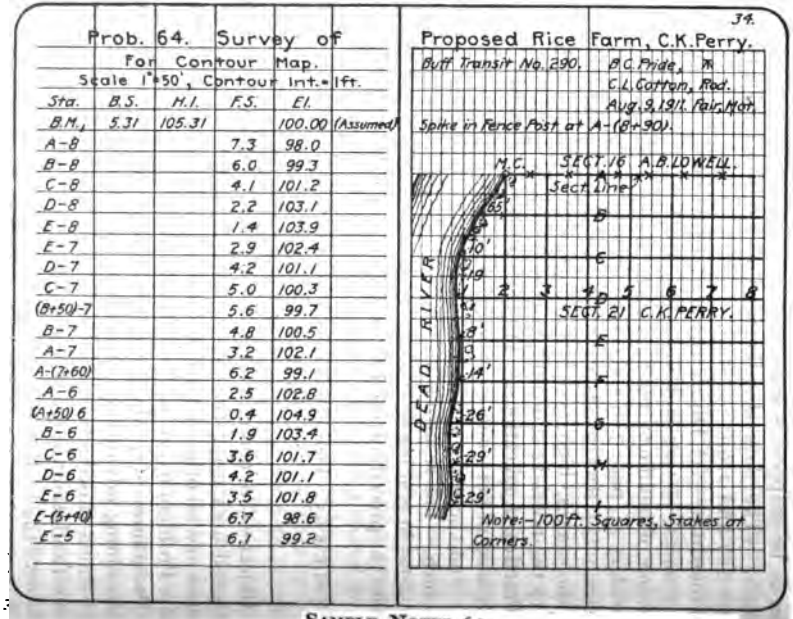
Procedure.—(1) Divide the tract to be mapped into 100-ft. squares, setting stakes at the corners. (2) Locate the position of buildings, roads, fences, etc., by offsets, ties, etc. (Problem 23, page 74), and show these measurements on the sketch. (3) Run levels over the area taking rod-readings at the stakes, high and low points, and changes in slope. Keep the notes as shown in Sample Notes 64, page 168.

Practical Applications.—The above method may be profitably used in flat country where the contour interval is 1 or 2 ft. and the scale of the map is from 20 ft. to 100 ft. to the inch. The resulting map may be the basis of a study of problems in grading or land drainage. It will be noticed that, unlike the preceding problem, contours are not located directly.

Problem 65. Topographic Survey with Plane-table (Large Scale)

Object.—To make a large-scale topographic map of an assigned area using the plane-table. The map is to be accurate and all other details as well as contours are to be shown, as in Problem 63, p. 165.

Procedure.—(1) Make a reconnaissance for the purpose of determining advantageous positions for plane-table stations. (2) Run a closed azimuth traverse between these points with transit and stadia as in Problem 59, p. 153. (*Allowable error of closure*, $\frac{1}{500}$.) Run differential levels over the course establishing bench-marks at or near traverse points. (*Allowable error of closure*, 0.1 *ft.* $\times \sqrt{\textit{distance in miles}}$.) (3) Plot the traverse on the plane-table sheet; show the elevations of bench-marks. (4) At any station orient the plane-table by sighting at an adjacent station as described in Problem 40, Procedure (3), page 108. (5) Determine the height of instrument above the station occupied or other bench-mark. (6) Observe the vertical angle and stadia reading to a change



Prob. 64. Survey of
For Contour Map.
Scale 1"=50', Contour Int.= 1ft.

Sta.	B.S.	H.I.	F.S.	El.	
B.M.$_1$	5.31	105.31		100.00	(Assumed)
A-8			7.3	98.0	
B-8			6.0	99.3	
C-8			4.1	101.2	
D-8			2.2	103.1	
E-8			1.4	103.9	
E-7			2.9	102.4	
D-7			4.2	101.1	
C-7			5.0	100.3	
(B+50)-7			5.6	99.7	
B-7			4.8	100.5	
A-7			3.2	102.1	
A-(7+60)			6.2	99.1	
A-6			2.5	102.8	
(A+50) 6			0.4	104.9	
B-6			1.9	103.4	
C-6			3.6	101.7	
D-6			4.2	101.1	
E-6			3.5	101.8	
E-(5+40)			6.7	98.6	
E-5			6.1	99.2	

34.

Proposed Rice Farm, C.K. Perry.

Buff Transit No. 290. B.C. Pride, C.L. Cotton, Rod. Aug. 9, 19??. Fair, Hot.

Spike in Fence Post at A-(8+90).

M.C. SECT. 16 A.B. LOWELL.

Sect. Line

65'

10'

19

1 2 3 4 5 6 7 8

SECT. 21 C.K. PERRY.

ALLEN

DEAD

8'

14'

26'

29'

29'

Note:- 100 ft. Squares, Stakes at Corners.

SAMPLE NOTES 64.

of slope, land line, fence, road, building, or other necessary point; draw a line along the straight-edge of the alidade that will include the point to be plotted. (7) Deduce the elevation (if on the ground) and horizontal distance, using the stadia slide-rule or reduction diagram. Plot the point and mark its elevation in an unmistakable manner. (8) Locate other points in like manner. Sketch in all details except contours as soon as they are located using conventional signs (Figs. 44–47, pages 218–222). Sketch in contours by interpolating between the elevations of plotted points (Problem 89, page 227), when a sufficient number of ground points have been located to show the trend of the contours. (9) When all the topography that can conveniently be taken from one station has been plotted, proceed to the next point in the traverse and continue the work in the same manner. If it is found that transit traverse stations have been so located that portions of the area can not be mapped, run short secondary traverses with the plane-table (Problem 41, page 110), and carry elevations forward from transit traverse stations by vertical angles and stadia. From the secondary stations locate the omitted topography in the usual manner. (11) When the map is completed make a neat title showing name and location of area mapped, purpose of map, scale, contour interval, date, and members of surveying party.

Hints and Precautions.—(1) If the scale of the map is very large, the point on the paper representing the station occupied should be approximately over the corresponding point on the ground. The table should be roughly oriented before any attempt is made to fulfill the above condition. Many plane-tables have a plumbing arm for suspending a plumb-bob beneath the point on the plane-table sheet. If no such device is at hand, the relative positions of the two points may be determined with sufficient precision by dropping a pebble from beneath the estimated position of the point on the paper. (2) Care should be taken to keep the plane-table as nearly level as possible, and it should be low enough to allow sketching without resting elbows or body on the board. (3) It is especially desirable that the board should be level where the vernier is fixed, as is the case with some alidades, for when not level the index error will vary for every direction of the telescope; but however much care is taken, it will be found, unless the table is unusually rigid, that as the

alidade is moved about on the sheet, the board will spring somewhat. For this reason, no vertical angle should be taken without ascertaining the index error if the vernier is fixed or without making sure that such error has been eliminated if the vernier is movable. (4) If the vernier has an attached bubble, it should be centered before reading each angle; if the vernier is movable, the zeros should be brought together with the telescope level before taking each sight; if the vernier is not movable, the index error should be determined and the final vernier reading should be corrected accordingly. (5) Use direct leveling when possible; it simplifies computations and is more accurate. (6) If it is possible, measure vertical angles by sighting at a point as far from the foot of the rod as the instrument is above the point from which the height of the instrument is determined. (7) Before leaving a plane-table station check the orientation by sighting to an adjacent station or other mark, the position of which has been correctly plotted. Any movement between initial and final sights will be indicated by the distant point, its plotted position, and the plotted position of the station occupied not falling on a straight line. (8) Do not take long sights; the topographer can not accurately judge topographic details far from the station occupied. (9) If the scale is larger than 50 ft. = 1 in., contours should be exactly located by direct leveling unless the ground is very even, and distances to important details should be measured with the tape. When the contour interval is as small as 1 or 2 ft. and the ground is uneven the engineers' level may well be used in conjunction with the plane-table, one rodman serving both instruments at the same time. The rodman is directed to a particular contour by the leveler; the plane-table operator observes distance and direction and plots the point with its elevation. This method makes it possible to cover more ground from a single plane-table station, particularly if slopes are steep. (10) If the scale is as small as 100 ft. = 1 in., the contour interval is as great as 5 ft., and the area to be mapped is small, horizontal control may be carried forward, as the work of taking topography progresses, by plane-table traverse using stadia (Problem 41, p. 110); and vertical control, by trigonometric leveling with the plane-table. (11) Note the hints and precautions of Problems 39, 40, and 41 which apply to this problem.

Practical Applications.—The plane-table has not been used to any great extent on large-scale work. This may be due to the scarcity of surveyors who understand its use and possibilities, or to the quite prevalent idea that it is fit only for rough surveys and large contour intervals. It would seem that it might properly be used with as good results as could be obtained by any method and at less cost, regardless of the scale, providing proper vertical and horizontal control were established.

Problem 66. Topographic Survey with Plane-table (Medium Scale)

Object.—To make a medium-scale topographic map of an assigned area using the plane-table. It is assumed that horizontal control has been established by a system of triangulation, or by transit traverses, or by both; that vertical control has been established by direct leveling; and that signals, which may be seen for long distances, have been erected at these control stations.

Procedure.—(1) Plot the position of triangulation or traverse points and bench-marks on the plane-table sheet. (2) Choose a favorable site for plane-table station, and locate the position of the station and orient the board by one of the methods of resection (Problems 43–47, pages 112–122). Determine the elevation of the station by trigonometric leveling from a bench-mark. (3) Take stadia readings and vertical angles to governing points, as along ridge and valley lines and at sharp changes in slope. (4) Reduce and plot the observations as in the previous problem. Note the general form of the slopes and sketch in the contours accordingly, interpolating between points of known elevation (Problem 89, page 227). (5) Take stadia readings or draw intersection lines (Problem 40, p. 108) to other important objects, such as buildings, road intersections, land lines, etc., but sketch less important details without measurement. (6) When all topography has been sketched about the station, move the plane-table to another favorable point and proceed as before. (7) If it is not practicable at any desirable plane-table station to locate the station by resection upon control points, run a traverse (Problem 41, p. 110) to include the proposed station, between control points or to stations that may be located by

resection, carrying elevations forward by trigonometric leveling. Check the traverse and distribute the error, assuming that the control points or stations located by resection are correct. (8) When the map is completed, make a neat title showing name of area mapped, purpose of survey, scale, date, and members of party.

Hints and Precautions.—(1) Often certain details, such as buildings or woods, would if placed on the plane-table sheet so crowd certain areas as to seriously interfere with contour sketching. Such details may well be plotted on a piece of tracing cloth temporarily fastened over the plane-table sheet. The tracing cloth may then be removed and not used again until these particular kinds of details are encountered. (2) When vertical angles are small many topographers prefer to determine elevations by the "stepping" or "interval" method as explained in Problem 59, Hints and Precautions (15), page 158. (3) It goes without saying that many of the horizontal control points will be favorable plane-table stations and may be used as such, orientation being obtained by sighting to any signal which can be identified. (4) Often the number of primary control points will not furnish sufficient control for the area to be mapped. It is then customary to locate secondary control stations by graphical triangulation with the plane-table, using the method of intersection or methods of resection. At the same time elevations are determined by vertical angles and scaled distances. Signals are, of course, erected at these secondary points. Thus it is seen that any point located by intersection or resection may become secondary control, providing its strength of determination will so warrant. (5) The sketching of contours, from what can be seen by the topographer and a few elevations of governing points, so that the map will be a truthful representation of the country is the most difficult part of the work, and the only method of learning this art seems to be through experience. The topographer should form the habit of judging distances and slopes and he should check these estimations against more precise measurements whenever possible. It may be taken as a rule that all ground should be viewed from at least two points, and preferably more, before the contour or other object is finally located by estimation. The beginner will find the hand-level very useful for tracing out the trend of contours between determined points,

and the clinometer very useful for judging slopes. The rodman should be consulted when there is doubt; he may prepare rough sketches and determine slopes of the ground about points remote from the plane-table. (6) When expansion of the system of horizontal control is achieved through graphical triangulation, secondary vertical control is usually obtained by determining differences of elevation from vertical angles and scaled distances, through the relation that

$$h = D \tan \alpha$$

in which h is the difference in elevation between center of instrument and point sighted, D is the corresponding horizontal distance scaled from the paper, and α is the vertical angle. Since vertical angles will in general be small, it will be seen that a considerable error in the scaled length D will introduce only an insignificant error in the computed value of h. The topographer is furnished with a list of elevations of easily distinguishable points on signals (for example, the elevations of tops of masts) as well as the elevations of bench-marks on the ground. When the plane-table is set up at a station which has been located on the plane-table sheet but the elevation of which is unknown, the topographer measures the vertical angles to two or more of the points on signals contained in his list, scales the corresponding distances, and computes the differences in elevation by the preceding equation. These, when added to or subtracted from the elevations of the corresponding points, will furnish as many independent determinations of the elevation of the instrument as there are vertical angles taken. The work of observing and computing is checked if the discrepancies are reasonably small, and the mean of the elevations thus determined is the accepted elevation of the instrument. (7) If the sights are long, due allowance should be made for the effect of curvature of the earth and refraction of the atmosphere. The correction for curvature and refraction is approximately expressed by the equation

$$h' = 0.57K^2$$

in which K is the distance in miles and h' is the correction in feet to be added to or subtracted from the difference in elevation according as the vertical angle is positive or negative. This correction will be unnecessary for a horizontal distance of less than one mile.

Practical Applications.—The methods of this problem are practically the same as those used by the U. S. Coast and Geodetic Survey on a scale of 1 in 10,000 to 1 in 20,000 and a contour interval of 10 ft. to 50 ft. They are particularly adapted to rough or rolling country such as is often found along the seacoast, and must necessarily be varied to suit topographic conditions. Such methods have been successfully used for a much larger scale and smaller contour interval for constructing topographic maps of cities, drainage areas, and water storage basins.

On small-scale surveys, such as those conducted by the U. S. Geological Survey in which scales of $\frac{1}{48,000}$ or smaller are used, primary control points are relatively few. In rough country, points of secondary control are located by graphic triangulation; in level or wooded country, by plane-table traverses along the roads closing on primary control points, elevations being determined either by trigonometric leveling or very rapid spirit leveling. Distances are often obtained by pacing or by the revolutions of a buggy wheel. Most of the sketching is done from roads already traversed, the positions of only a very few critical points being determined by the topographer. Frequently the topographer has no rodman, and therefore has to depend more upon triangulation for the location of critical details away from the line. In rough country where a large contour interval is used, the elevations of points along the topographical route are determined with the aneroid barometer, it being set at proper elevation at one point of secondary control or bench-mark and checked at the next. On small-scale work, since the number of known points for a given area is very much smaller, the skill of the topographer must be relatively great if a correct interpretation of the relief is to be obtained.

Problem 67. Base-line Preparation

Object.—To prepare a short base-line for precise measurements with the steel or invar tape. It is assumed that the base-line site has already been chosen and that permanent marks have been established at the end points.

Procedure.—(1) Set 3″ × 5″ stakes along the base-line at intervals of the length of tape with which measurements are to be taken. These may be called "*station*" stakes. Use the

transit for lining, making sure that the line cuts the middle of the top of each stake, and measure distances with sufficient precision to insure the ends of the base-line tape always falling on stakes when the base is measured continuously from one end, a full tape length at a time. (2) If the soil is yielding, brace each station stake with a smaller stake driven into the ground at an angle of 45° with the horizontal, with top nailed to the larger stake. The braces should be so placed as to best resist movement of the top of the station stake in the direction of the line. (3) Over each end of the base-line build a table by driving four 2″ × 2″ stakes well into the ground (forming a rectangle) and nailing a piece of board to their tops. Brace the table by nailing small diagonal strips to adjacent legs. (4) Tack ½″ × 5″ strips of zinc or copper on the top of each of the tables, and with the transit, *carefully* project each end point of the base-line to the metal strip above. Tack similar strips to the intermediate stakes. (5) At regular intervals between base-line stations set small stakes for supporting the tape on line, making sure that they project from the ground a sufficient distance to be cut by a uniform grade line joining the tops of adjacent station stakes. (6) "Shoot in" grade lines (Problem 31, page 91) setting up the transit at every other station along the base. Drive 10d. nails into the small intermediate stakes at grade, bending them so that they will readily support the tape. (7) Run levels over the line, determining the elevations of tops of station stakes and end tables.

Hints and Precautions.—(1) Because of the great variation of temperature in layers of air very close to the earth's surface, the height of the station stakes should be such that at no place will the grade line come closer to the ground than 1 ft. providing measurements are to be taken in the sunlight; for the sake of convenience in measuring the base-line the station stakes should project from the ground no more than 3 ft. (2) The nails in the intermediate stakes should be driven to form a guide over which the tape may be dragged without slipping off, but should not be sharply inclined else the tape may become pinched between nail and stake. If the intermediate stakes are driven with tops to grade, two nails ½ in. apart, driven upright, will serve as a guide for the tape. (3) The end points of the base-line may be projected to the metal strips on end tables as follows: Set up

the transit a short distance, say 25 ft., from the table, and sight to the end point on the ground. Elevate the telescope, and mark two points a few inches apart on the metal strip above. Scratch a straight line between these two points. Now move the transit to another position such that when a sight is taken to the end point the line of sight will form nearly a right angle with the line scratched on the metal strip. Sight at the end point, elevate the telescope, and mark the point of intersection of the line of sight and the scratch. This point will be over the point on the ground. (4) If the base-line is along a railroad tangent, the preparation may consist of placing zinc strips on boards nailed to the ties at a uniform distance from the rail, and of running levels over the line. In this case the tape will be supported by the ground for its full length.

Problem 68. Base-line Measurement

Object.—To measure a given base-line with the steel or invar tape, by such methods that the probable error of the corrected length shall not exceed $\frac{1}{250,000}$. It is assumed that the base-line has already been laid out as explained in the previous problem. (The data of this problem may be used in Problem 91, page 230.) See Sample Notes 68, page 178.

Parties for base-line measurement are usually made up of four (sometimes six) persons; two tapemen who hold the tape and exert the required pull with spring balances attached to each end; a rear observer who reads a thermometer, records all observations, and signals when the rear of the tape is at the mark; and a forward observer who scratches a zinc plate opposite the forward end of the tape at the above signal from the rear observer, and reads a second thermometer.

Procedure.—(1) Stretch the tape from the end of the base-line from which preliminary measurements were begun, making sure that it is supported at all intermediate stakes. Suspend two thermometers, one near each end of the tape, so that the bulbs will be at a height approximately that of the grade line above the ground. (2) Exert the required pull through a spring balance at each end, and when there is exact coincidence between the initial point and the rear end of the tape, the rear observer signals to the forward observer. (3) The forward observer carefully scratches the position of the forward end of the tape on the plate at Station 1, using a needle

or other sharp-pointed instrument. (4) Release the tension until the tape shows a considerable sag between supports for its full length. Again apply the required pull and make a second scratch on the plate at Station 1. (5) Read the thermometers, and record the temperature. (6) Proceed to the next station, and repeat the operation, holding the rear end of the tape at each of the scratches made at Station 1. (7) At the end of the course, since there will generally be a fractional part of a tape length, mark on the plate at the division that falls nearest the end point, and measure the remaining distance with a finely divided scale. (8) In the preceding manner measure the base-line at least four times.

Hints and Precautions.—(1) If the balance is heavy, proper allowance should be made for having it horizontal instead of in its usual vertical position, unless it is already adjusted for such condition. Assuming that the balance reads zero when freely suspended with the hook down, the reading with balance horizontal corresponding to a required pull may be determined by the equation

$$T = T' - \tfrac{1}{2}(W - E)$$

in which T' is the required pull, W the correct weight of the balance, and E the reading when the balance is freely suspended from the hook. (2) When the required tension has been applied the tape should be set in vibration so that the amount of tension will become uniform throughout its length. The scratch should not be made until all vibration has ceased. (3) The apparent shortening of the tape due to sag varies directly as the square of the distance between supports and inversely as the square of the pull, and friction between supports and tape will have a relatively greater effect upon the sag when the tape is under low tension. Practice seems to indicate that a pull of 12 lb. may be used with good results with supports 25 ft. apart or less, but for intervals in excess of this distance the pull should be correspondingly increased. Thus with the tape supported every 50 ft. a tension of 20 lb. may well be applied. (4) If an invar tape is used, good results may be expected regardless of weather conditions providing the air is calm. If a steel tape is used, good results can not be attained when the tape is exposed to the glaring rays of the sun; for this reason measurements should be taken on foggy or cloudy days or after sundown. (5) The tapeman should take particular care

35.

Prob. 68. Base-Line Measurment.

Sta.	Thermometers Rear	Thermometers Forward	Dist. in feet. No. 1.	Dist. in feet. No. 2.	Mean Temp.
		△ 40 to △ 41			
		Trials 1 & 2.			
△ 40-1	79.0°F	80.4°F	300.000	300.000	
1-2	80.1	79.2	"	"	
2-3	78.0	77.2	"	"	77.9°F
3-4	78.1	76.2	"	"	
4-△ 41	76.0	75.0	225.2863	225.2808	
			1425.2863	1425.2808	
		Trials 3 & 4			
△ 40-1	70.4	72.0	300.000	300.000	
1-2	71.0	70.5	"	"	
2-3	70.0	70.7	"	"	70.6°F
3-4	70.0	70.7	"	"	
4-△ 41	70.0	70.8	225.3203	225.3843	
			1425.3203	1425.3243	
		Trials 5 & 6			
△ 40-1	71.4	71.6	300.000	300.000	
1-2	72.5	73.3	"	"	
2-3	72.5	72.5	"	"	72.2°F
3-4	72.8	72.9	"	"	
4-△ 41	71.5	71.5	225.3000	225.3008	
			1425.3000	1425.3008	

July 28, 1913. 7 P.M.
300-ft. Std. Steel Tape. Fair. Calm.
Calences. Tape + M. Rayries.
Stretcher. E. Raymond.
W. Leffiche.
P. Gregory.

July 29, 1913. 4 A.M.
Fair. Calm.

Note:
Supports 25 Ft. Apart.
12 lb. pull.

SAMPLE NOTES 68.

to hold the spring balance horizontal when applying tension. If it is not counterbalanced he should support it with his hand so that the slide to which the hook is attached will move freely and without friction. It should be held at a height that will barely allow the tape to rest on the adjacent support. (8) There are numerous tape-stretching devices for use with the spring balance, all of them depending upon the principle of the lever. If none of these is to be had, an oak stick 4 ft. long, pointed at one end, may be used in a similar capacity. The balance may be tied to the stick so that it can be readily adjusted to any desired height, and leverage may be obtained by sticking the point in the ground. (9) Care should be taken not to lean against the station stakes or disturb them in any way. (10) An invar tape is very easily damaged. It should, therefore, be handled with great care, and should never be wound except on its special reel. (11) The reductions are made according to the methods of Problem 91, page 230.

Practical Applications.—In any triangulation system it is necessary that the length of at least one side of one triangle in the system be measured before the other sides can be computed. Because of their length and position it is usually impracticable to measure any of the sides of the main triangles of the system. A relatively short line in a favorable location for measurement and for connection with the main triangulation scheme is therefore chosen; and it is measured with much greater precision than is expected to be attained in the triangulation, because the number of operations necessary to expand it through the base-net into one of the long sides causes much of its precision to be dissipated. The above methods have been used on many large surveys and it has been found that the tape when used with proper precautions will furnish the desired accuracy at a cost less than that of any other method.

On *precise traverses* in which the allowable error is less than $\frac{1}{15,000}$, as in city surveying, it is necessary to take temperature and pull into consideration. On such surveys no supporting stakes are set, often tape lengths are marked on the pavement, and much of the chaining is done on the slope. The tapes used on such work are usually short in comparison with those used for base-line measurement.

CHAPTER VIII

OFFICE PROBLEMS

Problem 69. Area of Field Surveyed with Tape

Object.—To determine the area of field surveyed with the tape by subdividing it into triangles. The data of Problem 4, page 31, may be used. The area is to be calculated by each of the three methods outlined therein.

Procedure.—(1) Decide upon a convenient and systematic form for each method, using logarithms where possible, and transcribe the necessary data from field to office book. Check the transcription. (2) Calculate the area of the triangles and the total area in square feet and acres, using four-place logarithms and following each method through before beginning another. Check the results with a slide-rule. (3) Note the time devoted to each method and record this time at one side of the corresponding computation. (4) When the problem is computed compare the three methods and criticise the computations with regard to neatness, clearness, and completeness. (5) Underline final results with red ink.

Problem 70. Area Between Meander Line and Curved Boundary—Offsets at Irregular Intervals

Object.—To calculate the area between meander line and irregular or curved boundary from notes of Problem 5, page 34, or similar data, rectangular offsets having been taken at irregular intervals.

Rule.—Multiply the distance along the course, of each intermediate offset from the first, by the difference between the two adjacent offsets, always subtracting the following from the preceding. Also multiply the distance of the last offset from the first by the sum of the last two offsets. The sum of these products divided by two is the required area.

Procedure.—(1) Choose a form similar to that of the accompanying sample computations. Transcribe the data

and check the copy. (2) Calculate the area according to the above rule. Reduce to acres. (3) Check results with the slide-rule. (4) If the data of Problem 5 are used, calculate the areas of triangles as in the preceding problem and determine the total area of the field.

Hints and Precautions.—(1) Show all computations and make sure that they are clear and complete. (2) Do not neglect to prefix to each quantity its proper sign; if the difference

18.

Area Between Meander Line and Bog Brook
on
Brigham Farm.

Field Notes
Drawer 11,
Book T6,
Pages 37-39.

Computations
Aug. 16, 1911.
Comp't'd by J. E. C.
Checked by R. Q. S.

Line E - F.

Dist. from E ft.	Length of Offset, ft.	Difference	Products −	Products +
0 = E	0.0			
20	14.3	− 23.1	462.0	
65	23.1	+ 4.7		305.5
87	9.6	+ 18.5		1609.5
100	4.6	− 8.1	810.0	
131	17.7	− 13.9	1820.9	
148	16.5	− 19.4	2871.2	
160	37.1	− 6.5	1040.0	
200	25.0	+ 12.9		2580.0
225	24.2	+ 2.0		450.0
250	23.0	+ 5.8		1450.0
300	18.4	+ 5.0		1500.0
350	18.0	− 20.4	7140.0	
385	38.8	+ 9.3		3580.5
400	6.7	+ 38.8		15520.0
413.7 = F	0.0	+ 8.7		3599.2
			14,144.1	30,594.7
				14,144.1
			2	16,450.6
				8,225 sq. ft.
			or	0.1888 Ac.

Note: For Area Computations Within Traverse See Page 17 of This Book.

SAMPLE COMPUTATIONS 70.

between adjacent offsets is negative, the resulting double area will be negative. Thus the total double area may come out a negative quantity.

Practical Applications.—This method of determining areas may frequently be used in land surveying where the boundary is a stream, lake, or road. A meander line is run as near the

irregular boundary as is practicable, and rectangular offsets are taken from this line at points where the boundary changes direction. The small areas between offset lines are considered as trapezoids, which, although not quite true, gives results well within the allowable error.

Problem 71. Area Between Curved Boundary and Line—Offsets at Regular Intervals (Trapezoidal Rule)

Object.—To compute the area between a curved boundary and traverse line when rectangular offsets from the line to the boundary have been taken at regular intervals, assuming that the areas between offset lines are trapezoids. (The data of Problem 5, page 34, may be used.)

Formula.—*Total Area*

$$= \frac{d}{2}[(h_1 + h_n) + 2(h_2 + h_3 + \ldots . h_{(n-1)})]$$

in which d is the common distance between offsets, and $h_1, h_2, \ldots . h_n$ are the offsets.

Procedure.—(1) Record the data and calculate the area by the above formula. If different intervals are used along different parts of the boundary, treat each of such parts as a problem by itself. (2) Check all computations. (3) If the data of Problem 5 are used, calculate the areas of the triangles and the total area of the field.

Practical Applications.—The practical applications are the same as those of the preceding problem. It should be noted that with a boundary convex away from the traverse line this method will give results too small; convex toward the line, results too large.

Problem 72. Area by Simpson's One-third Rule

Object.—To compute the area between irregular or curved boundary and traverse line when rectangular offsets have been taken at regular intervals, by using Simpson's one-third rule. (The data of Problem 5, page 34, may be used.)

Formula.—$Area = \frac{d}{3}[h_1 + h_n + 2(h_3 + h_5 + \ldots h_{(n-2)}) + 4(h_2 + h_4 + \ldots . h_{(n-1)})]$

in which d is the common interval, h_1, h_2, etc., are the offsets, and n is an odd number.

Procedure.—From the data of the previous problem compute the area by the above formula. (2) Check all computations. (3) Compare the result with that of the previous problem.

Hints and Precautions.—(1) SIMPSON'S one-third rule can only be applied to an odd number of offsets taken at regular intervals. The base-line will therefore be divided into an even number of parts. Should the date under consideration be for an odd number of parts, the area of the first or last part may be computed as a trapezoid and the remaining area by the above rule.

Practical Applications.—This method is based on the assumption that the boundary is a parabola. It is held by some that it will give results more nearly correct than those obtained by the trapezoidal rule, but unless the boundary is of very definite character, and therefore located by precise measurements, it seems doubtful if there is any added advantage in its use.

PROBLEM 73. AREA BY DOUBLE MERIDIAN DISTANCE METHOD

Object.—To calculate the area within a closed compass or transit traverse by the double meridian distance method. (The data of Problem 9, 20, or 21, pages 43, 65, and 69 may be used.)

Rules.—Latitude = Distance × cos bearing angle.

Departure = Distance × sin bearing angle.

{Latitudes / Departures} are positive or negative according as they are {north / east} or {south / west}.

In any closed traverse the algebraic sum of the {latitudes / departures} must equal zero.

Compass Rule for Balancing.—The correction to be applied the {latitude / departure} of any course is to the total error in {latitude / departure} as the length of the course is to the perimeter of the field.

Transit Rule for Balancing.—The correction to be applied to the $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ of any course is to the total error in $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ as the $\left\{\begin{matrix}\text{latitude}\\\text{departure}\end{matrix}\right\}$ of that course is to the arithmetical sum of all the $\left\{\begin{matrix}\text{latitudes}\\\text{departures}\end{matrix}\right\}$.

Rules for Double Meridian Distances.—(1) The *D.M.D.* of the first course equals the departure of that course. (2) The *D.M.D.* of any other course equals the *D.M.D.* of the preceding course plus the departure of the preceding course plus the departure of the course itself. (3) The *D.M.D.* of the last course is numerically equal to the departure of that course, but with opposite sign.

The double area of any trapezoid equals the product of its *D.M.D.* and its corrected latitude.

Procedure.—(1) Transcribe necessary data from the field-book into a form similar to that shown in Sample Computations 73, page 186. Check the copy. (2) Calculate the latitude and departure of each course, using logarithms as shown in sample computations. Check results with the slide-rule. (3) Determine the total error in latitude and in departure, and compute the error of closure. (4) Determine the latitude and departure corrections by one of the preceding rules for balancing, using the slide-rule. (5) Apply these corrections, and check by taking the algebraic sum of the corrected latitudes and the algebraic sum of the corrected departures. Each of these sums should equal zero. (6) From the corrected departures compute the *D.M.D.'s*, using the preceding rules and starting from the most westerly point in the survey. If the last *D.M.D.* is not numerically equal to the last corrected departure, it will indicate that a mistake in addition has been made. (7) Compute double areas by the preceding rule, using logarithms and paying special attention to signs. Check computations. (8) Sum up the double areas, divide by two, and transform into acres. (9) Note the time taken for computation, and compare this method with that of Problem 69, page 180.

Hints and Precautions.—(1) Use logarithmic tables with number of places consistent with the precision of the field measurements (Art. 20, page 15). If the bearings have been determined with the surveyors' compass, four-place logarithms

will be sufficient; if angles have been taken to the nearest minute (in error less than 30″) with the transit, five-place tables should be used. (2) Checks should be applied after each of the steps in the computations. An absolute check on the work can, of course, only be had by re-computation, using methods that will give as many significant figures in the final result as did the original computations. However, the slide-rule will furnish an approximate check, which is very desirable. (3) If, after having calculated the latitudes and departures and after having checked them against large errors, the error of closure is found to be larger than that allowable, the computer may frequently locate the mistake, whether it be in computations or fieldwork, through the relation of total error in latitudes and total error in departures. Thus, if the mistake is in the length of one line and there are no other large errors, the ratio of the total error in departures to the total error in latitudes will approximately express the tangent of the bearing angle of that line, or if a mistake has been made in the latitude of a line, the departures may nearly close. The computer should, therefore, conduct a critical examination of results, and should then re-compute those values which seem most likely to contain the mistake. If the mistake is not brought to light when all latitudes and departures have been re-checked then, and only then, may he be warranted in concluding that the mistake occurred in the field. (4) The Compass Rule or Transit Rule will be used for balancing latitudes and departures according as the error is assumed to be as much in angles as in distances or as the error is assumed to be mostly due to erroneous lengths. (5) When the error of closure is small, the latitudes and departures may usually be balanced by inspection, without computing the corrections by either of the preceding rules. When the computer knows the conditions surrounding the fieldwork, he may often distribute the error according to his own judgment rather than by any fixed rule. (6) Often neither calculated nor magnetic bearings of lines are shown in the transit notes. If deflection or interior angles were taken, it will be convenient to assume one of the lines in the traverse as the meridian and calculate the bearings of other lines accordingly. If magnetic bearings are recorded in the field notes, they should not be confused with calculated bearings and used as the basis of computations, for their precision will not warrant such use. (7) Corrections for erroneous

Prob. 73. Area of Balsam Park, Island Pond, Vermont By D.M.D. Method.

47.

Field Notes Book No.3 Page 47.

Computations Aug. 17, 1913. Computed and Checked by J. Q. M.

Line	Cal. Bear.	Dist. 66 ft. Ch.	Latitudes N	Latitudes S	Departures E	Departures W	Corrected Lats.	Corrected Deps.	D.M.Ds.	Double Areas +	Double Areas −
A–B	S 80° 29½ W	34.464		5.694		33.991	− 5.693	−33.990	61.812		351.89
B–C	S 33 04 W	25.493		21.364		13.911	−21.361	−13.911	13.911		297.15
C–D	S 33 46½ E	33.934		28.205	18.867		−28.201	+18.867	18.867		532.06
D–E	N 87 58½ E	28.625	1.013		28.607		+ 1.013	+28.608	66.342	67.21	
E–A	N 027 E	54.235	54.234		0.426		+54.242	+ 0.426	95.376	5173.51	
		176.751	55.247	55.263	47.900	47.902	ΣL=0	ΣD=0		5240.72	1181.10
				55.247		47.900				1181.10	
				.016		.002				2 4059.62	
										2029.81 Sq. Ch.	
										or 202.981 Ac.	

E. of C. = $\frac{.016}{176.751} = \frac{1}{11,000}$

$E = \sqrt{.016^2 + .002^2} = 0.016$ Chains.

Line	A–B	B–C	C–D	D–E	E–A
Lat.	5.694	21.364	28.205	1.013	54.234
Log. Lat.	0.75542	1.32968	1.45032	0.00584	1.73427
Log. Cos.	9.21805	9.92326	9.91969	8.54899	9.99999
Log. Dist.	1.53737	1.40642	1.53063	1.45614	1.73428
Log. Sin.	9.99399	9.73689	9.74509	9.99973	7.89535
Log. Dep.	1.53136	1.14331	2.27572	1.45647	9.62964
Dep.	33.991	13.911	18.867	28.607	0.426
Log. Cor. Lat.	0.75534	1.32962	1.45026	0.00584	1.73434
Log. D.M.D.	1.79107	1.14336	1.27570	1.82179	1.97944
Log. D.A.	2.54641	2.47298	2.72596	1.82763	3.71378
Dble Area	351.89	297.15	532.06	67.21	5173.51

Note: Survey Balanced by Transit Rule.

length of chain or tape should not be overlooked. Constant errors of this sort will have no effect on the error of closure. (8) By starting with the most westerly point in the survey all the *D.M.D.'s* become positive; it is not necessary for the solution of the problem that this point be chosen, but it is customary.

Practical Applications.—The double meridian distance method of calculating the area *within a closed traverse* is universally used in preference to subdividing into triangles. It is generally agreed that it takes less time, is more systematic, and offers more easy checks; through the use of latitudes and departures, the error of closure is readily determined.

A method essentially the same as the preceding is that of double parallel distances used by some surveyors. In principle the two methods are identical, the only difference being that in double parallel distances (*D.P.D.'s*) the bases of trapezoids are along a line perpendicular to the meridian, while in double meridian distances they lie on the meridian itself. Thus, the rules for finding *D.M.D.'s* may be changed to rules for finding *D.P.D.'s* by substituting the word "latitude" for "departure"; and the rule for finding double areas will then be as follows: *The double area of any trapezoid equals the product of its D.P.D. and its corrected departure.*

Problem 74. Omitted Measurements—Bearing and Length of One Side Unknown

Object.—To determine the length and bearing of one side of a closed traverse, these measurements not having been taken in the field. It is assumed that the error in the known lines of the traverse is so small as to be inconsiderate.

Procedure.—(1) Calculate the latitudes and departures of the known lines as in the previous problem, and find their totals. (2) On the preceding assumption and since the algebraic sum of latitudes and of departures for any closed traverse is zero, it follows that the latitude and departure of the unknown line are numerically equal to the sums of corresponding quantities for the known lines, but with opposite sign. Therefore, determine the bearing and length of the unknown line by the equations:

$$\textit{Tan bearing angle} = \frac{\textit{Departure of line}}{\textit{Latitude of line}}$$

and

$$\textit{Length of line} = \sqrt{Lat^2 + Dep^2} = \frac{\textit{Lat. of line}}{\textit{cos Bear. ang.}} = \frac{\textit{Dep. of line}}{\textit{sin Bear. ang.}}$$

Hints and Precautions.—(1) Observe the hints and precautions of the previous problem that are here applicable. (2) It should be noted that the omission of measurements makes checking fieldwork through computations impossible, and throws all error into the unknown length and bearing. (3) It is well to plot the traverse with scale and protractor, thus graphically determining the unknown length and bearing.

Problem 75. Omitted Measurements—Bearing of One Side and Length of Another Side Unknown

Object.—To determine the length of one side and the bearing of another side of a closed traverse, these measurements having been omitted in the field. It is assumed that the errors in the traverse measurements are so small as to be inconsiderate.

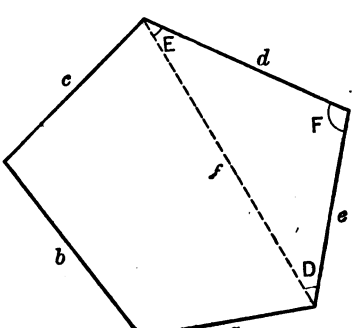


Fig. 33.

Procedure.—(1) Compute the latitudes and departures of known sides as in Problem 73, p. 183. (2) Suppose that *a*, *b*, and *c* (Fig. 33) are the known lines, *d* the line of unknown bearing, and *e* the line of unknown length. Then compute the bearing and length of the side *f* as in the previous problem, thus forming a closed traverse *abcf*. (3) The bearings of *e* and *f* being known, compute the angle *D*. (4) In the triangle *def*, two

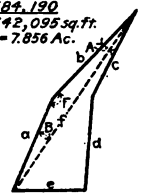
Black Addition.

Field Notes
Book No. 11
Page 4.

Computations.
Dec. 4, 1905.
Comp'd by F.K.R.
Checked by R.J.P.

Line.	Cal. Bear.	Dist. Ft.	Latitudes N	Latitudes S	Departures E	Departures W	D. M. Ds.	Double Areas. +	Double Areas. −
a	N 20° 00' E (N 45°00' E omitted)	800.0 (omitted)	584.50		212.73		212.73	124,350	
b		800.0	565.69		565.69		991.15	560,700	
c	S 30 00W	500.0		433.01		250.00	1306.84		565,700
d	S 5 04E	720.0		717.18	63.58		1120.42		803,540
e	W	592.0		0.00		592.00	592.00		
			Lat. f = +1150.19		63.58	842.00		685,050	1,369,240
						63.58			685,050
					Dep. f = +778.42				2|684,190

342,095 sq. ft.
= 7.856 Ac.



Line.	c	d	e	a	b		f
Lat.	433.01	717.18	0.00	584.50	565.69	Log Dep.	2.89121
Log Lat.	2.63650	2.85563		2.76678	2.75258	Log Lat.	3.06077
Log cos	9.93753	9.99830		9.97299	9.84949	Log Tan	9.83044
Log Dist.	2.69897	2.85733		2.79379	2.90309	Bear.	N 34°053E
Log Sin	9.69897	8.94603		9.53405	9.84949	Log cos	9.91812
Log Dep.	2.39794	1.80336		2.32784	2.75258	Log Dist.	3.14265
Dep.	250.00	63.58	592.00	212.74	565.69	Dist.	

Angle	B	f	A	Line	a	b
Bear. f	N 34°053E			Log f	3.14265	
Bear. a	N 20 00 E			Log sin A	9.27712	
Ang B	14°053			Log sin F	9.62593	
Log sin B		9.38637		Log a	2.79384	
Log f		3.14265		Dist.	622.0	
Log b		2.90309		Bear. a		N 20°00'E
Log sin F		9.62593		Ang(180-f)		+ 25 00
Ang.		155°00'	10°54.7	Bear		N 45°00E

Note:
Necessary Computations for Omitted Measurements Show in Vertical figures.
Further Computations Necessary to Determine Area Shown in *Slope* figures.

sides d and f, and the angle D are known; solve for the remaining unknown quantities, and from the bearing of f and angle E, compute the bearing of d. (5) Calculate the latitudes and departures of the sides d and e. As a check, total the latitudes and departures of the whole traverse; their sums should equal zero. Record in the form shown in Sample Computations 75, page 189.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 73 and 74 that are applicable to this problem. (2) It will be a considerable help to the computer if he first solves the problem graphically. Special attention

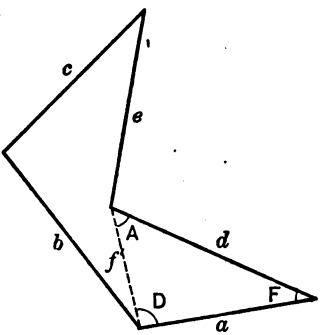


FIG. 34.

should be paid to signs, and *bearings should not be confused with angles*. (3) If the lines of omitted direction or length are not adjacent, they may be treated as though they were; for it is evident that a closed traverse will always close regardless of the order of the lines in that traverse. For example, suppose a and d (Fig. 33, page 188) are lines with omitted measurements in bearing or distance. Then in Fig. 34 are plotted the known lines parallel to their former positions beginning with b and going around the traverse in a clockwise direction. Then $bcef'$ is a closed traverse. The bearing and length of f' may be found as were the corresponding quantities for f in Fig. 33, and the triangle $af'd$ may be as readily solved for the unknowns as was the corresponding triangle when the lines containing omitted measurements were adjacent. From Fig. 34 it will be seen that if f' becomes zero a and d will be the same length but running in opposite directions. (4) *By method*

similar to the preceding, a solution may generally be performed when the lengths of two lines are omitted, or when the bearings of two lines are omitted. In the latter case there are two solutions, hence the approximate direction of the lines must be known. The solution, whether for two sides or two bearings unknown, becomes weak as the closing side f (Fig. 33) or f' (Fig. 34) approaches zero. If the closing side becomes zero, as it may where the sides containing omitted measurements are not adjacent, the problem becomes indeterminate by this method.

Problem 76. Subdivision of Land by a Line Starting from a Known Point on the Boundary

Object.—To cut off a required area from a closed traverse by a straight line starting from a known point on the traverse. (The data of Problem 9, 20, or 21, pages 43, 65 and 69 may be used.) See Sample Computations 76, page 192.

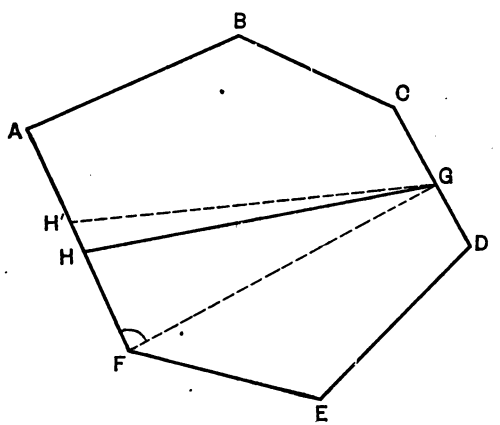


Fig. 35.

Procedure.—(1) Calculate the area within the traverse by method of double meridian distances (Problem 73, page 183). Plot the figure approximately to scale. Let this be represented by Fig. 35, and let G represent the known point from which the subdividing line must start. Draw a line GF to that corner of the traverse which, from inspection

Prob. 76. Computations for Balsam Park Division to Divide in two Equal Parts by a Line through "A".

55.

Field Notes
Book No. 3, Page 47.
Area Computations
Page 7 this Book.

Total Area (see P. 7).

Compt'd by W.O.F. Aug. 17, 1913.
Checked " H.W.C. " " " .

Line	Cal. Bear	Dist. 66 ft. Ch.	Latitudes N	Latitudes S	Departures E	Departures W	D.M.Ds.	Double Areas +	Double Areas −
A-B	S 80° 29½' W	34.46		5.69		33.99	61.81		351.89
B-C	S 33 04 W	25.49		21.36		13.91	13.91		297.15
C-D	S 33 46½ E	33.93		28.20	18.87		18.87		532.06
D-E	N 87 58½ E	28.63	1.01		28.61		66.34	67.21	
E-A	N 0 27 E	54.24	54.23		0.43		95.38	5173.51	
							Total	5240.72	1181.10
								Area 202.98 Ac.	
					Area	A-B-C-D.			
A-B				5.69		33.99	61.81		351.89
B-C				21.36		13.91	13.91		297.15
C-D				28.20	18.87		18.87		532.06
D-A	N 27 43 E	62.41	55.25		29.03		66.77	3688.78	
							Area A-B-C-D	125.38 Ac.	1181.10
					Area	A-B-C-F.			
A-B				5.69		33.99	61.81		351.89
B-C				21.36		13.91	13.91		297.15
C-F	S 33° 46½' E	25.22		20.96	14.03		14.03		294.08
F-A	N 35 12 E	58.75	48.01		33.87		61 93	2973.10	
									943.12
							Area A-B-C-F	101.50 Ac.	Check.

Formulas.

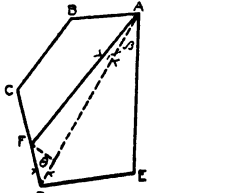
$$F\text{-}D = \frac{2\,A\text{-}D\text{-}F}{AD \sin\theta}$$

$$\tan\beta = \frac{F\text{-}D \sin\theta}{D\text{-}A - D\text{-}F \cos\theta}$$

$$F\text{-}A = \frac{F\text{-}D \sin\theta}{\sin\beta}$$

Area A-B-C-D = 125.38 Ac.
Half Park = 101.49
Area A-D-F = 23.89 Ac.

C-D = 33.93
F-D = 8.712
C-F = 25.22



Line						Line	C-F	F-A
Dist.	D-A 62.412	Log 2 A·D·F	2.67928	Log D·A·D·F cos θ	1.76534			
Log Dist.	1.79527	Log D-A	1.79527	Log D·F sine	0.88401			
Log cos Br	9.94704	Log sin θ	9.94391	Log tan β	9.11867	Log Dep.	1.14687	1.52978
Log Lat.	1.74231	Log D·F	0.94010	β		7° 29'		
Log D.M.D.	1.82457	D-F 8.712		Bear. D·A		N 27° 43' E	Log Dist.	1.40178	1.76903
Log D. Ar.	3.56688	Log D·F sin θ	0.88401	Bear. F-A		N 35° 12' E	Log cos	9.91969	9.91230
D. Area	3688.76	Log cos θ	9.67860	Log F-D	0.94010	Log Lat.	1.32147	1.68133
Log Dep	1.46291	Log D·F cos θ	0.61870	Log sin θ	9.94391	Latitude	20.961	48.015
Log tan Br.	9.72060	D-F cos θ	4.156	Log sin β	9.11498	Log D.M.D.	1.14700	1.79189
Bear.	N 27° 43' E	D-A	62.412	Log A-F	1.76903	Log D. Ar.	2.46847	3.47382
		D·A-D·F cos θ	58.256	A-F	58.753	D. Area	294.08	2973.10

of the figure, will come nearest being on the required line of division. (3) Calculate the latitude and departure of the line *CG*, and by methods of Problem 74, page 187, compute the latitude, departure, length, and bearing of *FG*. (4) By double meridian distances compute the area of *FABCG*, the amount cut off by the line *FG*. Find the difference between this area and that required. In the figure it is assumed that *FABCG* has an area greater than the desired amount, *GH* being the correct position of the dividing line. Then the triangle *GFH* represents this excess area, and, since angle *F* may be calculated from known bearings, compute

$$HF = \frac{2 \times \text{area } GFH}{FG \sin F},$$

the angle at *G*, and the bearing and length of the line *GH*. (5) As a check, compute the latitudes and departures of *GH* and *HA* and the area of *ABCGH*. This should equal the required area if the computations contain no mistake.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 73 and 74. (2) Often a line drawn from the known point in the boundary to any corner will nowhere near cut off the required area. Under such conditions or when the traverse has a large number of sides, it is advisable to plot the traverse with protractor and scale and to establish a trial line of subdivision such as *GH'* in Fig. 35, page 191. The planimeter (Problem 78, page 195) may be used to good advantage for finding the approximate area cut off by this trial line, and the line may be shifted until the area cut off agrees closely with that required. The scaled distance *AH'* may be used in the computations. It will be seen that the method of solution will now be identical with that used when the trial line was drawn to a corner.

Problem 77. Subdivision of Land by a Line Running in a Given Direction

Object.—To cut off a required area from a closed traverse by a straight line running in a given direction. (The data of Problem 9, 20, or 21, pages 43, 65 and 69 may be used.)

Procedure.—(1) Calculate the area within the closed traverse by double meridian distances (Problem 73, page 183).

(2) Plot the traverse to scale. Let this be represented by Fig. 36. Through the corner that seems likely to be nearest the line cutting of the required area draw a trial line DG in the given direction. (3) In the figure $ABCDG$, the latitudes and departures of AB, BC, and CD, and the bearings of DG and GA are known. By the methods of Problem 75, page 188, compute the unknown lengths, latitudes, and departures of DG

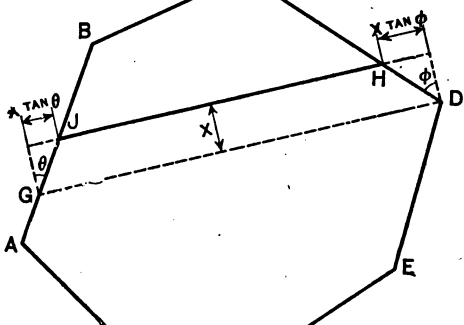


FIG. 36.

and GA. (4) By double meridian distances calculate the area cut off by the trial line. The difference between this area and that required is represented by the trapezoid $DGJH$, in which the side DG is known and the angles at D and G may be calculated from the known bearings of adjacent sides. (5) Calculate the angles θ and ϕ. Then

$$\textit{Area of trapezoid} = GD.x + \frac{x^2}{2}(\tan\theta + \tan\phi),$$

in which tan θ or tan ϕ is positive or negative according as θ or ϕ lies within or without the trapezoid. Solve this quadratic equation for x, or by trial find a value that will satisfy the equation. (6) Find values of GJ, HD, and HJ, and compute the area of $BCHJ$ by double meridian distance

method. If this and the required area are the same, the computations for subdivision will be correct.

Hints and Precautions.—(1) Observe the hints and precautions of Problems 73 and 75. (2) It should be noticed that the sign of the squared term in the above equation will depend upon the direction of the lines *GJ* and *DH* with respect to the base of the trapezoid. For example, had these lines formed obtuse angles with the base, the two end triangles would have been added. The above definition of signs for tan θ and tan ϕ makes the equation a general formula. (3) The computations should be recorded in systematic form as in the preceding problem.

Problem 78. Area with Planimeter

Object.—To determine area with the polar planimeter. It is assumed that the figure, the area of which is to be found, is plotted to scale. The data of Problem 32, page 92, may be used as in Problem 83, page 205.

Procedure.—(1) Set the tracing arm so that a complete revolution of the measuring wheel will bear some simple relation to the given scale and unit of measurement. For example, it may be convenient because of the plotted size of the figure to have a complete revolution represent 10 sq. in.; then set the index to the mark on the tracing arm stamped "10 sq. in." (2) Test the accuracy of the setting by constructing a figure of known dimensions, say a 2″ × 5″ rectangle, and with the pole or fixed point in position *outside* the figure, set the tracing point over one corner. Read and record to four places the score on the measuring wheel and disc. (3) Carefully and slowly move the tracing point completely around the boundary *in a clockwise direction* to the point of starting. Again read and record the score of the measuring wheel and disc. (4) It is evident that for the example stated above, the difference between the initial and final readings should equal one complete revolution. If this difference is too small, slightly reduce the length of the tracing arm by means of the slow motion screw, and *vice versa*. (5) In like manner repeat the tests until the desired relation is obtained. (6) Having adjusted the planimeter, traverse the perimeter of the figure the area of which is to be determined, using the same method of procedure as with the figure of known area. Check the operation. (7) Transform the difference between readings into area.

Hints and Precautions.—(1) The reason for moving the tracing arm about the figure in a clockwise direction is that by so doing readings of wheel will increase, or the area will always be positive, when the pole is outside the area. (2) If the pole is placed inside the area, and this is sometimes advantageous with large figures, the difference between initial and final readings will be positive or negative according as the area of the figure is greater or less than that of the *zero circle* (the circle about which the tracing point might be moved without causing the measuring wheel to roll), the direction of motion of tracing point still being clockwise. When the pole is inside the figure, the area of the zero circle must always be algebraically added to the area indicated by the score of the measuring wheel; and the result will always be a positive area, direction of motion being as indicated. (3) The area of the zero circle can be found by measuring any given area with the pole inside and outside of the figure. The area indicated by the difference between the two scores will be the area of the zero circle. (4) If it is of such shape that its area can not be determined with one setting of the planimeter, the figure should be divided into several parts. The area of each of these subdivisions may then be determined as though the figures were separate. (5) The planimeter should be manipulated very carefully, and no result should be accepted as correct until checked. The paper should be stretched on a level surface so that it is free from wrinkles, the contact edge of the measuring wheel should be bright and free from dirt, and the mechanism of the planimeter should be so adjusted that it will move with the utmost freedom, yet without lost motion. The position of the pole should be so chosen that the measuring wheel will stay on the paper as the tracing point is moved about the figure. (6) It is not necessary that a simple relation exist between revolutions of wheel and unit area; the tracing arm may be set at any convenient length and the revolutions of the wheel determined for a known area. Then by proportion, *the known area, is to the revolutions for known area, as any area, is to revolutions for that area.* The first part of the proportion is a constant as long as the length of the tracing arm remains unchanged. (7) On profiles the vertical and horizontal scales are often dissimilar; this will make no difference in finding the area. For example, with a horizontal scale of 100 ft. = 1 in.

and a vertical scale of 10 ft. = 1 in. the value of a square inch on the paper is 1000 sq. ft.

Practical Applications.—The polar planimeter is a specially useful instrument for finding the areas of irregular cross-sections, profiles, and land between curved or irregular boundaries and transit lines, its precision generally being sufficient for such quantities. It is extremely useful as a means of approximately checking areas determined by more precise methods.

Problem 79. Volume of Borrow-pit

Object.—To calculate the volume of earth removed from a borrow-pit. It is assumed that the computer has two sets of level notes, one set giving elevations of ground points before the earth was removed and the other set giving elevations at corresponding points after excavation has been completed.

Procedure.—(1) From the level notes calculate the depth of layer of earth removed at the various points. In the computation book make a sketch showing the plan of the borrow-pit and the amount cut at the various points. Let such sketch be represented by Fig. 37, page 198. (2) Then compute the volumes of the various rectangular, triangular, or trapezoidal truncated prisms by the following rules:

Volume Truncated Triangular Prism (as *abc*) =

$$\frac{A}{3}(h_1 + h_2 + h_3),$$

in which A is the horizontal cross-sectional area and h_1, h_2, and h_3 the corner heights.

Volume Truncated Rectangular Prism (as *defg*) =

$$\frac{A}{4}(h_1 + h_2 + h_3 + h_4).$$

The volume of a truncated trapezoidal prism is the sum of the volumes of the two truncated triangular prisms into which it may be divided (as *jklm*). Figures with more than four sides may be divided into rectangles, trapezoids, and triangles (as *nopqrs*). (3) If several adjacent rectangular prisms have the same cross-sectional area, compute them as one solid as follows: Multiply each corner height by the number of prisms of same cross-section in which it occurs; sum up these

results, and divide by four. Multiply this quantity by the cross-sectional area of a single prism. (4) Sum up the individual areas, and transform into cubic yards.

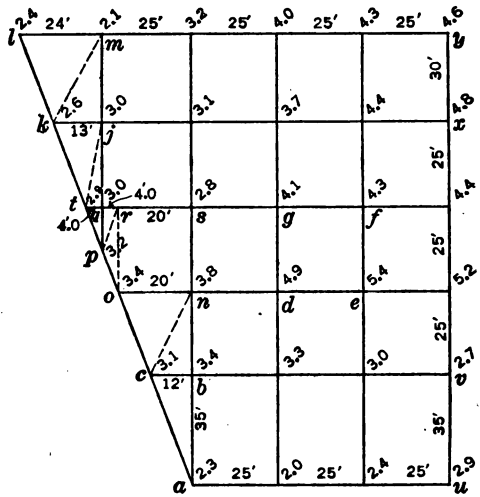


FIG. 37.—BORROW-PIT.

Figure	Cor. hts.	Area		Volume	
uvba	32.7	25 x 35	x ¼	7158	
vxjqsb	160.9	25 x 25	x ¼	25141	
xymj	59.9	25 x 30	x ¼	11231	
nors	13.0	25 x 20	x ¼	1625	
klm	7.1	12 x 30	x ⅓	852	
jkm	7.7	6.5 x 30	x ⅓	501	
kjt	8.5	6.5 x 25	x ⅓	460	
jqt	8.9	2 x 25	x ⅓	148	
tqp	9.1	2 x 12	x ⅓	73	
pqr	9.2	2 x 12	x ⅓	74	
opr	9.6	2 x 25	x ⅓	160	
con	10.8	10 x 25	x ⅓	858	
bcn	10.8	6 x 25	x ⅓	515	
abc	8.8	6 x 35	x ⅓	616	
				49407	cu. ft.
			or	1830	cu. yds.

Hints and Precautions.—(1) Data and results may be tabulated as shown in the computations below Fig. 37. (2) Frequently the second set of level notes does not give elevations of

points having the same horizontal position as those of the first set. When this is the case the volume can best be found by plotting the two profiles (one, of the ground before excavation, the other, of the ground after) of each line on cross-section paper. The area between original and final profiles is the cross-sectional area of the earth removed, and the volume between adjacent stations is a prismoid. The methods of calculating such areas and volumes is taken up in detail in the next problem.

Problem 80. Plotting Cross-sections; and Quantities of Earthwork

Object.—To plot cross-sections from field notes and to calculate quantities of earthwork. It is assumed that the cross-section notes are for railroad (Problem 30, page 88), highway, or similar work, and give cut or fill rather than elevations.

Formulas.—For a three-level section (levels at slope-stakes and center).

$$Area = \frac{h_c(d_r + d_l) + W/2\ (h_r + h_l)}{2}$$

in which h_c, h_l, and h_r are center, left, and right cuts or fills; d_l and d_r are the distances left and right from center to slope-stakes; and W is the width of the roadbed.

For volume between cross-sections the *End-area Formula* is

$$V = \frac{l}{2}(A_1 + A_2)$$

in which A_1 and A_2 are the areas of end bases or cross-sections, l is the distance between cross-sections, and V is the volume in cubic feet.

The *Prismoidal Formula* for volume between cross-sections is

$$V' = \frac{l}{6}(A_1 + 4A_m + A_2)$$

with the same nomenclature as above, except A_m which is the middle area or area half-way between the end sections and which is found *by averaging corresponding dimensions of the end sections*, A_1 and A_2, *not by averaging areas* A_1 and A_2.

Procedure.—(1) Beginning near the top and left-hand end of the sheet or roll of cross-section paper, choose convenient

heavy horizontal and vertical lines as grade and center lines. With these as co-ordinates plot the cross-section notes of the first station, counting the number of spaces out from the center and up from the grade line corresponding to these distances in the notes. Usually the scale used on such work is 1 in. = 10 ft. or one space equals 1 ft., but it may be larger or smaller. Mark the plotted points with dimensions identical with those of corresponding points in the notes. Draw straight lines showing roadbed and side slopes of cut or fill and the original ground, thus enclosing the section. Outside and just below the cross-section, and near the center line mark the station number. (2) At a convenient distance below and on the same center line, plot the next section in similar manner. (3) When the bottom of the sheet is reached, plot the next section a little farther to the right and at the top of the sheet; and in this way continue until all plotting is done. (4) Calculate the area of each three-level section by the preceding rule (if it is irregular, by dividing into triangles), and show its value within the section (as 123 sq. ft.). (5) Compute volumes by one of the preceding rules, and show the volume of each prismoid between its end sections (as 97 cu. yd.). (6) Find the total yardage in each cut and fill, and mark these totals conspicuously. (7) Make an appropriate title.

Hints and Precautions.—(1) All plotting and computations should be checked, preferably by someone other than the original computer. In plotting, two men can work together to better advantage than one alone, one calling off field notes while the other plots. When checking the plotted sections it is well to change positions and to reverse the operation by calling out measurements as they appear on the plotted cross-section. (2) Cross-section sheets may be regarded as working drawings, and as such should not receive undue elaboration. Titles should be free-hand, and should show the name of the work, the location of the sections by station numbers, the number of the field note-book and pages containing the original notes, the date of plotting, and the names of computer and checker. It is customary to ink the outlines of the sections in black ink. Areas and volumes should be shown in ink only after having been checked. (3) When cross-sections are plotted continuously throughout a roll of cross-section paper, which is a common method on extended surveys, a title should be placed on each end of the sheet, on the back

and near the lower edge of the paper, so that when the roll is filed it may be identified without unrolling. (4) The areas of other and more irregular forms of cross-sections such, for example, as cross-sections of borrow-pit (Problem 79, page 197) may be conveniently determined by plotting on cross-section paper. The method of procedure will be similar to the above, except that elevations must often be plotted instead of cut or fill. For each section it will be convenient to let a heavy horizontal line represent the multiple of 10 ft. nearest the elevations shown in the notes of that section. Elevations can then be plotted by counting spaces from this line. (5) The planimeter can often be used to good advantage in determining the areas of irregular cross-sections (Problem 78, page 195), but the areas of regular three-level sections can be computed by the given formula quicker than with the planimeter. If a planimeter is not to be had, irregular sections must be divided into rectangles, triangles, or trapezoids, and the area of each of the smaller figures must be computed. (6) The *Prismoidal Formula* for volume is generally considered as giving results more nearly correct than the *End-area Formula;* however the latter is more generally used because it is simpler and the theoretical error introduced is small compared with the probable error due to changes of slope between sections which are unnoticed or disregarded in the field. As an example of the magnitude of the error in volume introduced through apparently insignificant variations of the surface, suppose that between two cross-sections 50 ft. apart a sag takes place gradually until at a point midway between the sections it amounts to a depth of 0.5 ft. over a width of 20 ft., thus forming two pyramids of error each with a base of 10 sq. ft. and an altitude of 25 ft. The volume of these pyramids will be $\frac{1}{3} \times 20 \times 0.5 \times 50 = 167$ cu. ft. = 6 cu. yd. The preceding example also shows the uselessness of computing volumes of earthwork closer than to the nearest cubic yard, unless the sections are very close together and the fieldwork is performed with more than the usual care.

Practical Applications.—The methods of computing the volume of earthwork outlined in this problem are the most common in use. Cross-section paper greatly facilitates the plotting, and area computation of irregular sections is thereby simplified. It is not necessary that three-level sections, or

other forms of sections not considered in this book but common on railroad and highway work, be plotted in as far as the computations of their total areas are concerned, but these plotted cross-sections are frequently made the basis of preliminary estimates of material excavated, surface lines at the end of each month being plotted on the original section. At any station the area between original surface and the surface at the end of any month as plotted on the cross-section paper shows the cross-sectional area of material moved. If the material is not to grade at any given station at the time of the monthly estimate, that is, if the excavation or embankment is only partially completed, the cross-sectional area of the material will form an irregular figure which cannot be readily computed unless plotted. Frequently, for the purpose of showing the progress of the work, the areas between lines showing monthly progress are tinted with water colors, a different color for each month.

Problem 81. Plotting Profile

Object.—To plot a profile from level notes, and to fix the grade line for railroad, highway, pipe-line, or similar work. (The data of Problem 28 or 29, pages 85 and 87, may be used.)

Procedure.—(1) Choose a horizontal and vertical scale in keeping with the purpose of the profile. If it is for a railroad or highway, and later field measurements will give data for calculating volumes of earthwork, a scale of 1 in. = 400 ft. (horizontal) and 1 in. = 20 ft. (vertical) may be conveniently used with Plate "A" profile paper; if the profile is to be made the basis of earthwork computations, as it may be when it is for a sewer, a scale as large as 1 in. = 40 ft. (horizontal) and 1 in. = 4 ft. (vertical) may be required. (2) Examine the field notes to determine the range between points of maximum and minimum elevation. Number each of the heaviest horizontal lines with its elevation. These numbers, for convenience in plotting the profile, should be multiples of 50 ft. when the vertical scale is small, 10 ft. when the vertical scale is large, and they should be such that a maximum amount of the profile may be plotted without having a break in its continuity. For example, in Fig. 38 the profile may range between elevations of 450 and 650 ft. without leaving the

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sheet. Near the foot of heavy vertical lines record the 100-ft. station numbers, these numbers increasing from left to right, and being multiples of ten for small scales. (3) From the profile level notes plot the profile. The numbers on heavy horizontal and vertical lines will serve as a guide. First locate the vertical line of the station (if it is a plus station the nearest vertical line), then run up this line until the horizontal line corresponding nearest to the elevation is reached, and finally by estimation plot the point. (Using Fig. 38 for example, suppose a point of chainage 642 + 25 and elevation 543.4 is to be plotted. Look along on the profile paper to station 640, then at the second line beyond run up to eight spaces below the heavy line representing elevation 650, and finally plot the point by estimation one-fourth the distance to the next vertical line, and 0.4 the distance to the next horizontal line above.) Through the plotted points draw a free-hand curve. Show the names of streams and roads crossed, directly above their crossings on the profile. Check the profile, and ink it with *black ink*. (4) Fix the grade line. Its determination will depend on the purpose of the survey. If it is for a railroad or similar work, the cuts and fills should nearly balance and the rate of grade should not exceed a stipulated amount; if it is for a sewer, it must allow for drainage and must be below the frost line. At each change of grade draw a small circle, and indicate the elevation (and plus if it does not fall at a full station) of each of these points on a vertical line directly under the small circle. Draw a continuous line through the foot of these verticals, and on it record the rates of grade, as shown in Fig. 38. Ink, *in red*, the grade line and all lines and numbers referring to it. (5) Near the bottom of the paper indicate in *black* ink the horizontal alignment, using a scheme similar to that shown in the figure. (6) Make a title showing the name and location of the work, the horizontal and vertical scale, the date, and the name of the draftsman.

Hints and Precautions.—(1) The profile should be checked by reading elevations and stations from the profile, not the level notes. Two men can work together to good advantage —one reading the notes, while the other plots the profile; when checking the profile, they should exchange places. (2) It is a common mistake to read the elevations of turning-points and bench-marks as ground elevations. This may be

avoided by enclosing each of these elevations in the field note-book with a line. (3) When most of the elevations are for full 100-ft. stations, it is common practice for the "reader" not to call off station numbers except at plusses, but the notes and profile are usually tallied every ten stations. (4) A more uniform width of line can be obtained if the profile is inked with a ruling pen rather than with a lettering pen. The draftsman should not endeavor to make it a series of straight lines, for this will not be the condition of the actual profile; on the other hand, he should not round off the summits and depressions an undue amount, for such points will naturally be greatly exaggerated in sharpness on account of the relation between horizontal and vertical scales. (5) Printed profile paper may be obtained in three standard styles: Plate "A" having 4×20 spaces to the inch, Plate "B" having 4×30 spaces to the inch, and Plate "C" having 5×25 spaces to the inch. The use of Plate "A" paper is the most common; it may also be used for cross-section paper. (6) Sometimes the profile of a line is shown on the same sheet with the plan, the horizontal position of points on the profile being determined by projections from corresponding points on the map. Since the plan of a line is not usually straight and will not, therefore, be parallel to the horizontal lines of the profile, the plotted distance between 100-ft. stations on the profile will be a varying quantity depending upon the relative directions of plan and profile. This method of plotting makes comparison between plan and profile very simple, but it can not be used if the change in horizontal direction of line is great within the limits of the sheet. (7) Profiles are often plotted from contour maps. For example, an excellent study of the possible routes of a proposed railway through country that has been mapped by the U. S. Geological Survey can often be made by projecting these proposed routes on the Geological Survey maps. A profile of each route may then be plotted with scaled distances to contour crossings. (8) Profiles of a series of parallel lines on large-scale contour maps, plotted in the above manner, are frequently made the basis of grading estimates. Such profiles are but little different from cross-sections (Problem 80). (9) When profiles are in rolls, besides the title on the face of the drawing, there should be one at each end on the back of the paper, so that unrolling to determine the contents will be unnecessary.

Practical Applications.—Since the location of many lines depends very largely upon the relief of the country through which the lines pass, the profile is as important as the plan. It is made the basis of estimates of cost preliminary to the work of actual construction, and only through its use can grades be laid out to good advantage. The progress of construction can be more readily shown by profile than in any other way; the *progress profile* is usually tinted with water-colors to indicate the work completed, a different color being used for each month.

Problem 82. Profile and Quantities of Excavation for Pipe-line

Object.—To plot a profile of pipe-line, and to compute the quantity of excavation by two methods. (The data of Problem 29, page 87, are to be used.)

Procedure.—(1) Plot the profile and grade line from notes as explained in the preceding problem, using scales of 1 in. = 40 ft. (horizontal) and 1 in. = 4 ft. (vertical). (2) With the planimeter determine the area between profile and grade line as explained in Problem 78, page 195. (3) Compute the volume of excavation by multiplying this area by the estimated average width of trench. (4) From the cut as shown in the notes compute the cross-sectional area at each station. Compute the volume by method of average end areas (Problem 80, page 199). (5) Compare the results of the two methods.

Hints and Precautions.—(1) For pipe up to 18 in. in diam. the width of the bottom of the trench may be taken as 2 ft. 6 in. The width of trench for pipes of larger diameter may be assumed as 9 in. greater than the diameter of the pipe. The width of the top of the trench will be somewhat greater, depending upon the nature of the material excavated and the depth of excavation. (2) If Problems 78, 80, and 81 have not already been performed, study them carefully.

Problem 83. Profile and Cross-sections of Street

Object.—To plot the profile and cross-sections of a proposed street from notes of Problem 32, page 92, and to compute the quantities of earthwork.

Procedure.—(1) Plot the profile of center line, and determine the grade line according to methods of Problem 81, page

202; use Plate "A" profile paper and scales of 1 in. = 40 ft. (horizontal) and 1 in. = 4 ft. (vertical). (2) Plot the cross-sections on the same sheet, using scales of 1 in. = 20 ft. (horizontal) and 1 in. = 4 ft. (vertical). Use methods of Problem 80, page 199. Plot the sub-grade of sidewalks and street. (3) Find the areas of cross-sections with the planimeter (Problem 78, page 195) and compute the volume by average end-areas (Problem 80, page 199). (4) Make a neat and appropriate title.

Hints and Precautions.—(1) If Problems 78, 80, and 81 have not already been performed, study them carefully. (2) Arrange the profile and cross-sections as shown in Fig. 39.

Problem 84. Plotting by Tangents

Object.—To plot a given traverse by method of tangents. (The data of Problem 9, 20, or 21, pages 43, 65 and 69, may be used.)

Procedure.—(1) Tabulate angles and distances of the given traverse in the computation book. If angles are given as azimuths, change them to deflection angles. (2) Look up and record in tabulated form the natural tangent of each deflection angle less than 45°; the cotangent of each angle greater than 45°. Check the tabulated data. (3) Plot the traverse roughly to small scale using the protractor for angles; note its general form. (4) Carefully plot the first line on drawing paper to the required scale, estimating its position by means of the small-scale sketch. The line should lie so that the drawing will, when finished, be symmetrical with the sheet. Suppose this first line is AB, Fig. 40. (5) If the deflection angle is *less* than 45° prolong AB a distance of 10 in. to b, and carefully erect a perpendicular of indefinite length in the direction of the angle turned. Scale bb' equals 10 *in.* $\times$ *tan of deflection angle.* Draw line Bb'. The angle α equals the given deflection angle if no error has been made in plotting. Scale the distance BC. (6) If the deflection angle is *greater* than 45°, as for example β, lay off Cc 10 in. long and perpendicular to BC. At the extremity of this perpendicular erect a parallel to the line BC of indefinite length and scale off distance cc' equals 10 *in.* $\times$ *cot deflection angle.* The angle laid off from the perpendicular Cc is $(90° - \beta)$, and if no mistake has been made in plotting, its complement β is the same as the given deflection

angle. Plot the distance *CD*. (7) By the above methods plot the entire traverse, being very careful to verify all measurements as soon as they have been plotted. (8) If it is a closed traverse and does not close on the sheet, it may be due to errors in the fieldwork (unless it has been balanced by latitudes and departures) or in the plotting. Carefully go over the traverse, scaling all distances and testing all perpendiculars. (9) If errors are not discovered by such investigation, cut a strip of paper somewhat longer than the combined length of the traverse lines. Let *a*, *b*, *c*, *d*, etc., be points to be

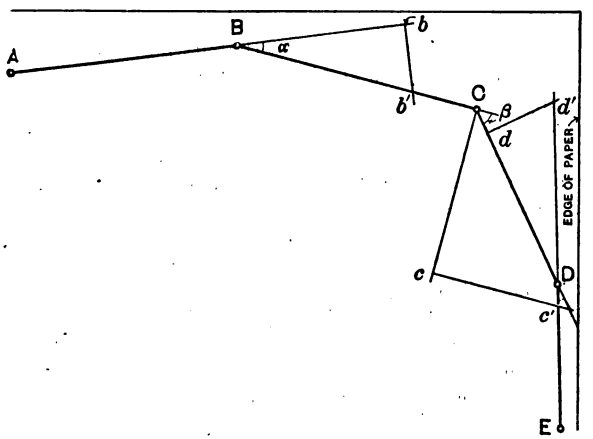


FIG. 40.—PLOTTING BY TANGENT OFFSETS.

marked on this strip corresponding to traverse points *A*, *B*, *C*, *D*, etc. Near one end mark point *a* on its edge. Place this mark at *A* in the traverse and with the edge of the strip along *AB*, mark point *b* on its edge opposite *B* on the traverse. Move the strip to *BC* with *b* opposite the corresponding point in the traverse, and mark point *c*. Continue in this way around the entire traverse. Scale the total length marked off on the strip. This scaled distance should agree very closely with the sum of the lengths in the given data. (10) If no error is discovered, apply the same scheme to tangent offsets. (11) If the traverse still fails to close, the error may be assumed to be in the fieldwork and it may be distributed among the different courses so that the traverse will close. (12) If it is not a

closed traverse perform tests (8), (9), and (10). In addition, if possible transfer lines by triangles, parallel ruler, or straight-edge to intersect other lines several courses in advance; scale the tangents of angles of intersection between these lines, and compare these scaled distances with the natural tangents of corresponding calculated angles. (13) Make a neat and appropriate title.

Hints and Precautions.—(1) Particular care should be taken in scaling the lengths of the lines of the traverse and the tangent distances. Points should be pricked with a needle. The scale should be between draftsman and line, and the eye should be above each point as it is plotted. Lines should be drawn with a sharp pencil held so that its point will lie close to the edge of triangle or straight-edge. (2) The perpendicular at any point should not be erected by placing the base of triangle along the base-line with the corner of the triangle at the point, but should be constructed as follows: Fit the hypotenuse of triangle to the base-line, and place a straight-edge in contact with the base of the triangle. Turn the triangle through 90°, and slide it along the straight-edge until its hypotenuse passes through the point. A line drawn along the hypotenuse will be perpendicular to the given base-line providing the triangle has edges that are 90° apart. A second triangle is often used instead of the straight-edge. (3) It is well to test each perpendicular by measuring the hypotenuse of a right triangle of which the perpendicular is one side and the base-line another; if distances of 10 in. are scaled along these sides, then the length of the hypotenuse should equal the square-root of 200 or 14.14 in. It is well to remember this number. (4) Parallels should be laid off with two triangles or with triangle and straight-edge by methods similar to those explained above for erecting perpendiculars. (5) There is no reason why a base of 10 in. should be chosen except for simplicity in computing the tangent offsets; but, although bases of shorter length ordinarily should not be used, because of the increased accuracy of plotted angles it is often advantageous and sometimes necessary on long traverses to use bases of greater length, say 20 in. The distance between graduations on the usual form of natural scale (ten spaces to the inch) make close estimation of tangent distances impossible or at least very uncertain. For this reason it is common practice to divide the tangent distances by two and to plot with

the 50 scale. (7) Often when the traverse approaches the edge of the sheet, the usual method of laying angles will not be practicable on account of the construction lines falling off the paper. The method shown in Fig. 40, page 207, for laying off the angle at D may generally be resorted to, the base-line Dd being measured back along the line CD and the tangent offset dd' being laid off as usual. The line $d'D$ is then prolonged beyond D, and E is located in the customary manner.

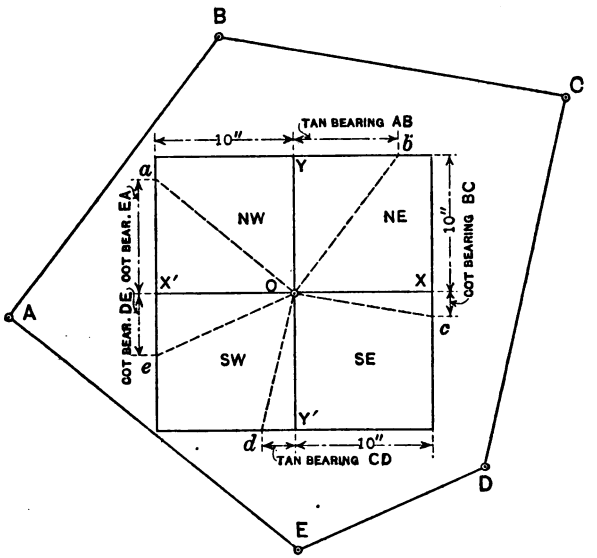


FIG. 41.—PLOTTING BEARINGS BY TANGENTS.

(8) Tangents increase rapidly for angles between 45° and 90°. It is for this reason that the cotangents or, what is the same thing, the tangents of the complements of these angles are plotted. (9) When the directions of lines are given by *bearings or azimuths*, some draftsmen prefer to use the method illustrated by Fig. 41. Four 10-in. squares with sides perpendicular or parallel to the meridian are constructed as shown, corresponding to the four quadrants in which bearings or azimuths may fall. The tangents of bearing angles of less than 45° or azimuths corresponding to such bearing angles are

scaled east or west from the line YY' along the outer sides of squares forming corresponding quadrants; in a similar manner the cotangents of bearing angles greater than 45° or azimuths corresponding to such angles are scaled north or south of the line XX'. Lines joining these points with the intersection of the X and Y axes will have the same bearings as corresponding lines of the traverse.

For example, the line AB has a bearing N 37° E. Its tangent offset is therefore scaled from YY' along the north side of the NE quadrant, thus locating b. The line ob must, therefore, be parallel to AB. The line BC has a bearing of S 71° E which is greater than 45°. The point c is, therefore, located by scaling the cotangent distance south from the line XX' along the east side of the SE quadrant and oc is parallel to BC.

In this way the angles of the whole traverse are laid off. The line ob is then transferred to its proper position with parallel ruler or triangles, and the distance AB is plotted to scale; oc is transferred until it passes through B, and the distance BC plotted to scale, etc.

Practical Applications.—Plotting by tangents is one of the most common methods of plotting traverses. In point of accuracy it is much superior to the protractor and is generally considered as slightly better than the chord method (Problem 85, page 211) but it is inferior to the method of total latitudes and departures (Problem 86, page 213). In point of simplicity of computations it is to be preferred to any other graphical method. Without doubt it is for the latter reason that the method is so commonly used. One of the principal disadvantages of the method is that the accuracy of the position of each line depends upon the accuracy of the position of the preceding line. For this reason it should not be used in plotting long traverses, whether they be closed or continuous. Plotting by tangents of bearing angles or azimuths partly overcomes this difficulty, since the direction of each line is independent of the directions of all other lines. However, there may be a considerable error introduced in transferring the lines from square to traverse, and erroneous lengths will have the same effect as in plotting by tangent offsets of deflection angles.

A very useful labor saving device for plotting by this method is known as the *tangent protractor*. It is nothing more than a scale subdivided into tangent distances for a 10-in. base

corresponding to angles of multiples of 10′ between 0° and 45°. The values of angles, not tangent distances, are numbered on the protractor, 45° being 10 in. from the zero end, 30° being 5.77 in. from the zero end, etc. Its use is identical with that of the ordinary scale except that points are plotted opposite given angles rather than opposite pre-determined tangent distances; the use of trigonometric tables is therefore unnecessary. To facilitate the plotting of angles between 45° and 90° by cotangent distances, a second set of angular values are placed under the first, the former decreasing from 90° to 45° as the latter increases from 0° to 45°. The tangent protractor can not be used conveniently for a base other than 10 in. in length.

Problem 85. Plotting by Chords

Object.—To plot a given traverse by method of chords. (The data of Problem 9, 20, or 21, pages 43, 65, and 69 may be used.)

This method is much like that of tangents explained in the preceding problem, except that instead of erecting a perpendicular at the end of a 10-in. base-line, an arc of 10-in. radius is struck. The chord distance for the given angle is then scaled from the point of intersection between arc and base-line to a point on the arc. It is evident that the length of this chord is $2R \sin \frac{1}{2}\alpha$, in which R is the radius of 10 in. and α is the given angle.

Procedure.—(1) Tabulate the given data in computation book in form indicated below. If angles are given as bearings or azimuths, change to deflection angles. (2) Compute the half angles and record their natural sines, and compute the

Station	Distance	Def. Ang.	½ Ang.	Sin ½ Ang.	Chord Length

chord lengths (2×10 in. $\times \sin \frac{1}{2}$ *ang.*). Check the computations. (3) Plot the traverse roughly to small scale, using the protractor for angles; note its general form. Carefully plot the first line of the traverse on drawing paper to the required scale, estimating its position by means of the small-scale sketch. The line should lie so that, when the drawing is finished, it will be symmetrical with the sheet. Let this first line be AB, Fig. 42, page 212. (4) Prolong AB an indefi-

nite length. About B as a center swing an arc of 10-in. radius through b in the direction of the angle. With the scale plot the point b', the distance bb' being made equal to the corresponding tabulated chord length. Draw a line through Bb', and plot the distance BC. (5) With the ordinary scale it will be inconvenient to plot distances in excess of 12 in. This chord length, for a radius of 10 in., corresponds to an

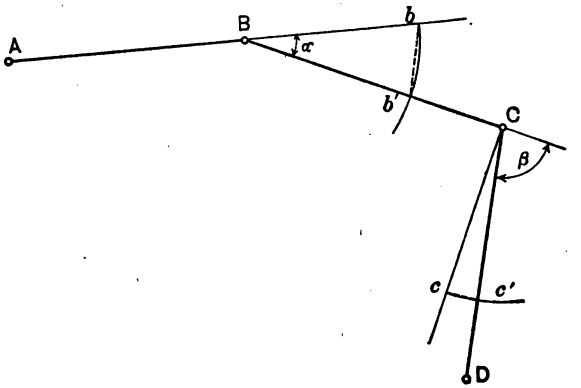


FIG. 42.—PLOTTING BY CHORDS.

angle of 73° 46′. For an angle in excess of this amount, compute the chord length corresponding to the complement of the angle. Let β be such angle. At C erect a perpendicular and swing an arc of 10-in. radius as before. From c scale the computed chord distance cc'. If no mistake has been made in plotting, Cc' is at the given angle with BC prolonged. Scale the distance CD. (6) By the above methods plot the entire traverse. (7) Check the traverse by methods similar to those of the preceding problem. (8) Make a neat and appropriate title.

Hints and Precautions.—(1) The whole of Problem 84 should be understood before attempting the chord method, for on account of the similarity of the two methods, much of the detail is omitted from the above description. (2) It is essential that the plotting and checking be done with as great care as with the tangent method. In swinging an arc with radius of 10 in. it is best to use beam compasses, for the ordinary dividers when extended to such length, even though

they have an extension arm, are too flexible to make the drawing of a true circular arc a certainty. A very good substitute for beam compasses is a thin strip of wood pierced by two very small holes, so that when a needle is inserted in one hole and the sharp point of a pencil in the other, the two points will be exactly 10 in. apart. (3) If a table of chords is available * the chord lengths may be obtained directly, and no computations are necessary. (4) When directions of lines are given by bearings or azimuths instead of deflection angles, some draftsmen prefer to use a method of plotting by chords corresponding to that explained for plotting by tangents (Problem 84, Hints and Precautions (9)), and illustrated by Fig. 41, page 209, except that instead of using a 20-in. square divided into four 10-in. squares, a 20-in. circle is divided into four quadrants. This method of plotting azimuths or bearings by chords has the same advantages and disadvantages as the corresponding method for plotting by tangents.

Practical Applications.—The chord method of plotting is not as extensively used as the tangent method, and is generally regarded as being less accurate. There seems to be no reason why its precision should be less, provided the arcs are properly constructed with beam compasses. On account of computations, it is not as practicable as the method of tangents, unless the draftsman has access to a table of chords.

Problem 86. Plotting by Total Latitudes and Departures

Object.—To plot a given traverse by the method of total latitudes and departures or co-ordinates. The data of Problem 9, 20, or 21, pages 43, 65, and 69 may be used.

Rule.—The { total latitude / total departure } of any point equals the algebraic sum of { latitudes / departures } of lines lying between that point and the { reference parallel / reference meridian } passing through the origin. { Total latitudes / Total departures } are positive or negative according as the corresponding points lie

{ north or south of the reference parallel. / east or west of the reference meridian. }

* See Trautwine's *Engineer's Pocket-book* (page 143, 1909 edition).

Procedure.—(1) Transpose the given data to computation book in form shown in the accompanying sample computations, page 216. If directions of lines are given in azimuths or deflection angles, calculate bearings referred either to the true or an assumed meridian. (2) Compute the latitudes and departures of the traverse lines, and if it is a closed traverse, balance the survey (Problem 73, page 183). Assume one of the traverse points as the origin of co-ordinates (*A* in computations), and calculate total latitudes and departures by the preceding rule. (4) Check all computations. (5) From the total latitudes and departures determine the size of the enclosing rectangle, the four sides of which pass through the extreme eastern, western, northern, and southern points of the traverse. (6) Roughly plot the enclosing rectangle and traverse to small scale, and note their relative positions. (7) On drawing paper plot the enclosing rectangle to the required scale, estimating its position on the sheet by means of the small-scale sketch. The traverse, not the rectangle, should be symmetrical with the sheet; therefore, the sides of the rectangle may or may not be parallel to the edges of the paper. (8) Test the accuracy of the plot by scaling the length of the diagonals. The scaled and calculated distances should agree very closely. Let *HJKL*, Fig. 43, be the enclosing rectangle of the traverse of which the computations are shown. (9) Plot the reference meridian (if it is not one of the sides of the enclosing rectangle) by scaling *JA* and *HM* equal corresponding computed total departure. Check by scaling *ML* and *AK*. In a similar manner plot and check the reference parallel. (10) If the drawing is of large size and there are a considerable number of points in the traverse, to facilitate plotting, construct other meridians and parallels so that the area will be divided into squares the sides of which are less than the length of the scale used (less than 12 in. with the ordinary scale) and represent some whole number of hundreds or thousands of feet at the given scale. For example, in the figure, meridians are drawn at intervals of 1000 ft. west of the reference meridian; parallels, at intervals of 1000 ft. north of the reference parallel. If the scale of the drawing is 100 ft. = 1 in., the actual length of the sides of the resulting squares is 10 in. Number each of these lines with its distance from the reference meridian or parallel. (11) Locate each traverse point by plotting its total latitude and departure.

Mark it with a fine dot enclosed in a small circle; draw traverse lines up to this circle, but not inside. Check by scaling the length of the preceding traverse line. For example, with origin at *A*, *F* has total N latitude of 1250 ft. and total E departure of 394 ft. (See Sample Computations 86, page 216.) Then, above parallel 1000 N, scale *ab* and *cd* equal 250 ft.,

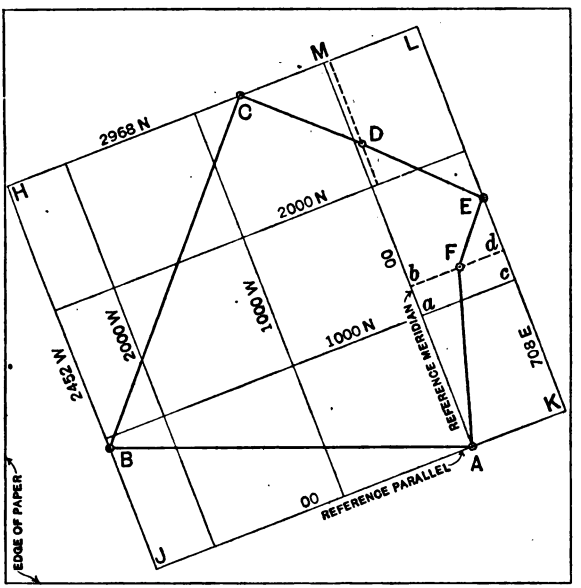


FIG. 43.—PLOTTING BY TOTAL LATITUDES AND DEPARTURES.

draw line *bd*, and scale *bF* equals 394 ft. Scale *EF*; it should equal the distance given in the notes, if no mistake has been made in plotting or computing. (12) Finish the drawing in pencil without erasing construction lines. Make a neat and appropriate title.

Hints and Precautions.—(1) Meridians and parallels should be accurately constructed. Parallel lines should not be drawn with the T-square by the common method of sliding its head along the edge of the drawing board, for such lines are seldom truly parallel. Having, by trial, constructed a true enclosing rectangle, the direction and position of all other

nes should be based entirely upon linear measurements. The T-square may be used as a straight-edge, but that is all. The greatest pains should be taken in plotting the enclosing ectangle, since upon its precision depends the accuracy of the

48.

Prob. 86. Carry Pond Traverse. Total Lats. and Deps.

Field Notes Book No. 4 P. 27
Log. Computations on P. 47 this Book.

Comp'd by W.E.L.
C'k'd by C.W.B.
Nov. 4, 1912.

Sta.	Cal. Bear.	Dist. Ft.	Latitudes N	Latitudes S	Departures E	Departures W	Adjusted Lats.	Adjusted Deps.	Total Lats. N	Total Lats. S	Total Deps. E	Total Deps. W
A	N 69°07' W	2622	934			2450	+ 936	-2454	0	0	0	0
B	N 41°50' E	2722	2030		1816		+2032	+1812	936			2452
C	S 46°20' E	946		652	686		- 652	+ 682	2968			642
D	S 41°52' E	950		672	670		- 670	+ 668	2316		40	
E	S 38°30' W	506		396		314	- 396	- 314	1646		708	
F	S 17°30' W	1312		1252		392	-1250	- 394	1250		394	
A		9058	2964	2972	3172	3156	±2968	±3162	0	0	0	0

E. of C. = 8/3058 = 1/383

2964 3156
8 16

SAMPLE COMPUTATIONS 86.

hole drawing. (2) If latitudes and departures are com-uted solely for the purpose of map construction, they should ot be determined with greater precision than can be used in lotting. For a scale of 1 in. = 100 ft. total latitudes and lepartures should be correct to the nearest foot; for a scale of

1 in. = 500 ft., to the nearest 5 ft. (3) It sometimes happens that the length of a line will check when its direction is slightly in error. This will occur when the angle between line and meridian or line and parallel is small; for this reason points upon which such lines depend should be checked by re-measuring the total latitude and departure as well as by scaling the length of the line. (4) Some draftsmen prefer to consider the southwest corner of the enclosing rectangle as the origin of co-ordinates, since by so doing all the total latitudes and departures will be positive. (5) Long traverses that do not form closed figures may often be more conveniently plotted without constructing the enclosing rectangle; in fact often as much of it would fall off as on the map. The general method is to construct reference meridian and parallel as accurately as possible, and with these as a basis to construct other meridians and parallels, forming squares of a convenient size only where such squares are necessary. (6) For small drawings, construction lines other than those of the enclosing rectangle will be unnecessary.

Practical Applications.—The method of plotting by total latitudes and departures is generally recognized as the best method for plotting the average traverse. Its single disadvantage as compared with the method of tangents or chords is in the increase in the amount of computations necessary. This is not, however, always true, since in the case of closed traverses it is often necessary to compute the area; latitudes and departures are therefore necessary, and the increased labor of calculating co-ordinates is very slight. The principal advantages of the method are that the size and shape of the drawing can be accurately determined beforehand, that the accuracy of location of any point does not depend upon the accuracy of previous lines in the traverse, that the method of checking is simple and unlike the method of plotting, and that in closed traverses, the survey is balanced before plotting is begun. It is the only practical method for large drawings with many traverse lines, whether such drawings be of closed or continuous traverses.

Problem 87. Conventional Signs

Object.—To construct a plate of the more common of the standard conventional signs shown in Figs. 44–47, pages 218–222.

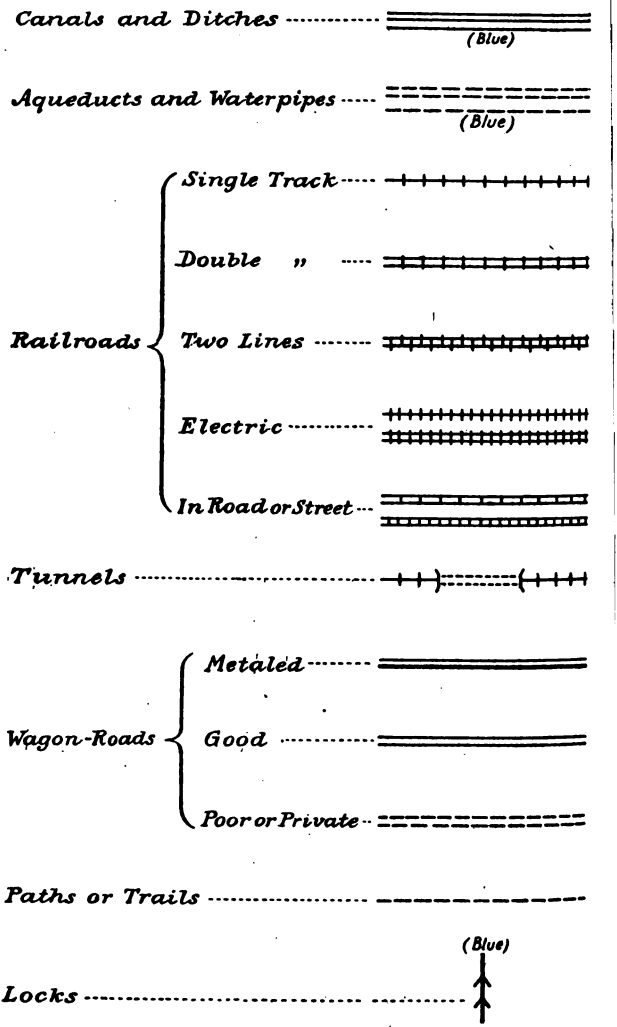


FIG. 44(*a*).—CONVENTIONAL SIGNS; PUBLIC AND PRIVATE WORKS.

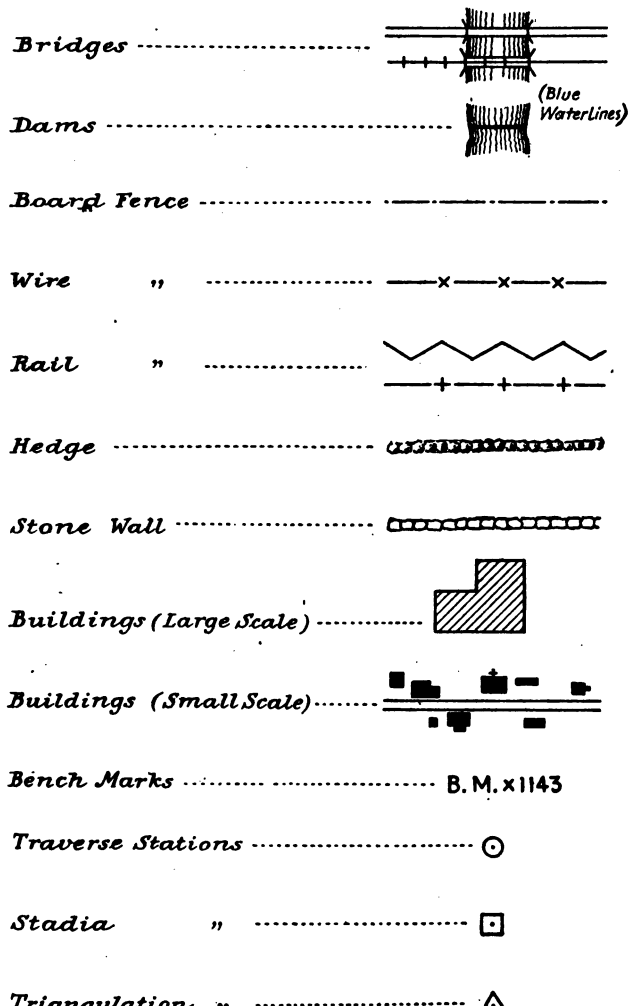
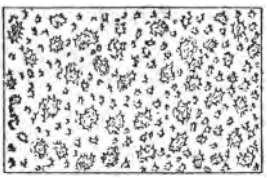
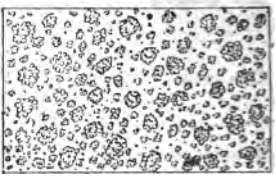


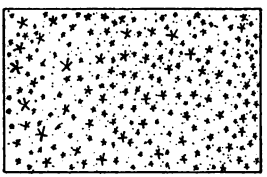
FIG. 44(*b*).—CONVENTIONAL SIGNS; PUBLIC AND PRIVATE WORKS.



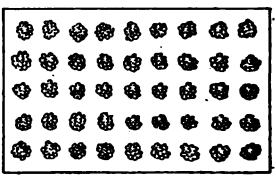
Deciduous Trees (Oak)



Deciduous Trees (Round Leaf)

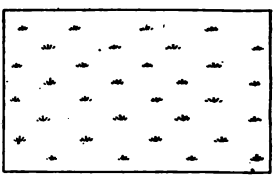


Evergreen Trees

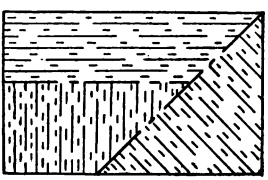


Orchard

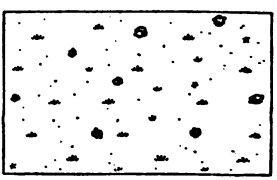
Clearing



Grass



Cultivated Land



Pasture

FIG. 45.—CONVENTIONAL SIGNS; CULTURE.

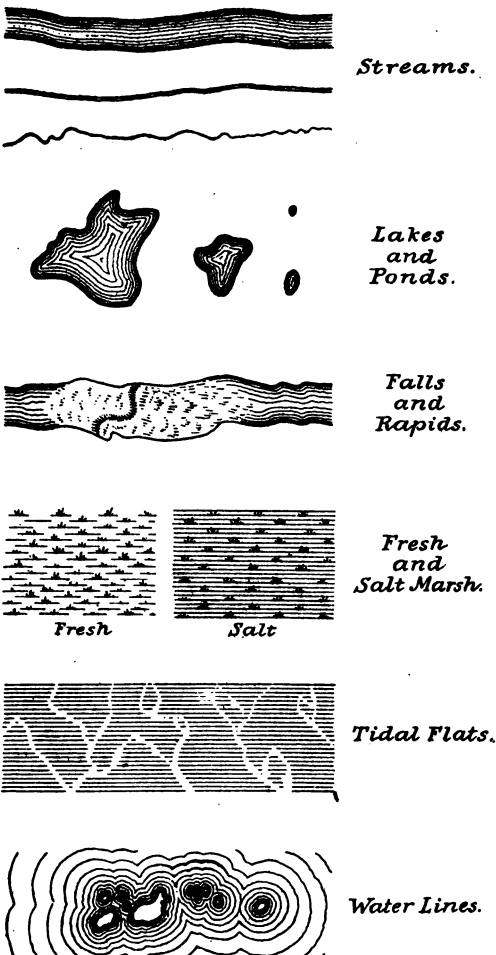


FIG. 46.—CONVENTIONAL SIGNS; HYDROGRAPHY.

Procedure.—(1) Divide a sheet of good quality drawing paper by lines forming rectangles or strips, and in pencil sketch such signs as those for fences, property lines, streets, paths, railroads, buildings, bridges, tunnels, etc., representing the works of man; deciduous and evergreen trees, meadow land, cultivated land, etc., representing culture of land; streams, lakes, fresh and salt marsh, tidal flats, etc., representing hydrography; and contours, depression contours, hachures, etc., representing hypsography or relief. (2) Opposite each

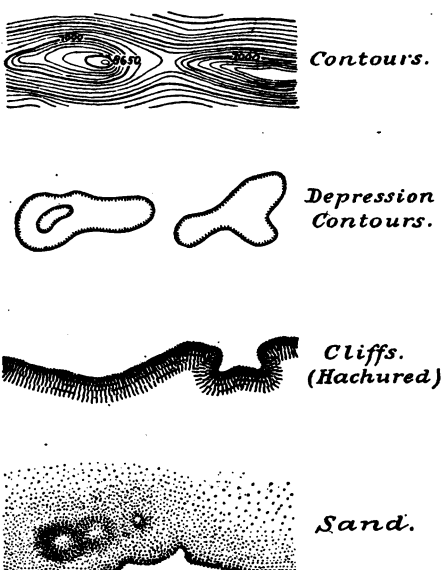


FIG. 47.—CONVENTIONAL SIGNS; HYPSOGRAPHY.

sign show its name in Italics (Fig. 49, page 224). (3) Ink, *in black*, the symbols representing works of man and culture of land; ink, *in blue*, the symbols representing hydrography; and ink, *in brown*, the symbols representing relief. (4) Make a neat and appropriate title in Roman letters (Fig. 49, page 224), and surround the plate with a simple border line.

Hints and Precautions.—(1) It is a common fault for inexperienced draftsmen to construct the symbols for trees, culti-

vated land, marsh, and meadow land much too large or too heavy. (2) Water-lines and other hydrographic features may be made with prussian blue water-color with a touch of burnt sienna; relief, with burnt sienna water-color. The general tendency is to mix the colors much too dark. (3) The construction of water-lines is perhaps the most difficult of any work the draftsman will be called upon to do. Water-lining can not be done hastily even by the expert draftsman; if the beginner is to obtain satisfactory results he will, therefore, have to proceed very slowly and carefully. Beginning

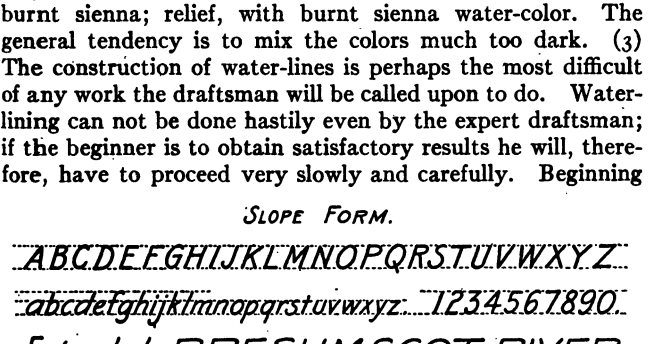


FIG. 48.—REINHARDT'S STYLE LETTERS.

at the shore line, the space between water-lines should *increase* while the width of the lines *diminishes*. Starting with the first water-line, the draftsman, with short, quick strokes of the pencil, should exactly parallel the shore line for its full length, and so on, line by line, except that as the distance from shore increases, the minor irregularities apparent in the shore line should become less and less noticeable. The pencil lines should be very light. The necessary weight is given to the lines when they are gone over with the pen, but when inked they should not be made unduly heavy. (4) The general tendency is to make contours too heavy. Every fifth or tenth contour, counting from the datum plane, is made

ROMAN

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

ITALIC OR STUMP WRITING

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

GOTHIC

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

INCLINED GOTHIC

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

HAIRLINE ANTIQUE

ABCDEFGHIJKLMNOPQRSTU
VWXYZ
abcdefghijklmnopqrstuvwxyz
1234567890

FIG. 49.—STYLES OF LETTERING.

heavier than the rest, and at intervals it is broken to allow showing its elevation. The breaks in the accentuated contours should not be made haphazardly. When there are several such contours the elevations should appear one above the other. Elevations on the same contour should appear at intervals sufficiently frequent to make reading the map a simple matter. Elevations should be recorded before the contour is drawn, otherwise the draftsman is likely to ink over the intended break; and the numbers should be of the same color as the contour. (5) The inking of water-lines or contours may be done with a contour pen or with a ruling pen; either one will give a more uniform width of line than the usual lettering pen. Some draftsmen prefer to use a fine lettering pen that has become well worn so that the nibs are slightly spread. Such a pen will produce a line of uniform width with but very little or no pressure on the part of the operator. (6) Tufts of grass, the conventional sign for meadow land, should not be placed too thickly; the blades of the individual tufts should be made to radiate from a common center some distance below. (7) A single line representing a stream should gradually increase in width from a line barely discernible at the source.

Practical Applications.—Ordinary maps usually show but few of the conventional signs, those for boundaries, fences, streams, roads, and buildings being the most common. Detail maps not showing the relief, sometimes called *flat topographic* maps, and contour or complete topographic maps will often show as many symbols as there are features in the area surveyed.

Problem 88. Map Construction

Object.—To plot a map from field notes of Problem 23, page 74, or similar data.

Procedure.—(1) Plot the traverse to given scale by method of tangents (Problem 84, p. 206), chords (Problem 85, p. 211), or total latitudes and departures (Problem 86, p. 213). (2) Plot details by methods the reverse of those used in the field. (3) By conventional signs represent features shown in Figs. 44–47, pages 218 to 222; represent objects not included in the figures by symbols, and on the map construct a legend or key to such symbols. Use a style of lettering in keeping with the

purpose of the map (Art. 27, page 22). (4) Show the direction of the true or magnetic meridian by a simple but neatly constructed meridian needle (Art. 26, page 21). (5) Make a neat and appropriate title of Gothic capitals (Art. 28, page 23).

Hints and Precautions.—(1) The protractor should be used for laying off angles to details. Where a number of angles have been taken from a single traverse point, if the protractor is not constructed so that angles and distances can be plotted at the same time, it is best to plot all the angles before scaling any of the distances. There are so many varieties of protractors that no attempt will be made to give explicit directions as

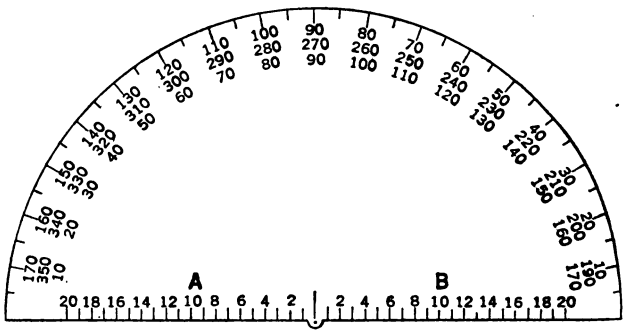


FIG. 50.—PROTRACTOR FOR PLOTTING DETAILS.

to their use. However, the student should first make certain of the location of the center of the protractor, no matter what kind he is using. (2) For plotting side shots a very good protractor can be made by pasting a half-sheet of polar coordinate paper to heavy drawing paper in the form shown in Fig. 50. It is well to have a small piece of transparent celluloid with a hole for a needle point, pasted at the center. The scale along the diameter should be plotted with fine lines and with spaces corresponding in length to those of the ordinary scale. The angle numbers are so arranged that either azimuths or deflection angles may be laid off by moving the protractor about the station as a center until the graduation on the protractor representing the given angle coincides with the reference line; the distance is then found on the diametrical scale. With such protractor, when angles are given in azimuths, a meridian line must be constructed through each

transit station. To lay off a line of given azimuth (from North) move the protractor about the station as a center until the graduation corresponding to the given azimuth coincides with the south end of the meridian line. The distance will then be found on scale *A* or *B* according as the azimuth is less or greater than 180°. (3) Points located by linear measurement from two traverse stations may be plotted by intersecting arcs of the given radii drawn with dividers. (4) When objects, such as roads and streams, have been located by perpendicular offsets from transit line, these offsets may usually be plotted by estimating the perpendiculars with the eye. (5) The location of each point or object should be checked. The scaled lengths of sides of buildings should compare with field measurements, and if the buildings are rectangular, the angles formed by sides should be tested with the protractor. It often happens that mistakes in the fieldwork give an object more than one location or that a portion of the field notes are confusing. In such case, it is best to proceed with the plotting of other portions of the survey, for these portions may help in clearing up the doubtful point. (6) Note the hints and precautions of Problem 87, page 217, concerning the construction of conventional signs.

Problem 89. Interpolation of Contours

Object.—To interpolate contours on a sheet upon which have been plotted streams, lakes, ridge and valley lines, and representative ground points with their elevations. (The data of Problem 64, page 167, may be used.)

Facts Concerning Contours.—(1) All points in any one contour line have the same elevation. (2) Every contour closes upon itself either within or without the limits of the map; in the latter case it is drawn to the edge of the sheet. (3) Two contours can not cross each other except at an overhanging cliff or cave, in which case they must cross each other *twice*, and the lower contour must be shown as a dotted line. (4) The area within a closed contour is either a summit or depression. In the latter case, *when there is no water in the depression*, the contours are marked as shown in Fig. 47, page 222. (5) The distance between contours varies inversely as the slope. (6) Contours are evenly spaced where the slope is uniform. (7) A single contour can not intervene between

two higher or two lower contours; that is, the maximum contours of ridge and minimum contours of valley must run in pairs if they do not close. (8) No contour should be drawn across a stream. As it approaches the stream it should turn up-stream and disappear in coincidence with the line defining the water's edge. Its re-appearance should be at a point directly across the stream. (9) The sharpest bends in contours occur at their intersection with ridge or valley lines.

Procedure.—(1) Interpolate points on contours assuming that the slope between adjacent known elevations is uniform. Give each interpolated point its elevation. (2) After numerous points have been located in this manner, draw in the contours, observing the preceding facts concerning them. Make each fifth or tenth contour heavier than the rest, and mark its elevation as explained in Problem 87, page 217.

Hints and Precautions.—(1) The interpolated position of points on contours may be computed by proportion. For

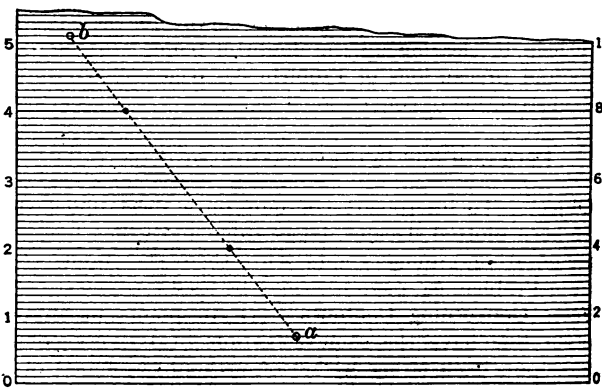


FIG. 51.

example, if a and b with elevations of 53 ft. and 63 ft. are 120 ft. apart, the 55-ft. contour will cross the line ab at a distance of $2/10 \times 120$ ft. $= 24$ ft. from a; the 60-ft. contour, at a distance of $7/10 \times 120$ ft. $= 84$ ft. from a. (2) There are a number of mechanical contrivances whereby contour points may be interpolated with greater rapidity and with less mental labor on the part of the draftsman than with the arithmetical method. One of these is shown in Fig. 51. A number of

parallel lines are drawn at equal intervals on tracing cloth, each fifth or tenth line being made heavier or of a different color than the rest and numbered as shown. Now if it be desired to interpolate the position of 52-ft. and 54-ft. contours between *a* with elevation of 50.7 and *b* with elevation of 55.1, the line corresponding to 0.7 ft. (left end) is placed over *a* and the tracing turned about *a* as a center until the line corresponding to 5.1 ft. (left end) covers *b*. The interpolated points are at the intersections of lines representing their elevations and the line *ab*, and may be pricked through the tracing cloth. Had the known points been much closer together, the figures at the right end of the tracing would have been used, or in other words, the value of each space would have been doubled; and if the scale were small, the contour interval large, and the topography rugged, each space might represent 1 ft. Thus it will be seen that, by assigning different values to the spaces, a single piece of tracing cloth prepared in this way can be made to suit a variety of conditions.

Problem 90. Topographic Map Construction

Object.—To construct a complete topographic map from field notes, relief being represented by contours. (The data of Problems 59, 60, 61, and 64, pages 153, 160, and 167 may be used.)

Procedure.—(1) If the skeleton of the survey is a traverse, plot by method of tangents (Problem 84, p. 206), chords (Problem 85, p. 211), or total latitudes and departures (Problem 86, p. 213); if the horizontal control is in the form of rectangles or squares, plot by methods used in Problem 86 for constructing meridians and parallels. (2) Plot the details of the map as in Problem 88, p. 225, using conventional signs (Problem 87, p. 217). (3) Mark each ground point by a dot and mark each elevation in such position that there will be no doubt as to which point it refers. (4) Interpolate contours as in the preceding problem. (5) Show the direction of the true or magnetic meridian by a neat but not elaborate meridian needle (Art. 26, page 21). (6) Place necessary notes so that they will not interfere with the map. (7) Make a neat and appropriate title (Art. 28, page 23). (8) Ink in the map, observing the hints and precautions of Problem 87.

Problem 91. Base-line Reduction

Object.—To compute the true length of a base-line from given field notes. The data of Problem 68, page 176, may be used.

Formulas.—

(1) *Correction for Temperature* (*for one tape length*) $= + Lk(T - T_0)$.

(2) *Correction for Pull* (*for one tape length*) $= + L\frac{(P - P_0)}{aE}$

(3) *Correction for Sag* (*for one tape length*) $= + \frac{L}{24}\left(\frac{w}{PP_0}\right)^2 (l_0^2P^2 - l^2P_0^2)$.

(4) *Correction for Grade* (*between grade intersections*) $= -\left(\frac{h^2}{2L'} + \frac{h^4}{8L'^3}\right)$, (approximate).

(4′) *Correction for Grade* (*between grade intersections* $= - 2L' \sin^2 \tfrac{1}{2}\theta$, (exact)

(5) *Correction to Reduce to Sea-level* (*for whole base-line*) $= - L''\frac{A}{R}$

L is length of tape in feet.

L' is length of uniform grade.

L'' is the length of the base-line in feet.

T_0 is the standard temperature.

T is the mean of the observed temperatures.

k is the coefficient of expansion of the tape for 1° F.

a is the cross-sectional area of the tape *in square inches.*

E is the modulus of elasticity of the material of the tape in pounds/in.^{2}

w is the weight of the tape *per linear foot.*

l_0 is standard distance in feet between supports.

l is the distance in feet between supports during observations.

P_0 is the standard pull.

P is the observed pull.

h is the difference in elevation between grade points at ends of length L'.

θ is the angle of inclination of base-line between grade intersections, $\sin \theta = \frac{h}{L'}$.

A is the mean altitude of the base above sea-level.

R is the radius of the earth in feet. (Mean radius equals 20,890,600 ft., Log R equals 7.31995.)

All the corrections are in feet.

Procedure.—(1) Transpose the necessary data to the computation book. (2) Decide upon a neat and systematic form for computations. (3) Compute the mean of the observed temperatures. (4) Calculate the correction for temperature, pull, sag, and erroneous length of tape for one tape length according to the preceding formulas. (5) Find the algebraic sum of the above corrections, and multiply by the number of tape lengths in the base-line. (6) Compute the total correction for grade. (7) Apply the total corrections for sag, pull, temperature, and grade to the measured length of the base. This gives its correct horizontal length. (8) Reduce this to the corresponding length at sea-level. (9) Check all work.

Hints and Precautions.—(1) The computer should be very careful to apply corrections with their proper signs which will not necessarily be the same as those prefixed to the correction in the formulas. For example, if the observed pull or temperature is less than the pull or temperature at which the tape was standardized, the corresponding correction as applied to the measured base will be negative. The fact to be always borne in mind is that conditions which tend to make the tape too long will necessitate a positive correction to the measured length, and *vice versa*. This applies to corrections for erroneous length as well as sag, pull, and temperature. The corrections for grade and sea-level will always be negative. (2) Tapes standardized by the Bureau of Standards at Washington are accompanied by a Certificate of Comparison, the contents of which depend upon the specifications for standardization made by the owner of the tape and the amount of the fee charged by the Bureau of Standards, but in all cases it will contain the length of the tape at a given temperature and pull either flat or supported at given intervals. A complete Certificate of Comparison also includes the coefficient of expansion and modulus of elasticity of the tape. These latter quantities are exceedingly important, if other than the standard pull is used and if the temperature at which measurements are taken is greatly different from that at which the tape was standardized, because assumed values

may contain a considerable error. However, if the tape is steel, and these values are unknown, the coefficient of expansion may be taken as 0.0000065 for 1° F., and the modulus of elasticity as 28,000,000. (3) The correction for pull becomes zero if $P = P_0$ in Formula (2). (4) If the tape was standardized flat, the correction for sag becomes $-\frac{L}{24}\left(\frac{wl}{P}\right)^2$. If the distance between supports and the pull are the same as they were during the standardization test, the corrections for sag and pull become zero. (5) The sea-level correction is usually omitted from the base-line calculations of the smaller triangulation systems, but in geodetic work it is always considered. (6) For ordinary grades, Formula (4) may be used without introducing appreciable error; for very steep grades Formula (4′) should be used. For grades of less than 2 per cent, Formula (4) may be reduced to $C_g = -\frac{h^2}{2L'}$ unless the required precision is very high. If the distances between succeeding grade intersections are equal, as when changes of grade occur at each tape length, Formula (4) may be written in the form:

Total correction for grade =

$$\left[\frac{(h_1^2+h_2^2+h_3^2+\ldots.h_n^2)}{2L'}+\frac{(h_1^4+h_2^4+h_3^4+\ldots.h_n^4)}{8L'^3}\right]$$

in which h_1, h_2, h_3, etc., are the differences in elevation between the ends of succeeding uniform grades. (7) The cross-sectional area of the tape can best be found by first computing the volume with weight of tape as basis (3.56 cu. in. of steel to the pound); the cross-sectional area is then determined by dividing the volume by the length of the tape. Since E (Formula (2)) is in pounds/in.^{2}, it is necessary that a be in *square inches*.

Problem 92. Triangulation Adjustments and Computations

Object.—To adjust the angles of a triangulation system, and to compute the lengths of sides and co-ordinates of triangulation points.

It is assumed that the angles about each point in the triangulation system have been accurately determined either

with repeating or direction theodolite, that the length of at least one line is known, that the true azimuth of one line is known, and that the relation between the area of the system and the required precision is such that the spheroidity of the earth may be neglected.

Procedure.—(1) Adjust the angles about each triangulation station so that their sum shall equal 360°. If all the angles have been taken with the same precision, which will usually be the case, the angular error of closure should be divided equally among the angles regardless of size of angle; if the field notes indicate that the conditions under which observations of certain angles were taken were unfavorable, or if a greater number of observations were taken on some angles than on others, the error of closure should be distributed accordingly. This is called the *Station* adjustment, and the resulting angles will be referred to as "station" angles. (2) Make a rough sketch of the triangulation system, and record the value of each angle according to the station adjustment. (3) Carefully study the figures formed by the various triangles in the system, and divide the system into quadrilaterals, polygons, and single triangles, but as far as possible avoid figures composed of a single triangle. Fig. 52, page 234, shows such subdivision of a triangulation system. The quadrilaterals *ABCD*, *DEFG*, *DGHJ*, and *GHKL* were first determined; this division left the polygon *EPONF* and the triangles *CDE*, *ADJ*, *LGM*, *MGF*, and *OPR*. (4) By inspection adjust each quadrilateral so that the sum of the angles of each of its four triangles is equal to 180° and the sum of its interior angles is 360°. *Change the values of the station angles as little as possible.* (5) In a similar manner adjust each polygon so that the sum of the angles in each of its triangles is 180°, the sum of its interior angles is $(n - 2) \times 180°$ (in which n is the number of sides), and the sum of the angles about the interior station (as Q in Fig. 52, page 234) is 360°. (6) Adjust each figure composed of a single triangle so that the sum of its interior angles is 180°. (7) Re-adjust the angles, if necessary, so that the sum of the angles about each interior station of the triangulation system is equal to 360°, the geometric conditions stated in (4), (5), and (6) are fulfilled, and the adjusted values will be altered the smallest amount. Make this adjustment by altering the values of angles of the weakest figures. In the majority of cases this

will mean the alteration of figures composed of single triangles, assuming the angles of adjusted quadrilaterals and polygons as correct.

As an example all angles about D (Fig. 52) except a and d have been determined through quadrilateral adjustments.

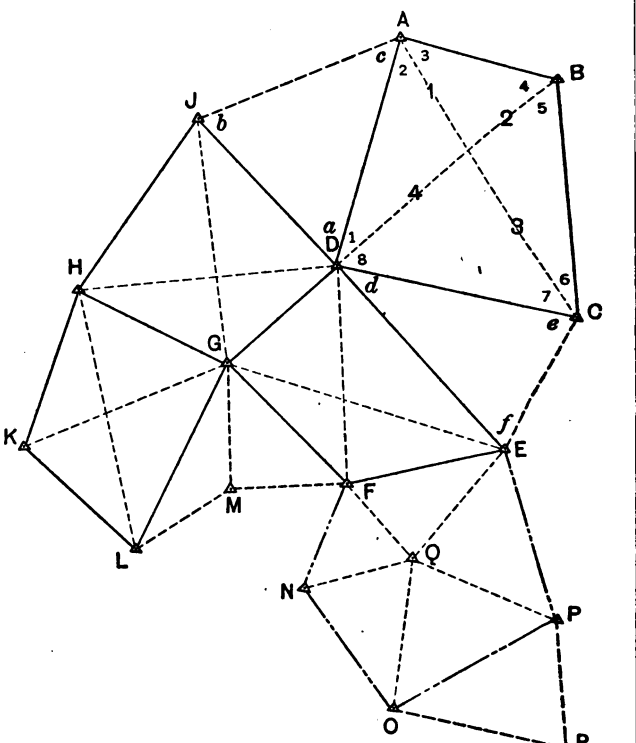


FIG. 52.—TRIANGULATION FIGURES.

Angles a and d have been determined through the adjustment of single triangles. Suppose the sum of the angles about D is 04″ in excess of 360°. Each of the angles a and d would be decreased by 02″ and angles b, c, e, and f, increased by 01″.

(8) Starting with the base-line or side of known length, compute the lengths of the other sides by the equation $\frac{a}{b} = \frac{\sin A}{\sin B}$.

Use logarithms and record in form shown in Fig. 53. Check, not by re-computation, but by making each computation depend upon the preceding until the length of some line has been determined by two independent series of computations. From the preceding statement it follows that each line must be

Quadrilateral DEFG

Lines Stations	Dists. Angles	Logs
ΔDFG		
DF	8778.28	3.943409
G	104° 44' 40"	0.014542
D	34 37 38	9.754527
F	40 37 42	9.813681
FG	5157.96	3.712478
GD	5910.61	3.771632
ΔDFE		
DF		3.943409
E	78° 08' 44"	0.009363
D	43 00 57	9.833912
F	58 50 19	9.932328
FE	6119.05	3.786684
ED	7675.38	3.885100
ΔFEG		
FE		3.786684
G	44° 23' 49"	0.155134
F	99 28 01	9.994045
E	36 08 10	9.770635
GE	8627.06	3.935863
GF	5157.67	3.712454
	FG = 5157.96	ED = 7675.38
	5157.67	7675.01
	5157.81 Mean	7675.19 Mean
ΔGED		
GE		3.935863
D	77° 38' 35"	0.010180
G	60 20 51	9.939040
E	42 00 34	9.825590
ED	7675.01	3.885083
DG	5910.62	3.771633
	GD = 5910.61	
	5910.62	
	5910.61 Mean	

FIG. 53.—COMPUTATION OF TRIANGLE SIDES.

computed by two independent solutions, or must be computed by one solution and made the basis of computations in another. (This will be impossible if the triangulation system is a chain of simple triangles.) If all conditions had been satisfied in the adjustment of angles, these independent

determinations should exactly agree. Since the adjustment of angles was made purely by inspection and all possible conditions are not fulfilled, this is not likely to be the case, but it may be safely assumed that the computations are correct if the discrepancy between these independent determinations is small.

In Fig. 53, page 235, are recorded the computations for each of the triangles of the quadrilateral $DGEF$ (Fig. 52, page 234), DF being the line of known length. Solutions 1 and 2 are sufficient to determine the sides of the quadrilateral, but none of the lengths is checked. Solution 3 is based upon the computed length of FE of solution 2; and through solution 3, FG of solution 1 is checked (the discrepancy being 0.29 ft.)

Symbols	G from F (F, G)		G from D (D, G)	
	Dists. and Azimuths	Logs	Dists. and Azimuths	Logs
l	5157.81	3.712466	5910.61	3.771632
Z	Log cos Z 48° 48′ 09″	9.818659	124°03′29″	9.748214
	Log sin Z	9.876474		9.918277
Lat.	+3397.23	3.531125	+3310.21	3.519846
Dep.	+3880.97	3.588940	-4896.77	3.689909
L_1	- 87.20		0.00	
M_1	-8777.85		0.00	
Tl. Lat. G	+3310.03		+3310.21	
Tl. Dep. G	-4896.88		-4896.77	

Mean Tl. Lat. G = +3310.12
" " Dep. G = -4896.82

l = length
Z = azimuth
L_1 = total lat. of known point
M_1 = total dep. " "

FIG. 54.—COMPUTATION OF CO-ORDINATES.

and GE is computed. Solution 4 is based on the computed length GE, and in the determination of ED and DG not only are the corresponding distances in solutions 1 and 2 checked very closely but the fact that this check is performed shows that the length of the line GE is correct as far as computations are concerned.

(9) Starting with the line of known azimuth, compute the azimuths of the other lines of the system (about one triangulation point at a time) using the *adjusted* angles. By so doing

the azimuth of every line will be computed from each end. Check by noting that opposite azimuths differ by 180°. Tabulate the results. (10) Calculate the latitudes and departures (Problem 73, page 183) of a sufficient number of the lines of the triangulation system so that the co-ordinates of each point may be determined by two solutions. When possible utilize lines the lengths of which have been determined by two solutions, and use the mean of the lengths. Fig. 54 illustrates a convenient form and shows computations for latitudes and departures of the lines *FG* and *DG*. (11) Choose some triangulation point as an origin, and compute the co-ordinates of each of the other stations (by two routes when possible) using the method of total latitudes and departures (Problem 86, page 213). If the latitudes and departures are computed correctly and no mistake has been made in the totals, the co-ordinates of a station determined by one route should agree very closely with the co-ordinates determined by any other route. Check by noting this agreement, and assume the correct co-ordinates as the mean of the corresponding computed values. Fig. 54 also shows computations for co-ordinates.

Hints and Precautions.—(1) The station adjustment is usually, though not always, performed in the field (Problem 18, page 62). This is as it should be, for the observer is in much better position to judge the reliability of the angles at this time than he or anyone else could possibly be later, even though elaborate notes were taken. Other conditions being the same it is common practice to assume that the angular precision varies as the number of observations with the direction instrument or as the number of sets of observations with the repeating instrument, although such assumption is not in accord with the theory of errors. (2) In making the figure adjustment it will be considerably easier to deal with the discrepancies of each figure rather than with the angles themselves, since the former will be much the smaller. These discrepancies may be recorded within the corresponding figures on the sketch and should be prefixed by a positive or negative sign according as the correction must be added to or subtracted from the station angles. Thus in the polygon *FNOPE* (Fig. 52, page 234) the sum of the station angles (not shown) of each triangle will give a discrepancy for that triangle, and the sum of the interior station angles of the polygon will give a dis-

crepancy for the polygon. By inspecting these small quantities the computer may readily estimate the correction that will least change the station angles and will at the same time satisfy the given conditions. (3) Below are given four equations which render the determination of the angle corrections of a quadrilateral very simple and which fulfill for each figure the geometric conditions previously cited:

$$V_1 = V_2 = \tfrac{1}{4}d_4 + \tfrac{1}{8}(d_1 - d_3)$$
$$V_3 = V_4 = \tfrac{1}{4}d_2 + \tfrac{1}{8}(d_1 - d_3)$$
$$V_5 = V_6 = \tfrac{1}{4}d_2 - \tfrac{1}{8}(d_1 - d_3)$$
$$V_7 = V_8 = \tfrac{1}{4}d_4 - \tfrac{1}{8}(d_1 - d_3)$$

In the above equations d_1, d_2, d_3, and d_4 are the discrepancies of the four *triangles* of which the quadrilateral is composed as numbered in $A\,B\,C\,D$ (Fig. 52), while the corrections are V_1, V_2, ... etc., the subscripts referring to *angles* numbered as in $A\,B\,C\,D$. Corrections satisfying these geometric conditions may be determined more readily by using these equations than by inspecting the figure. (4) Do not use logarithmic tables with more places than are necessary. In computing lengths of the sides, a six-place table will generally be sufficient if the angles are carried to seconds. See Arts. 12 and 21, pages 8 and 17. (5) It should be noted that the procedure outlined makes appreciable mistakes impossible by providing a check by independent solution of every line, azimuth, and co-ordinate.

Practical Applications.—This method of adjusting the angles and computing the co-ordinates of a triangulation system is sufficiently exact for all surveys except those of the highest precision. It will always furnish sufficiently precise horizontal control for mapping. For very exact or extended triangulation systems such as those established by the U. S. Coast and Geodetic Survey, the Great Lakes Survey, and some of the larger cities, the methods of this problem will not give sufficiently precise results.

A more exact method of adjustment of angles and one in which more conditions must be fulfilled than are here stated is based upon the theory of *least squares*, but because of the intricate and lengthy computations it is not practicable except on such surveys as those just mentioned. In the quadrilateral for which computations are shown (Fig. 53, page 235) the maximum difference between the length of a side computed from angles determined by the exact "least squares"

nethod and computed from angles determined by the four pre-:eding equations occurs in GE and is 0.16 ft. in a length of 627 ft. or about 1 in 50,000.

Since the measured angles are spherical angles, each tri-ngle will contain more than 180°. The amount greater han 180° is termed the *spherical excess* and is about one second or each 75.5 square miles of area of triangle. It is clear that 10 correction for spherical excess will be necessary unless the riangles are very large, and then only in the most precise vork.

TABLES

TABLE I.*—CORRECTION TO OBSERVED ALTITUDE OF THE SUN FOR REFRACTION AND PARALLAX

App't alt.	Temperature										App't alt.
	−10° C. +14° F.	−5° C. +23° F.	0° C. +32° F.	5° C. +41° F.	10° C. +50° F.	15° C. +59° F.	20° C. +68° F.	25° C. +77° F.	30° C. +86° F.	35° C. +95° F.	
°	′	′	′	′	′	′	′	′	′	′	°
10	5.52	5.42	5.30	5.20	5.10	5.00	4.92	4.83	4.75	4.67	10
11	5.02	4.92	4.82	4.73	4.63	4.55	4.47	4.38	4.32	4.23	11
12	4.60	4.50	4.42	4.33	4.25	4.17	4.10	4.03	3.97	3.88	12
13	4.23	4.15	4.07	4.00	3.92	3.85	3.78	3.72	3.65	3.58	13
14	3.92	3.83	3.77	3.70	3.62	3.55	3.50	3.45	3.37	3.32	14
15	3.65	3.58	3.50	3.43	3.37	3.32	3.25	3.20	3.13	3.08	15
16	3.43	3.35	3.30	3.23	3.17	3.12	3.07	3.00	2.95	2.90	16
17	3.22	3.15	3.10	3.03	2.98	2.92	2.88	2.82	2.77	2.72	17
18	3.02	2.95	2.90	2.85	2.80	2.75	2.70	2.65	2.60	2.55	18
19	2.83	2.78	2.73	2.68	2.63	2.58	2.53	2.48	2.43	2.40	19
20	2.68	2.63	2.58	2.53	2.48	2.43	2.38	2.33	2.30	2.27	20
21	2.53	2.48	2.43	2.38	2.35	2.30	2.27	2.22	2.17	2.13	21
22	2.38	2.35	2.30	2.25	2.22	2.18	2.13	2.08	2.05	2.02	22
23	2.28	2.25	2.20	2.15	2.12	2.08	2.03	1.98	1.95	1.93	23
24	2.17	2.13	2.08	2.05	2.02	1.98	1.93	1.88	1.87	1.83	24
25	2.07	2.03	1.98	1.95	1.92	1.88	1.83	1.80	1.77	1.75	25
26	1.99	1.95	1.90	1.87	1.83	1.80	1.75	1.72	1.70	1.67	26
27	1.88	1.85	1.82	1.78	1.75	1.72	1.68	1.63	1.62	1.60	27
28	1.80	1.77	1.72	1.70	1.67	1.63	1.60	1.57	1.53	1.52	28
29	1.72	1.68	1.65	1.63	1.60	1.57	1.53	1.50	1.47	1.46	29
30	1.65	1.62	1.58	1.57	1.53	1.50	1.47	1.45	1.42	1.40	30
32	1.53	1.50	1.47	1.45	1.42	1.38	1.35	1.33	1.30	1.28	32
34	1.41	1.37	1.35	1.32	1.30	1.27	1.25	1.23	1.20	1.18	34
36	1.30	1.27	1.25	1.22	1.20	1.18	1.15	1.13	1.10	1.08	36
38	1.20	1.18	1.15	1.13	1.12	1.10	1.07	1.05	1.02	1.02	38
40	1.11	1.10	1.07	1.05	1.03	1.02	0.98	0.97	0.95	0.93	40
42	1.03	1.00	0.98	0.97	0.95	0.93	0.90	0.88	0.87	0.87	42
44	0.96	0.93	0.92	0.90	0.88	0.87	0.85	0.83	0.82	0.80	44
46	0.89	0.88	0.87	0.85	0.83	0.82	0.80	0.78	0.77	0.75	46
48	0.83	0.82	0.80	0.78	0.77	0.75	0.73	0.72	0.70	0.68	48
50	0.77	0.75	0.73	0.72	0.70	0.68	0.67	0.67	0.65	0.63	50
55	0.63	0.62	0.60	0.60	0.58	0.57	0.57	0.55	0.53	0.52	55
60	0.52	0.52	0.50	0.50	0.48	0.47	0.47	0.45	0.45	0.43	60
65	0.42	0.40	0.40	0.40	0.38	0.38	0.37	0.37	0.35	0.33	65
70	0.32	0.32	0.32	0.30	0.30	0.30	0.28	0.28	0.28	0.27	70
75	0.23	0.23	0.23	0.22	0.22	0.22	0.20	0.20	0.20	0.18	75
80	0.15	0.15	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.12	80
85	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.05	0.05	0.05	85
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90

* From *Principal Facts of Earth's Magnetism*, Coast and Geodetic Survey.

TABLE II.—CORRECTION TO OBSERVED ALTITUDE OF A STAR FOR REFRACTION

App't alt.	Temperature										App't alt.
	C. −10° F. +14°	C. −5° F. +23°	C. 0° F. +32°	C. 5° F. +41°	C. 10° F. +50°	C. 15° F. +59°	C. 20° F. +68°	C. 25° F. +77°	C. 30° F. +86°	C. 35° F. +95°	
°	′	′	′	′	′	′	′	′	′	′	°
10	5.67	5.57	5.45	5.35	5.25	5.15	5.07	4.98	4.90	4.82	10
11	5.17	5.07	4.97	4.88	4.78	4.70	4.62	4.53	4.47	4.38	11
12	4.75	4.65	4.57	4.48	4.40	4.32	4.25	4.18	4.12	4.03	12
13	4.38	4.30	4.22	4.15	4.07	4.00	3.93	3.87	3.80	3.73	13
14	4.06	3.97	3.91	3.84	3.76	3.69	3.64	3.59	3.51	3.46	14
15	3.79	3.72	3.64	3.57	3.51	3.46	3.39	3.34	3.27	3.22	15
16	3.57	3.49	3.44	3.37	3.31	3.26	3.21	3.14	3.09	3.04	16
17	3.36	3.29	3.24	3.17	3.12	3.06	3.02	2.96	2.91	2.86	17
18	3.16	3.09	3.04	2.99	2.94	2.89	2.84	2.79	2.74	2.69	18
19	2.97	2.92	2.87	2.82	2.77	2.72	2.67	2.62	2.57	2.54	19
20	2.82	2.77	2.72	2.67	2.62	2.57	2.52	2.47	2.44	2.41	20
21	2.67	2.62	2.57	2.52	2.49	2.44	2.41	2.36	2.31	2.27	21
22	2.52	2.49	2.44	2.39	2.36	2.32	2.27	2.22	2.19	2.16	22
23	2.42	2.39	2.34	2.29	2.26	2.22	2.17	2.12	2.09	2.07	23
24	2.31	2.27	2.22	2.19	2.16	2.12	2.07	2.02	2.01	1.97	24
25	2.21	2.17	2.12	2.09	2.06	2.02	1.97	1.94	1.91	1.89	25
26	2.12	2.08	2.03	2.00	1.96	1.93	1.88	1.85	1.83	1.80	26
27	2.01	1.98	1.95	1.91	1.88	1.85	1.81	1.76	1.75	1.73	27
28	1.93	1.90	1.85	1.83	1.80	1.76	1.73	1.70	1.66	1.65	28
29	1.85	1.81	1.78	1.76	1.73	1.70	1.66	1.63	1.60	1.59	29
30	1.78	1.75	1.71	1.70	1.66	1.63	1.60	1.58	1.55	1.53	30
32	1.65	1.62	1.59	1.57	1.54	1.50	1.47	1.45	1.42	1.40	32
34	1.53	1.49	1.47	1.44	1.42	1.39	1.35	1.35	1.32	1.30	34
36	1.42	1.39	1.37	1.34	1.32	1.30	1.27	1.25	1.22	1.20	36
38	1.32	1.30	1.27	1.25	1.24	1.22	1.19	1.17	1.14	1.14	38
40	1.22	1.21	1.18	1.16	1.14	1.13	1.09	1.08	1.06	1.04	40
42	1.14	1.11	1.09	1.08	1.06	1.04	1.01	0.99	0.98	0.98	42
44	1.07	1.04	1.03	1.01	0.99	0.98	0.96	0.94	0.93	0.91	44
46	0.99	0.98	0.97	0.95	0.93	0.92	0.90	0.88	0.87	0.85	46
48	0.93	0.92	0.90	0.88	0.87	0.85	0.83	0.82	0.80	0.78	48
50	0.86	0.84	0.82	0.81	0.79	0.77	0.76	0.76	0.74	0.72	50
55	0.72	0.71	0.69	0.69	0.67	0.66	0.66	0.64	0.62	0.61	55
60	0.59	0.59	0.57	0.57	0.55	0.54	0.54	0.52	0.52	0.50	60
65	0.48	0.46	0.46	0.46	0.44	0.44	0.43	0.43	0.41	0.39	65
70	0.37	0.37	0.37	0.35	0.35	0.35	0.33	0.33	0.33	0.32	70
75	0.27	0.27	0.27	0.26	0.26	0.26	0.24	0.24	0.24	0.22	75
80	0.18	0.18	0.16	0.16	0.16	0.16	0.16	0.15	0.15	0.15	80
85	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.06	0.06	0.06	85
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	90

TABLE III.—REFRACTION CORRECTIONS TO BE APPLIED TO APPARENT DECLINATIONS

(To be used with solar attachment)

January			February			March			April			May			June		
Hour angle		Refraction correction lat. 40°	Hour angle		Refraction correction lat. 40°	Hour angle		Refraction correction lat. 40°	Hour angle		Refraction correction lat. 40°	Hour angle		Refraction correction lat. 40°	Hour angle		Refraction correction lat. 40°
		′ ″			′ ″			′ ″			′ ″			′ ″			′ ″
					See Jan. 30												
1	1	1 58	1		′ ″	1	1	1 03	1	3	0 57	1	3	0 39	1	4	0 44
2	2	2 16	2	3	2 13	2	2	1 10	2	4	1 19	2	4	0 55	2	5	1 11
3	3	3 04	3	4	3 41	3	3	1 27	3	5	2 18	3	5	1 30	3		
4	4	6 23	4			4	4	2 06	4			4			4	1	0 19
5			5	1	1 26	5			5	1	0 39	5	1	0 26	5	2	0 23
	1	1 54		2	1 37		1	0 59		2	0 44		2	0 30		3	0 30
6	2	2 11	6	3	2 04	6	2	1 06	6	3	0 54	6	3	0 37	6	4	0 43
7	3	2 59	7	4	3 21	7	3	1 21	7	4	1 14	7	4	0 53	7	5	1 10
8	4	6 01	8			8	4	1 56	8	5	2 08	8	5	1 26	8		
9			9	1	1 21	9	5	4 04	9			9			9	1	0 18
10			10	2	1 31	10			10	1	0 36	10	1	0 25	10	2	0 22
	1	1 50		3	1 56		1	0 55		2	0 41		2	0 29		3	0 29
11	2	2 06	11	4	3 04	11	2	1 02	11	3	0 51	11	3	0 36	11	4	0 43
12	3	2 49	12			12	3	1 15	12	4	1 10	12	4	0 51	12	5	1 09
13	4	5 33	13			13	4	1 47	13	5	1 58	13	5	1 22	13		
14			14	1	1 16	14	5	3 34	14			14			14	1	0 18
15			15	2	1 25	15			15	1	0 34	15	1	0 23	15	2	0 22
	1	1 46		3	1 48		1	0 52		2	0 38		2	0 27		3	0 29
16	2	2 01	16	4	2 47	16	2	0 58	16	3	0 48	16	3	0 34	16	4	0 42
17	3	2 40	17	5	8 39	17	3	1 10	17	4	1 06	17	4	0 49	17	5	1 08
18	4	5 00	18			18	4	1 39	18	5	1 49	18	5	1 18	18		
19			19	1	1 12	19	5	3 08	19			19			19	1	0 18
20			20	2	1 20	20			20	1	0 32	20	1	0 22	20	2	0 22
	1	1 42		3	1 40		1	0 48		2	0 36		2	0 26		3	0 29
21	2	1 56	21	4	2 31	21	2	0 54	21	3	0 45	21	3	0 33	21	4	0 42
22	3	2 31	22	5	6 49	22	3	1 05	22	4	1 02	22	4	0 47	22	5	1 08
23	4	4 35	23			23	4	1 32	23	5	1 42	23	5	1 15	23		
24			24	1	1 07	24	5	2 51	24			24			24	1	0 18
25	1	1 37	25	2	1 15	25			25	1	0 30	25	1	0 21	25	2	0 22
	2	1 50		3	1 33		1	0 45		2	0 34		2	0 25		3	0 29
26	3	2 22	26	4	2 18	26	2	0 50	26	3	0 42	26	3	0 32	26	4	0 42
27	4	4 07	27	5	5 29	27	3	1 01	27	4	0 58	27	4	0 46	27	5	1 08
28			28			28	4	1 25	28	5	1 36	28	5	1 13	28		
29			29		See Mar. 1	29	5	2 34	29			29			29	1	0 18
30	1	1 32				30			30	1	0 28	30	1	0 20	30	2	0 22
31	2	1 44				31	1	0 42		2	0 32	31	2	0 24		3	0 29
							2	0 47					3	0 31			

For latitudes other than 40° multiply by latitude coefficient, Table III(a).

TABLE III (*Cont'd*).—REFRACTION CORRECTIONS TO BE APPLIED TO APPARENT DECLINATIONS

(*To be used with solar attachment*)

July	Hour angle	Refraction correction lat. 40°	August	Hour angle	Refraction correction lat. 40°	September	Hour angle	Refraction correction lat. 40°	October	Hour angle	Refraction correction lat. 40°	November	Hour angle	Refraction correction lat. 40°	December	Hour angle	Refraction correction lat. 40°
		′ ″			′ ″			′ ″			′ ″			′ ″			′ ″
1	4	0 43	1	5	1 22	1	1	0 39	1	1	0 56	1			1	1	1 54
2	5	1 09	2			2	2	0 44	2	2	1 06	2	1	1 26	2	2	2 11
3			3	1	0 26	3	3	0 54	3	3	1 21	3	2	1 37	3	3	2 59
4	1	0 19	4	2	0 30	4	4	1 14	4	4	1 56	4	3	2 04	4	4	6 01
5	2	0 23	5	3	0 37	5	5	2 08	5	5	4 04	5	4	3 21	5		
	3	0 30		4	0 53												
6	4	0 43	6	5	1 26	6	1	0 42	6	1	1 03	6	1	1 32	6	1	1 58
7	5	1 10	7			7	2	0 47	7	2	1 10	7	2	1 44	7	2	2 16
8			8	1	0 28	8	3	0 57	8	3	1 27	8	3	2 13	8	3	3 04
9	1	0 20	9	2	0 32	9	4	1 19	9	4	2 06	9	4	3 41	9	4	6 23
10	2	0 24	10	3	0 39	10	5	2 18	10	5	4 39	10			10		
	3	0 31		4	0 55												
11	4	0 44	11	5	1 30	11	1	0 45	11	1	1 07	11	1	1 37	11	1	2 00
12	5	1 11	12			12	2	0 50	12	2	1 15	12	2	1 50	12	2	2 19
13			13	1	0 30	13	3	1 01	13	3	1 33	13	3	2 22	13	3	3 09
14	1	0 21	14	2	0 34	14	4	1 25	14	4	2 18	14	4	4 07	14	4	6 43
15	2	0 25	15	3	0 42	15	5	2 34	15	5	5 29	15			15		
	3	0 32		4	0 58												
16	4	0 46	16	5	1 36	16	1	0 48	16	1	1 12	16	1	1 42	16	1	2 01
17	5	1 13	17			17	2	0 54	17	2	1 20	17	2	1 56	17	2	2 20
18			18	1	0 32	18	3	1 05	18	3	1 40	18	3	2 31	18	3	3 11
19	1	0 22	19	2	0 36	19	4	1 32	19	4	2 31	19	4	4 35	19	4	6 47
20	2	0 26	20	3	0 45	20	5	2 51	20	5	6 49	20			20		
	3	0 33		4	1 02												
21	4	0 47	21	5	1 42	21	1	0 52	21	1	1 16	21	1	1 46	21	1	2 01
22	5	1 15	22			22	2	0 58	22	2	1 25	22	2	2 01	22	2	2 20
23			23	1	0 34	23	3	1 10	23	3	1 48	23	3	2 40	23	3	3 11
24	1	0 23	24	2	0 38	24	4	1 39	24	4	2 47	24	4	4 59	24	4	6 49
25	2	0 27	25	3	0 48	25	5	3 08	25	5	8 39	25			25		
	3	0 34		4	1 06												
26	4	0 49	26	5	1 49	26	1	0 55	26	1	1 21	26	1	1 50	26	1	2 00
27	5	1 18	27			27	2	1 02	27	2	1 31	27	2	2 06	27	2	2 19
28			28	1	0 36	28	3	1 15	28	3	1 56	28	3	2 49	28	3	3 09
29	1	0 25	29	2	0 41	29	4	1 47	29	4	3 04	29	4	5 33	29	4	6 43
30	2	0 29	30	3	0 51	30	5	3 34	30	5	11 01	30			30		
31	3	0 36	31	4	1 10				31	Se	e Nov.				31	Se	e Jan. 1
	4	0 51		5	1 58						1						

For latitudes other than 40° multiply by latitude coefficient, Table III(a).

TABLE III(a).—LATITUDE COEFFICIENTS

Latitude	Coefficient	Latitude	Coefficient	Latitude	Coefficient
15°	0.30	30°	0.65	45°	1.20
16	0.32	31	0.68	46	1.24
17	0.34	32	0.71	47	1.29
18	0.36	33	0.75	48	1.33
19	0.38	34	0.78	49	1.38
20	0.40	35	0.82	50	1.42
21	0.42	36	0.85	51	1.47
22	0.44	37	0.89	52	1.53
23	0.46	38	0.92	53	1.58
24	0.48	39	0.96	54	1.64
25	0.50	40	1.00	55	1.70
26	0.53	41	1.04	56	1.76
27	0.56	42	1.08	57	1.82
28	0.59	43	1.12	58	1.88
29	0.62	44	1.16	59	1.94

To obtain the refraction correction (to be applied to declination) for any other latitude than 40° multiply the refraction correction for latitude 40° (Table III) by the coefficient corresponding to the latitude of observation.

TABLE IV*.—LOCAL MEAN (ASTRONOMICAL) TIME OF THE CULMINATIONS AND ELONGATIONS OF POLARIS IN THE YEAR 1915

(Computed for latitude 40° north and longitude 90° or 6h west of Greenwich)

Date	East elongation.	Upper culmination	West elongation.	Lower culmination.
1915	h m	h m	h m	h m
January 1	0 51.7	6 46.9	12 42.1	18 44.9
January 15	23 52.5	5 51.6	11 46.8	17 49.6
February 1	22 45.3	4 44.5	10 39.7	16 42.5
February 15	21 50.1	3 49.2	9 44.4	15 47.2
March 1	20 54.8	2 54.0	8 49.2	14 52.0
March 15	19 59.6	1 58.8	7 54.0	13 56.8
April 1	18 52.7	0 51.9	6 47.1	12 49.9
April 15	17 57.7	23 52.9	5 52.0	11 54.8
May 1	16 54.8	22 50.0	4 49.2	10 52.0
May 15	15 59.9	21 55.1	3 54.2	9 57.0
June 1	14 53.3	20 48.5	2 47.6	8 50.4
June 15	13 58.5	19 53.7	1 52.8	7 55.6
July 1	12 55.9	18 51.1	0 50.2	6 53.0
July 15	12 01.1	17 56.3	23 51.5	5 58.2
August 1	10 54.5	16 49.7	22 44.9	4 51.7
August 15	9 59.8	15 55.0	21 50.2	3 56.9
September 1	8 53.2	14 48.4	20 43.6	2 50.3
September 15	7 58.3	13 53.5	19 48.7	1 55.4
October 1	6 55.5	12 50.7	18 45.9	0 52.7
October 15	6 00.6	11 55.8	17 51.0	23 53.8
November 1	4 53.7	10 48.9	16 44.1	22 46.9
November 15	3 58.6	9 53.8	15 49.0	21 51.8
December 1	2 55.6	8 50.8	14 46.0	20 48.8
December 15	2 00.4	7 55.6	13 50.8	19 53.6

A. *To refer the above tabular quantities to years other than* 1915.

For year	m	For year	m
1914	subtract 1.5	1922	add 3.1
1916	add 1.6 up to Mar. 1; subtract 2.3 on and after Mar. 1	1923	add 4.5
1917	subtract 0.7	1924	add 5.9 up to Mar. 1; add 2.0 on and after Mar. 1
1918	add 0.9	1925	add 3.3
1919	add 2.5	1926	add 4.6
1920	add 4.0 up to Mar. 1; add 0.1 on and after Mar. 1	1927	add 5.9
1921	add 1.6	1928	add 7.2 up to Mar. 1; add 3.3 on and after Mar. 1

* Table IV was kindly furnished for this publication by Mr. O. H. Tittman, Superintendent of the U. S. Coast and Geodetic Survey.

TABLE IV.— *Concluded*

B. *To refer to any calendar day other than the first and fifteenth of each month* SUBTRACT *the quantities below from the tabular quantity for the* PRECEDING DATE.

Day of month	Minutes	No of days elapsed
2 or 16	3.9	1
3 17	7.8	2
4 18	11.8	3
5 19	15.7	4
6 20	19.6	5
7 21	23.5	6
8 22	27.4	7
9 23	31.4	8
10 24	35.3	9
11 25	39.2	10
12 26	43.1	11
13 27	47.0	12
14 28	51.0	13
29	54.9	14
30	58.8	15
31	62.7	16

C. *To refer the table to Standard time and to the civil or common method of reckoning:*

(*a*) ADD to the tabular quantities four minutes for every degree of longitude the place is west of the Standard meridian and SUBTRACT when the place is east of the Standard meridian.

(*b*) The astronomical day begins twelve hours after the civil day, *i.e.*, begins at noon on the civil day of the same date, and is reckoned from zero to twenty-four hours. Consequently an astronomical time less than twelve hours refers to the same civil day, whereas an astronomical time greater than twelve hours refers to the morning of the next civil day.

It will be noticed that for the tabular year two eastern elongations occur on January 14 and two western elongations on July 13. There are also two upper culminations on April 14 and two lower culminations on October 14. The lower culmination either follows or precedes the upper culmination by $11^h\ 58^m.0$.

D. *To refer to any other than the tabular latitude between the limits of* 10° *and* 50° *north:* ADD to the time of west elongation $0^m.10$ for every degree south of 40° and SUBTRACT from the time of west elongation $0^m.16$ for every degree north of 40°. Reverse these operations for correcting times of east elongation.

E. *To refer to any other than the tabular longitude:* ADD $0^m.16$ for each 15° east of the ninetieth meridian and SUBTRACT $0^m.16$ for each 15° west of the ninetieth meridian.

TABLE V.*—AZIMUTH OF POLARIS WHEN AT ELONGATION FOR ANY YEAR BETWEEN 1914 AND 1928

Latitude	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
10	I 10.3	I 10.0	I 09.6	I 09.3	I 09.0	I 08.7	I 08.4	I 08.1	I 07.8	I 07.4	I 07.2	I 06.8	I 06.5	I 06.2	I 05.9
11	10.5	10.2	09.9	09.6	09.2	08.9	08.6	08.3	08.0	07.7	07.4	07.0	06.7	06.4	06.1
12	10.8	10.4	10.1	09.8	09.5	09.2	08.9	08.6	08.2	07.9	07.6	07.3	07.0	06.7	06.4
13	11.0	10.7	10.4	10.1	09.8	09.4	09.1	08.8	08.5	08.2	07.8	07.6	07.2	06.9	06.6
14	11.3	11.0	10.7	10.4	10.0	09.7	09.4	09.1	08.8	08.5	08.2	07.8	07.5	07.2	06.9
15	11.6	11.3	11.0	10.7	10.4	10.0	09.7	09.4	09.1	08.8	08.5	08.1	07.8	07.5	07.2
16	12.0	11.7	11.4	11.0	10.7	10.4	10.1	09.8	09.4	09.1	08.8	08.5	08.2	07.8	07.5
17	12.4	12.0	11.7	11.4	11.1	10.8	10.4	10.1	09.8	09.5	09.2	08.8	08.5	08.2	07.9
18	12.8	12.4	12.1	11.8	11.5	11.1	10.7	10.5	10.2	09.8	09.5	09.2	08.9	08.6	08.2
19	13.2	12.8	12.5	12.2	11.9	11.6	11.2	10.9	10.6	10.2	09.9	09.6	09.3	09.0	08.6
20	13.6	13.3	13.0	12.7	12.3	12.0	11.7	11.4	11.0	10.7	10.4	10.0	09.7	09.4	09.1
21	14.1	13.8	13.5	13.1	12.8	12.5	12.2	11.8	11.5	11.2	10.8	10.5	10.2	09.8	09.5
22	14.6	14.3	14.0	13.6	13.3	13.0	12.6	12.3	12.0	11.6	11.3	11.0	10.6	10.3	10.0
23	15.2	14.8	14.5	14.2	13.8	13.5	13.2	12.8	12.5	12.2	11.8	11.5	11.2	10.8	10.5
24	15.8	15.4	15.1	14.7	14.4	14.1	13.7	13.4	13.0	12.7	12.4	12.0	11.7	11.4	11.0
25	16.4	16.0	15.7	15.3	15.0	14.7	14.3	14.0	13.6	13.3	13.0	12.6	12.3	11.9	11.6
26	17.0	16.6	16.3	16.0	15.6	15.3	14.9	14.7	14.2	13.9	13.6	13.2	12.9	12.5	12.2
27	17.7	17.3	17.0	16.6	16.3	15.9	15.6	15.2	14.9	14.6	14.2	13.9	13.5	13.2	12.8
28	18.4	18.0	17.7	17.3	17.0	16.6	16.3	15.9	15.6	15.2	14.9	14.6	14.2	13.8	13.5
29	19.1	18.8	18.4	18.1	17.7	17.4	17.0	16.6	16.3	16.0	15.6	15.2	14.9	14.6	14.2
30	19.9	19.6	19.2	18.8	18.5	18.1	17.8	17.4	17.0	16.7	16.4	16.0	15.6	15.3	14.9
31	20.7	20.4	20.0	19.7	19.3	18.9	18.6	18.2	17.9	17.5	17.2	16.8	16.4	16.1	15.7
32	21.6	21.2	20.9	20.5	20.1	19.8	19.4	19.1	18.7	18.3	18.0	17.6	17.2	16.9	16.5
33	22.5	22.1	21.8	21.4	21.0	20.7	20.3	19.9	19.6	19.2	18.8	18.5	18.1	17.8	17.4
34	I 23.5	I 23.1	I 22.7	I 22.4	I 22.0	I 21.6	I 21.2	I 20.9	I 20.5	I 20.1	I 19.8	I 19.4	I 19.0	I 18.6	I 18.3

TABLE V.*—AZIMUTH OF POLARIS WHEN AT ELONGATION FOR ANY YEAR BETWEEN 1914 AND 1928—*Concluded*

Latitude	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
35	1 24.5	1 24.1	1 23.7	1 23.3	1 23.0	1 22.6	1 22.2	1 21.8	1 21.5	1 21.1	1 20.7	1 20.4	1 20.0	1 19.6	1 19.2
36	25.5	25.2	24.8	24.4	24.0	23.6	23.3	22.9	22.5	22.1	21.7	21.4	21.0	20.6	20.2
37	26.7	26.3	25.9	25.3	25.1	24.7	24.3	24.0	23.6	23.2	22.8	22.4	22.0	21.6	21.3
38	27.8	27.4	27.0	26.6	26.2	25.9	25.5	25.1	24.7	24.3	23.9	23.5	23.2	22.8	22.4
39	29.0	28.6	28.2	27.8	27.5	27.1	26.7	26.3	25.8	25.5	25.1	24.7	24.3	23.9	23.5
40	30.3	29.9	29.5	29.1	28.7	28.3	27.9	27.5	27.1	26.7	26.3	25.9	25.5	25.1	24.7
41	31.7	31.3	30.9	30.4	30.0	29.6	29.1	28.8	28.4	28.0	27.6	27.2	26.8	26.4	26.0
42	33.1	32.7	32.3	31.9	31.5	31.0	30.6	30.2	29.8	29.4	29.0	28.6	28.2	27.8	27.3
43	34.6	34.2	33.8	33.4	32.9	32.5	32.1	31.8	31.2	30.8	30.4	30.0	29.6	29.1	28.7
44	36.2	35.8	35.3	34.9	34.5	34.1	33.6	33.2	32.8	32.4	31.9	31.5	31.1	30.6	30.2
45	37.8	37.4	37.0	36.6	36.1	35.7	35.3	34.8	34.4	34.0	33.5	33.1	32.6	32.2	31.8
46	39.6	39.2	38.7	38.3	37.8	37.4	37.0	36.5	36.1	35.6	35.2	34.8	34.3	33.9	33.4
47	41.5	41.0	40.6	40.1	39.7	39.2	38.8	38.3	37.9	37.4	37.0	36.5	36.1	35.6	35.2
48	43.4	43.0	42.5	42.0	41.6	41.1	40.7	40.2	39.8	39.3	38.8	38.4	37.9	37.4	37.0
49	45.5	45.0	44.5	44.1	43.6	43.1	42.7	42.2	41.7	41.3	40.8	40.3	39.9	39.4	38.9
50	1 47.7	1 47.2	1 46.7	1 46.2	1 45.7	1 45.3	1 44.8	1 44.3	1 43.8	1 43.4	1 42.9	1 42.4	1 41.9	1 41.4	1 41.0

* Table V was kindly furnished for this publication by Mr. O. H. Tittman, Superintendent of the U. S. Coast and Geodetic Survey.

The above table was computed with the mean declination of Polaris for each year. A more accurate result will be had by applying to the tabular values the following correction, which depends on the difference of the mean and the apparent place of the star. The deduced azimuth will, in general, be correct within o′.3.

For middle of	Correction	For middle of	Correction	For middle of	Correction
January	−o.5	May	+o.1	September	−o.1
February	−o.4	June	+o.2	October	−o.4
March	−o.3	July	+o.2	November	−o.6
April	o.o	August	+o.1	December	−o.8

TABLE VI.*—TOTAL SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, ON THE YEARLY AVERAGE, AT PROMINENT PLACES IN NORTH AMERICA

[*A + sign indicates a deflection of the north-seeking end of the magnet toward the east, a — sign the contrary direction*]

Local mean time	1 Key West, Fla.	2 Los Angeles, Cal.	3 Washington, D. C.	4 Philadelphia, Pa.	5 Madison, Wis.	6 Toronto, Canada	7 Sitka, Alaska	8 Uglamie, Point Barrow	9 Plover Point, Point Barrow	10 Fort Rae, Great Slave Lake	11 Kingua Fjord, Cumberland Sound	12 Fort Conger, Grinnell Land	Average values, stations 1 to 6, inclusive
	′	′	′	′	′	′	′	′	′	′	′	′	′
1 a.m.	+0.0	+0.0	+0.7	+0.6	+0.1	+0.6	+0.2	−12.8	− 8.0	−11.0	+11.7	+43.2	+0.35
2 a.m.	−0.0	+0.1	+0.7	+0.5	0.0	+0.5	+1.0	− 4.9	− 1.9	− 6.6	+15.8	+45.1	+0.05
3 a.m.	+0.1	+0.2	+0.9	+0.6	+0.2	+0.8	+1.4	+ 3.3	+ 3.6	+ 0.8	+18.0	+41.2	+0.07
4 a.m.	+0.2	+0.3	+1.2	+1.0	+0.5	+1.2	+2.0	+ 6.2	+10.9	+ 7.4	+19.1	+25.7	+0.75
5 a.m.	+0.4	+0.6	+1.7	+1.5	+1.0	+1.8	+2.9	+14.3	+16.6	+13.6	+19.3	+31.6	+1.19
6 a.m.	+1.0	+1.3	+2.1	+2.1	+1.4	+2.7	+4.2	+21.6	+19.3	+21.0	+20.1	+19.7	+1.79
7 a.m.	+2.1	+2.4	+2.8	+3.3	+2.6	+3.5	+5.3	+26.1	+27.1	+26.2	+19.9	+26.6	+2.80
8 a.m.	+2.5	+3.1	+3.2	+3.5	+3.2	+3.8	+6.0	+26.7	+27.0	+29.4	+17.4	+18.7	+3.24
9 a.m.	+2.2	+2.6	+2.3	+2.8	+3.0	+3.0	+5.3	+26.1	+19.9	+25.5	+10.8	+ 1.2	+2.67
10 a.m.	+1.1	+1.1	+0.9	+0.8	+1.7	−0.8	+3.0	+ 9.9	+ 9.3	+16.8	+ 3.7	−12.7	+1.09
11 a.m.	−0.2	−0.8	−1.3	−1.6	−0.7	−2.0	+0.6	+ 1.4	− 0.4	+ 8.0	+ 1.3	−21.4	−1.08
Noon	−1.4	−2.2	−3.2	−3.4	−2.5	−4.2	−2.1	− 5.9	− 8.2	− 0.9	− 9.0	−40.7	−2.80
1 p.m.	−2.1	−2.7	−4.3	−4.3	−3.5	−5.0	−3.2	− 7.3	−10.7	− 4.0	−15.1	−45.6	−3.63
2 p.m.	−2.2	−2.6	−4.3	−4.1	−3.5	−4.8	−4.2	− 7.7	− 9.8	− 8.1	−21.2	−49.2	−3.56
3 p.m.	−1.9	−2.0	−3.5	−3.1	−2.6	−3.8	−4.6	− 7.3	− 9.9	−10.6	−20.4	−45.8	−2.80
4 p.m.	−1.3	−1.1	−2.5	−2.2	−1.6	−2.5	−4.6	− 9.1	− 9.8	−11.3	−20.6	−53.7	−1.85
5 p.m.	−0.8	−0.5	−1.5	−1.0	−0.7	−1.3	−3.8	− 9.9	−10.2	−12.1	−23.6	−23.7	−0.95
6 p.m.	−0.4	−0.2	−0.8	−0.4	−0.2	−0.3	−3.2	− 9.9	− 9.7	−12.9	−19.4	−17.3	−0.36
7 p.m.	−0.2	−0.0	0.0	+0.0	+0.2	+0.2	−2.4	− 8.4	− 8.4	−12.5	−16.1	−27.2	+0.05
8 p.m.	+0.1	+0.1	+0.6	+0.8	+0.2	+0.7	−1.4	− 6.0	− 9.0	−11.0	−15.5	− 3.5	+0.44
9 p.m.	+0.2	+0.1	+1.0	+0.6	+0.6	+1.2	−0.8	− 8.1	− 7.5	−12.0	− 8.8	+ 3.5	+0.64
10 p.m.	+0.2	+0.1	+1.1	+1.2	+0.7	+1.3	−0.4	−10.9	− 7.9	−11.9	− 0.6	+22.4	+0.79
11 p.m.	+0.2	+0.1	+1.1	+0.7	+0.2	+1.2	−0.6	− 9.1	−11.5	−11.9	+ 3.9	−30.0	+0.60
Midnight.	+0.1	+0.0	+1.0	+0.6	+0.1	+0.8	−0.6	−13.4	−10.8	−12.0	+ 9.2	+32.6	+0.45
Range	4.7	5.8	7.5	7.8	6.7	8.8	10.6	40.1	38.6	41.4	43.7	98.8	6.9

* From Appendix No. 9, Coast and Geodetic Survey Report of 1890.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′										Correction for 1′ increase in declination of Polaris	
	Latitude 10°	Latitude 11°	Latitude 12°	Latitude 13°	Latitude 14°	Latitude 15°	Latitude 16°	Latitude 17°	Latitude 18°	Latitude 19°	Latitude 20°	Latitude 10°	Latitude 20°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0 15	0 04 36	0 04 37	0 04 38	0 04 39	0 04 41	0 04 42	0 04 43	0 04 45	0 04 47	0 04 48	0 04 50	− 4	− 4
0 30	0 09 11	0 09 13	0 09 15	0 09 17	0 09 20	0 09 23	0 09 25	0 09 29	0 09 32	0 09 36	0 09 39	− 8	− 8
0 45	0 13 43	0 13 46	0 13 49	0 13 53	0 13 57	0 14 01	0 14 05	0 14 10	0 14 15	0 14 20	0 14 26	−12	−13
1 00	0 18 12	0 18 16	0 18 20	0 18 25	0 18 30	0 18 35	0 18 41	0 18 47	0 18 54	0 19 01	0 19 09	−16	−17
1 15	0 22 36	0 22 41	0 22 46	0 22 52	0 22 58	0 23 05	0 23 12	0 23 20	0 23 28	0 23 37	0 23 46	−20	−21
1 30	0 26 54	0 27 00	0 27 06	0 27 13	0 27 21	0 27 29	0 27 37	0 27 46	0 27 56	0 28 07	0 28 18	−24	−25
1 45	0 31 05	0 31 12	0 31 19	0 31 27	0 31 36	0 31 45	0 31 55	0 32 06	0 32 17	0 32 29	0 32 42	−27	−29
2 00	0 35 09	0 35 16	0 35 24	0 35 33	0 35 43	0 35 53	0 36 04	0 36 16	0 36 29	0 36 43	0 36 57	−31	−32
2 15	0 39 03	0 39 11	0 39 20	0 39 30	0 39 41	0 39 52	0 40 04	0 40 18	0 40 32	0 40 47	0 41 03	−34	−36
2 30	0 42 47	0 42 56	0 43 06	0 43 17	0 43 28	0 43 41	0 43 54	0 44 09	0 44 24	0 44 40	0 44 58	−37	−39
2 45	0 46 19	0 46 29	0 46 40	0 46 52	0 47 04	0 47 18	0 47 32	0 47 48	0 48 04	0 48 22	0 48 41	−40	−42
3 00	0 49 40	0 49 51	0 50 02	0 50 15	0 50 28	0 50 42	0 50 58	0 51 15	0 51 33	0 51 52	0 52 12	−43	−45
3 15	0 52 48	0 52 59	0 53 11	0 53 24	0 53 39	0 53 54	0 54 11	0 54 28	0 54 47	0 55 07	0 55 29	−46	−48
3 30	0 55 43	0 55 54	0 56 07	0 56 21	0 56 36	0 56 52	0 57 09	0 57 28	0 57 47	0 58 09	0 58 31	−49	−51
3 45	0 58 22	0 58 34	0 58 48	0 59 02	0 59 18	0 59 35	0 59 53	1 00 12	1 00 33	1 00 55	1 01 18	−51	−53
4 00	1 00 47	1 01 00	1 01 13	1 01 28	1 01 44	1 02 02	1 02 21	1 02 41	1 03 02	1 03 25	1 03 49	−53	−55
4 15	1 02 56	1 03 09	1 03 23	1 03 38	1 03 55	1 04 13	1 04 33	1 04 53	1 05 15	1 05 39	1 06 04	−55	−57
4 30	1 04 49	1 05 02	1 05 17	1 05 33	1 05 50	1 06 08	1 06 28	1 06 49	1 07 12	1 07 36	1 08 02	−56	−59
4 45	1 06 25	1 06 39	1 06 53	1 07 10	1 07 27	1 07 46	1 08 06	1 08 28	1 08 51	1 09 15	1 09 42	−58	−61
5 00	1 07 44	1 07 58	1 08 13	1 08 29	1 08 47	1 09 06	1 09 26	1 09 48	1 10 12	1 10 37	1 11 03	−59	−62
5 15	1 08 46	1 08 59	1 09 15	1 09 31	1 09 49	1 10 08	1 10 29	1 10 51	1 11 15	1 11 40	1 12 07	−60	−62
5 30	1 09 30	1 09 43	1 09 59	1 10 15	1 10 33	1 10 52	1 11 13	1 11 35	1 11 59	1 12 25	1 12 52	−60	−63
5 45	1 09 56	1 10 09	1 10 25	1 10 41	1 10 59	1 11 18	1 11 39	1 12 01	1 12 25	1 12 51	1 13 18	−61	−63
6 00	1 10 04	1 10 17	1 10 32	1 10 49	1 11 07	1 11 26	1 11 47	1 12 09	1 12 33	1 12 58	1 13 26	−61	−64
6 15	1 09 54	1 10 07	1 10 22	1 10 39	1 10 55	1 11 15	1 11 36	1 11 58	1 12 22	1 12 47	1 13 14	−61	−63
6 30	1 09 26	1 09 39	1 09 54	1 10 10	1 10 28	1 10 46	1 11 07	1 11 29	1 11 52	1 12 17	1 12 44	−60	−63
6 45	1 08 40	1 08 53	1 09 08	1 09 23	1 09 41	1 09 59	1 10 19	1 10 41	1 11 04	1 11 29	1 11 55	−60	−62

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′										Correction for 1′ increase in declination of Polaris	
	Latitude 10°	Latitude 11°	Latitude 12°	Latitude 13°	Latitude 14°	Latitude 15°	Latitude 16°	Latitude 17°	Latitude 18°	Latitude 19°	Latitude 20°	Latitude 10°	Latitude 20°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
7 00	1 07 37	1 07 50	1 08 05	1 08 19	1 08 36	1 08 54	1 09 14	1 09 35	1 09 58	1 10 22	1 10 47	−59	−61
7 15	1 06 16	1 06 29	1 06 42	1 06 57	1 07 14	1 07 32	1 07 51	1 08 11	1 08 33	1 08 57	1 09 22	−58	−60
7 30	1 04 39	1 04 51	1 05 04	1 05 19	1 05 35	1 05 52	1 06 10	1 06 30	1 06 52	1 07 15	1 07 39	−56	−58
7 45	1 02 44	1 02 56	1 03 09	1 03 23	1 03 38	1 03 55	1 04 13	1 04 32	1 04 53	1 05 15	1 05 39	−54	−56
8 00	1 00 34	1 00 45	1 00 58	1 01 11	1 01 26	1 01 42	1 01 59	1 02 18	1 02 38	1 02 59	1 03 22	−52	−55
8 15	0 58 09	0 58 19	0 58 31	0 58 44	0 58 58	0 59 13	0 59 30	0 59 47	1 00 06	1 00 27	1 00 49	−50	−53
8 30	0 55 28	0 55 38	0 55 49	0 56 02	0 56 15	0 56 29	0 56 45	0 57 02	0 57 20	0 57 39	0 58 00	−48	−50
8 45	0 52 33	0 52 43	0 52 53	0 53 05	0 53 18	0 53 31	0 53 46	0 54 02	0 54 19	0 54 37	0 54 57	−45	−48
9 00	0 49 25	0 49 33	0 49 44	0 49 55	0 50 07	0 50 19	0 50 33	0 50 48	0 51 04	0 51 21	0 51 40	−42	−45
9 15	0 46 05	0 46 13	0 46 22	0 46 32	0 46 43	0 46 55	0 47 08	0 47 21	0 47 36	0 47 52	0 48 09	−40	−42
9 30	0 42 32	0 42 40	0 42 48	0 42 57	0 43 07	0 43 18	0 43 30	0 43 43	0 43 57	0 44 11	0 44 27	−37	−38
9 45	0 38 49	0 38 56	0 39 03	0 39 12	0 39 21	0 39 31	0 39 42	0 39 53	0 40 06	0 40 19	0 40 33	−34	−35
10 00	0 34 56	0 35 02	0 35 09	0 35 16	0 35 24	0 35 33	0 35 43	0 35 53	0 36 04	0 36 16	0 36 29	−31	−31
10 15	0 30 54	0 30 59	0 31 05	0 31 12	0 31 19	0 31 27	0 31 35	0 31 44	0 31 54	0 32 05	0 32 16	−28	−28
10 30	0 26 44	0 26 48	0 26 54	0 26 59	0 27 06	0 27 12	0 27 20	0 27 28	0 27 36	0 27 45	0 27 55	−24	−24
10 45	0 22 27	0 22 31	0 22 35	0 22 40	0 22 45	0 22 51	0 22 57	0 23 04	0 23 11	0 23 18	0 23 27	−20	−20
11 00	0 18 04	0 18 08	0 18 11	0 18 15	0 18 19	0 18 24	0 18 29	0 18 34	0 18 40	0 18 46	0 18 52	−16	−16
11 15	0 13 37	0 13 40	0 13 42	0 13 45	0 13 48	0 13 52	0 13 56	0 14 00	0 14 04	0 14 09	0 14 13	−12	−12
11 30	0 09 07	0 09 08	0 09 10	0 09 12	0 09 14	0 09 16	0 09 19	0 09 22	0 09 25	0 09 28	0 09 31	− 8	− 8
11 45	0 04 34	0 04 35	0 04 36	0 04 37	0 04 38	0 04 39	0 04 40	0 04 41	0 04 43	0 04 44	0 04 46	− 4	− 4
Elongation: Azimuth	1 10 04	1 10 18	1 10 33	1 10 49	1 11 07	1 11 26	1 11 47	1 12 09	1 12 33	1 12 58	1 13 26	−61	−64
	h m s	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*s*	*s*
Hour angle	5 59 11	5 59 06	5 59 02	5 58 56	5 58 51	5 58 46	5 58 41	5 58 36	5 58 30	5 58 25	5 58 19	+ 2	+ 2

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′										Correction for 1′ increase in declination of Polaris	
	Latitude 20°	Latitude 21°	Latitude 22°	Latitude 23°	Latitude 24°	Latitude 25°	Latitude 26°	Latitude 27°	Latitude 28°	Latitude 29°	Latitude 30°	Latitude 20°	Latitude 30°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
0 15	0 04 50	0 04 52	0 04 57	0 04 59	0 05 02	0 05 04	0 05 07	0 05 10	0 05 13	0 05 17		− 4	− 5
0 30	0 09 39	0 09 43	0 09 48	0 09 52	0 09 57	0 10 02	0 10 07	0 10 13	0 10 19	0 10 25	0 10 31	− 8	− 9
0 45	0 14 26	0 14 32	0 14 38	0 14 45	0 14 52	0 15 00	0 15 07	0 15 16	0 15 25	0 15 34	0 15 44	−13	−14
1 00	0 19 09	0 19 16	0 19 25	0 19 34	0 19 43	0 19 53	0 20 04	0 20 15	0 20 26	0 20 39	0 20 51	−17	−18
1 15	0 23 46	0 23 56	0 24 07	0 24 18	0 24 29	0 24 42	0 24 55	0 25 08	0 25 23	0 25 38	0 25 54	−21	−23
1 30	0 28 18	0 28 29	0 28 42	0 28 55	0 29 09	0 29 24	0 29 39	0 29 55	0 30 12	0 30 30	0 30 49	−25	−27
1 45	0 32 42	0 32 55	0 33 10	0 33 25	0 33 41	0 33 58	0 34 16	0 34 34	0 34 54	0 35 15	0 35 37	−29	−31
2 00	0 36 57	0 37 12	0 37 29	0 37 46	0 38 04	0 38 23	0 38 43	0 39 04	0 39 26	0 39 50	0 40 15	−32	−35
2 15	0 41 03	0 41 20	0 41 38	0 41 57	0 42 17	0 42 38	0 43 00	0 43 24	0 43 49	0 44 14	0 44 42	−36	−39
2 30	0 44 58	0 45 16	0 45 36	0 45 57	0 46 19	0 46 42	0 47 06	0 47 32	0 47 59	0 48 27	0 48 57	−39	−43
2 45	0 48 41	0 49 01	0 49 22	0 49 45	0 50 08	0 50 33	0 51 00	0 51 27	0 51 57	0 52 27	0 53 00	−42	−46
3 00	0 52 12	0 52 33	0 52 56	0 53 20	0 53 45	0 54 12	0 54 40	0 55 10	0 55 41	0 56 14	0 56 48	−45	−50
3 15	0 55 29	0 55 51	0 56 15	0 56 41	0 57 08	0 57 36	0 58 06	0 58 37	0 59 10	0 59 45	1 00 22	−48	−53
3 30	0 58 31	0 58 55	0 59 20	0 59 47	1 00 15	1 00 45	1 01 16	1 01 49	1 02 24	1 03 01	1 03 40	−51	−56
3 45	1 01 18	1 01 43	1 02 09	1 02 37	1 03 07	1 03 38	1 04 11	1 04 46	1 05 22	1 06 00	1 06 41	−53	−58
4 00	1 03 49	1 04 15	1 04 43	1 05 12	1 05 42	1 06 15	1 06 49	1 07 25	1 08 02	1 08 42	1 09 24	−55	−61
4 15	1 06 04	1 06 31	1 06 59	1 07 29	1 08 01	1 08 34	1 09 09	1 09 46	1 10 25	1 11 06	1 11 49	−57	−63
4 30	1 08 02	1 08 29	1 08 58	1 09 29	1 10 01	1 10 35	1 11 12	1 11 50	1 12 30	1 13 12	1 13 56	−59	−64
4 45	1 09 42	1 10 10	1 10 39	1 11 10	1 11 44	1 12 19	1 12 55	1 13 34	1 14 15	1 14 58	1 15 44	−61	−66
5 00	1 11 03	1 11 32	1 12 02	1 12 34	1 13 07	1 13 43	1 14 20	1 15 00	1 15 41	1 16 25	1 17 11	−62	−68
5 15	1 12 07	1 12 36	1 13 06	1 13 38	1 14 12	1 14 48	1 15 26	1 16 06	1 16 48	1 17 32	1 18 19	−62	−69
5 30	1 12 52	1 13 21	1 13 51	1 14 24	1 14 58	1 15 34	1 16 13	1 16 53	1 17 35	1 18 20	1 19 07	−63	−69
5 45	1 13 18	1 13 47	1 14 18	1 14 50	1 15 25	1 16 01	1 16 39	1 17 19	1 18 02	1 18 47	1 19 34	−63	−69
6 00	1 13 26	1 13 54	1 14 25	1 14 57	1 15 32	1 16 08	1 16 46	1 17 26	1 18 09	1 18 53	1 19 41	−64	−70
6 15	1 13 14	1 13 43	1 14 13	1 14 45	1 15 19	1 15 55	1 16 33	1 17 13	1 17 55	1 18 40	1 19 26	−63	−69
6 30	1 12 44	1 13 12	1 13 42	1 14 14	1 14 48	1 15 23	1 16 01	1 16 40	1 17 22	1 18 06	1 18 52	−63	−68
6 45	1 11 55	1 12 23	1 12 52	1 13 24	1 13 57	1 14 32	1 15 09	1 15 48	1 16 29	1 17 12	1 17 58	−62	−67

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′										Correction for 1′ increase in declination of Polaris	
	Latitude 20°	Latitude 21°	Latitude 22°	Latitude 23°	Latitude 24°	Latitude 25°	Latitude 26°	Latitude 27°	Latitude 28°	Latitude 29°	Latitude 30°	Latitude 20°	Latitude 30°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
7 00	1 10 47	1 11 15	1 11 44	1 12 15	1 12 47	1 13 22	1 13 58	1 14 36	1 15 16	1 15 59	1 16 44	−61	−66
7 15	1 09 22	1 09 49	1 10 17	1 10 47	1 11 19	1 11 53	1 12 28	1 13 05	1 13 45	1 14 26	1 15 10	−60	−65
7 30	1 07 39	1 08 05	1 08 32	1 09 02	1 09 33	1 10 05	1 10 40	1 11 16	1 11 54	1 12 35	1 13 17	−58	−64
7 45	1 05 39	1 06 04	1 06 30	1 06 59	1 07 28	1 08 00	1 08 33	1 09 08	1 09 46	1 10 24	1 11 06	−56	−62
8 00	1 03 22	1 03 46	1 04 11	1 04 38	1 05 07	1 05 38	1 06 10	1 06 43	1 07 19	1 07 57	1 08 36	−55	−60
8 15	1 00 49	1 01 11	1 01 36	1 02 02	1 02 30	1 02 59	1 03 29	1 04 02	1 04 36	1 05 12	1 05 49	−53	−57
8 30	0 58 00	0 58 22	0 58 45	0 59 10	0 59 36	1 00 04	1 00 33	1 01 04	1 01 36	1 02 10	1 02 46	−50	−54
8 45	0 54 57	0 55 17	0 55 40	0 56 03	0 56 28	0 56 54	0 57 22	0 57 51	0 58 21	0 58 53	0 59 27	−48	−51
9 00	0 51 40	0 51 59	0 52 20	0 52 41	0 53 04	0 53 29	0 53 55	0 54 22	0 54 51	0 55 21	0 55 53	−45	−48
9 15	0 48 09	0 48 27	0 48 46	0 49 07	0 49 28	0 49 51	0 50 15	0 50 40	0 51 07	0 51 35	0 52 05	−42	−45
9 30	0 44 27	0 44 43	0 45 01	0 45 20	0 45 40	0 46 01	0 46 22	0 46 46	0 47 11	0 47 37	0 48 04	−38	−42
9 45	0 40 33	0 40 48	0 41 04	0 41 21	0 41 39	0 41 58	0 42 19	0 42 40	0 43 02	0 43 26	0 43 51	−35	−38
10 00	0 36 29	0 36 43	0 36 57	0 37 12	0 37 29	0 37 46	0 38 04	0 38 23	0 38 43	0 39 04	0 39 27	−31	−34
10 15	0 32 16	0 32 28	0 32 41	0 32 54	0 33 08	0 33 24	0 33 40	0 33 57	0 34 14	0 34 34	0 34 53	−28	−30
10 30	0 27 55	0 28 05	0 28 16	0 28 28	0 28 40	0 28 53	0 29 07	0 29 22	0 29 37	0 29 53	0 30 10	−24	−26
10 45	0 23 27	0 23 35	0 23 44	0 23 54	0 24 05	0 24 16	0 24 27	0 24 39	0 24 52	0 25 06	0 25 20	−20	−22
11 00	0 18 52	0 18 59	0 19 07	0 19 15	0 19 23	0 19 32	0 19 41	0 19 51	0 20 01	0 20 12	0 20 24	−16	−18
11 15	0 14 13	0 14 19	0 14 24	0 14 30	0 14 37	0 14 43	0 14 50	0 14 58	0 15 05	0 15 14	0 15 22	−12	−13
11 30	0 09 31	0 09 34	0 09 38	0 09 42	0 09 46	0 09 51	0 09 56	0 10 01	0 10 06	0 10 11	0 10 17	− 8	− 9
11 45	0 04 46	0 04 48	0 04 50	0 04 52	0 04 54	0 04 56	0 04 58	0 05 00	0 05 03	0 05 06	0 05 09	− 4	− 4
Elongation:													
Azimuth	1 13 26	1 13 55	1 14 25	1 14 58	1 15 32	1 16 08	1 16 46	1 17 27	1 18 09	1 18 54	1 19 41	−64	−69
	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s	s	s
Hour angle	5 58 19	5 58 14	5 58 08	5 58 03	5 57 57	5 57 51	5 57 45	5 57 39	5 57 33	5 57 27	5 57 21	+ 2	+ 2

* From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′																						Correction for 1′ increase in declination of Polaris	
	Latitude 30°		Latitude 31°		Latitude 32°		Latitude 33°		Latitude 34°		Latitude 35°		Latitude 36°		Latitude 37°		Latitude 38°		Latitude 39°		Latitude 40°		Latitude 30°	Latitude 40°
h m	° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″		° ′ ″		″	″
0 15	0 05 17		0 05 20		0 05 23		0 05 27		0 05 31		0 05 35		0 05 40		0 05 44		0 05 49		0 05 54		0 06 00		− 5	− 5
0 30	0 10 31		0 10 38		0 10 45		0 10 53		0 11 01		0 11 09		0 11 18		0 11 27		0 11 37		0 11 47		0 11 57		− 9	− 10
0 45	0 15 44		0 15 54		0 16 04		0 16 16		0 16 27		0 16 40		0 16 53		0 17 07		0 17 21		0 17 36		0 17 52		− 14	− 16
1 00	0 20 51		0 21 05		0 21 19		0 21 34		0 21 50		0 22 06		0 22 24		0 22 42		0 23 01		0 23 21		0 23 42		− 18	− 21
1 15	0 25 54		0 26 11		0 26 28		0 26 47		0 27 06		0 27 27		0 27 48		0 28 11		0 28 34		0 28 59		0 29 26		− 23	− 26
1 30	0 30 49		0 31 09		0 31 30		0 31 52		0 32 15		0 32 40		0 33 05		0 33 32		0 34 00		0 34 30		0 35 01		− 27	− 31
1 45	0 35 37		0 36 00		0 36 24		0 36 49		0 37 16		0 37 44		0 38 14		0 38 44		0 39 17		0 39 51		0 40 27		− 31	− 36
2 00	0 40 15		0 40 41		0 41 08		0 41 37		0 42 07		0 42 38		0 43 12		0 43 47		0 44 23		0 45 02		0 45 42		− 35	− 40
2 15	0 44 42		0 45 11		0 45 41		0 46 13		0 46 46		0 47 21		0 47 58		0 48 37		0 49 18		0 50 00		0 50 45		− 39	− 45
2 30	0 48 57		0 49 29		0 50 02		0 50 37		0 51 13		0 51 52		0 52 32		0 53 14		0 53 59		0 54 46		0 55 35		− 43	− 49
2 45	0 53 00		0 53 34		0 54 10		0 54 47		0 55 27		0 56 08		0 56 52		0 57 37		0 58 25		0 59 16		1 00 09		− 46	− 53
3 00	0 56 48		0 57 25		0 58 03		0 58 43		0 59 25		1 00 10		1 00 56		1 01 45		1 02 37		1 03 31		1 04 28		− 50	− 57
3 15	1 00 22		1 01 01		1 01 41		1 02 24		1 03 08		1 03 55		1 04 45		1 05 37		1 06 31		1 07 29		1 08 29		− 53	− 60
3 30	1 03 40		1 04 20		1 05 03		1 05 48		1 06 35		1 07 24		1 08 16		1 09 11		1 10 08		1 11 09		1 12 12		− 56	− 63
3 45	1 06 41		1 07 23		1 08 08		1 08 54		1 09 44		1 10 35		1 11 30		1 12 27		1 13 27		1 14 30		1 15 36		− 58	− 66
4 00	1 09 24		1 10 08		1 10 54		1 11 43		1 12 34		1 13 28		1 14 24		1 15 23		1 16 26		1 17 31		1 18 40		− 61	− 69
4 15	1 11 49		1 12 35		1 13 23		1 14 13		1 15 06		1 16 01		1 16 59		1 18 00		1 19 05		1 20 12		1 21 23		− 63	− 72
4 30	1 13 56		1 14 43		1 15 32		1 16 23		1 17 18		1 18 14		1 19 14		1 20 17		1 21 23		1 22 32		1 23 45		− 64	− 74
4 45	1 15 44		1 16 31		1 17 21		1 18 14		1 19 09		1 20 07		1 21 08		1 22 12		1 23 20		1 24 31		1 25 45		− 66	− 75
5 00	1 17 11		1 18 00		1 18 51		1 19 44		1 20 40		1 21 39		1 22 41		1 23 46		1 24 55		1 26 07		1 27 23		− 68	− 76
5 15	1 18 19		1 19 08		1 19 59		1 20 54		1 21 50		1 22 50		1 23 53		1 24 59		1 26 08		1 27 21		1 28 38		− 69	− 77
5 30	1 19 07		1 19 56		1 20 48		1 21 42		1 22 40		1 23 40		1 24 43		1 25 49		1 26 59		1 28 12		1 29 30		− 69	− 78
5 45	1 19 34		1 20 23		1 21 15		1 22 10		1 23 07		1 24 08		1 25 11		1 26 17		1 27 27		1 28 41		1 29 58		− 69	− 78
6 00	1 19 41		1 20 30		1 21 22		1 22 16		1 23 13		1 24 14		1 25 17		1 26 23		1 27 33		1 28 47		1 30 04		− 70	− 78
6 15	1 19 26		1 20 15		1 21 07		1 22 01		1 22 58		1 23 58		1 25 01		1 26 07		1 27 17		1 28 30		1 29 46		− 69	− 78
6 30	1 18 52		1 19 41		1 20 32		1 21 26		1 22 22		1 23 21		1 24 24		1 25 29		1 26 38		1 27 50		1 29 06		− 68	− 77
6 45	1 17 58		1 18 46		1 19 36		1 20 29		1 21 25		1 22 23		1 23 24		1 24 29		1 25 37		1 26 48		1 28 03		− 67	− 76

*From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′												Correction for 1′ increase in declination of Polaris	
	Latitude 30°	Latitude 31°	Latitude 32°	Latitude 33°	Latitude 34°	Latitude 35°	Latitude 36°	Latitude 37°	Latitude 38°	Latitude 39°	Latitude 40°		Latitude 30°	Latitude 40°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″		″	″
7 00	1 16 44	1 17 31	1 18 20	1 19 12	1 20 06	1 21 04	1 22 04	1 23 07	1 24 14	1 25 24	1 26 37		−66	−75
7 15	1 15 10	1 15 56	1 16 44	1 17 35	1 18 28	1 19 24	1 20 23	1 21 25	1 22 30	1 23 38	1 24 50		−65	−73
7 30	1 13 17	1 14 02	1 14 49	1 15 38	1 16 30	1 17 24	1 18 21	1 19 21	1 20 25	1 21 31	1 22 41		−64	−72
7 45	1 11 06	1 11 49	1 12 34	1 13 22	1 14 12	1 15 05	1 16 00	1 16 58	1 17 59	1 19 04	1 20 11		−62	−69
8 00	1 08 36	1 09 18	1 10 01	1 10 47	1 11 36	1 12 26	1 13 20	1 14 16	1 15 14	1 16 16	1 17 21		−60	−66
8 15	1 05 49	1 06 29	1 07 11	1 07 55	1 08 41	1 09 30	1 10 21	1 11 14	1 12 11	1 13 10	1 14 12		−57	−64
8 30	1 02 46	1 03 24	1 04 04	1 04 46	1 05 29	1 06 16	1 07 04	1 07 55	1 08 49	1 09 45	1 10 44		−54	−61
8 45	0 59 27	1 00 03	1 00 40	1 01 20	1 02 01	1 02 45	1 03 31	1 04 19	1 05 10	1 06 03	1 06 59		−51	−58
9 00	0 55 53	0 56 26	0 57 02	0 57 39	0 58 18	0 58 59	0 59 43	1 00 27	1 01 14	1 02 04	1 02 57		−48	−54
9 15	0 52 05	0 52 36	0 53 09	0 53 43	0 54 20	0 54 58	0 55 38	0 56 20	0 57 04	0 57 50	0 58 39		−45	−50
9 30	0 48 04	0 48 33	0 49 03	0 49 35	0 50 08	0 50 43	0 51 20	0 51 58	0 52 39	0 53 22	0 54 07		−42	−46
9 45	0 43 51	0 44 17	0 44 44	0 45 13	0 45 44	0 46 16	0 46 49	0 47 24	0 48 01	0 48 40	0 49 21		−38	−42
10 00	0 39 27	0 39 50	0 40 15	0 40 41	0 41 08	0 41 37	0 42 07	0 42 39	0 43 12	0 43 47	0 44 24		−34	−38
10 15	0 34 53	0 35 14	0 35 35	0 35 58	0 36 22	0 36 48	0 37 14	0 37 42	0 38 12	0 38 43	0 39 15		−30	−34
10 30	0 30 10	0 30 28	0 30 47	0 31 07	0 31 28	0 31 50	0 32 12	0 32 37	0 33 02	0 33 29	0 33 57		−26	−29
10 45	0 25 20	0 25 35	0 25 51	0 26 08	0 26 25	0 26 43	0 27 03	0 27 23	0 27 44	0 28 07	0 28 30		−22	−24
11 00	0 20 24	0 20 36	0 20 49	0 21 02	0 21 16	0 21 31	0 21 46	0 22 03	0 22 20	0 22 38	0 22 57		−18	−20
11 15	0 15 22	0 15 31	0 15 41	0 15 51	0 16 02	0 16 13	0 16 24	0 16 37	0 16 50	0 17 03	0 17 17		−13	−15
11 30	0 10 17	0 10 23	0 10 29	0 10 36	0 10 43	0 10 51	0 10 58	0 11 07	0 11 15	0 11 24	0 11 34		− 9	−10
11 45	0 05 09	0 05 12	0 05 15	0 05 19	0 05 22	0 05 26	0 05 30	0 05 34	0 05 38	0 05 43	0 05 47		− 4	− 5
Elongation: Azimuth.	1 19 41	1 20 30	1 21 22	1 22 17	1 23 14	1 24 14	1 25 18	1 26 24	1 27 34	1 28 48	1 30 05		−69	−78
Hour angle.	h m s 5 57 21	h m s 5 57 14	h m s 5 57 08	h m s 5 57 01	h m s 5 56 54	h m s 5 56 47	h m s 5 56 39	h m s 5 56 32	h m s 5 56 24	h m s 5 56 16	h m s 5 56 08		s + 2	s + 3

*From Appendix No. 10, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Continued*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′												Correction for 1′ increase in declination of Polaris	
	Latitude 40°	Latitude 41°	Latitude 42°	Latitude 43°	Latitude 44°	Latitude 45°	Latitude 46°	Latitude 47°	Latitude 48°	Latitude 49°	Latitude 50°		Latitude 40°	Latitude 50°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″		″	″
0 15	0 06 00	0 06 05	0 06 11	0 06 17	0 06 24	0 06 31	0 06 38	0 06 46	0 06 54	0 07 03	0 07 12		− 5	− 6
0 30	0 11 57	0 12 09	0 12 21	0 12 33	0 12 46	0 13 00	0 13 14	0 13 30	0 13 46	0 14 03	0 14 21		−10	−13
0 45	0 17 52	0 18 09	0 18 27	0 18 45	0 19 05	0 19 25	0 19 47	0 20 10	0 20 34	0 21 00	0 21 27		−16	−19
1 00	0 23 42	0 24 04	0 24 28	0 24 52	0 25 18	0 25 45	0 26 14	0 26 45	0 27 17	0 27 51	0 28 27		−21	−25
1 15	0 29 26	0 29 53	0 30 22	0 30 53	0 31 25	0 31 59	0 32 34	0 33 12	0 33 52	0 34 34	0 35 18		−26	−32
1 30	0 35 01	0 35 34	0 36 08	0 36 45	0 37 23	0 38 03	0 38 46	0 39 30	0 40 18	0 41 08	0 42 01		−31	−38
1 45	0 40 27	0 41 05	0 41 45	0 42 27	0 43 11	0 43 57	0 44 46	0 45 38	0 46 33	0 47 30	0 48 31		−36	−43
2 00	0 45 42	0 46 25	0 47 10	0 47 57	0 48 47	0 49 39	0 50 35	0 51 33	0 52 35	0 53 40	0 54 49		−40	−49
2 15	0 50 45	0 51 33	0 52 22	0 53 15	0 54 10	0 55 08	0 56 09	0 57 14	0 58 22	0 59 35	1 00 51		−45	−54
2 30	0 55 35	0 56 26	0 57 21	0 58 18	0 59 18	1 00 22	1 01 29	1 02 40	1 03 54	1 05 13	1 06 37		−49	−59
2 45	1 00 09	1 01 05	1 02 04	1 03 06	1 04 11	1 05 20	1 06 32	1 07 48	1 09 09	1 10 34	1 12 04		−53	−64
3 00	1 04 28	1 05 28	1 06 30	1 07 36	1 08 46	1 10 00	1 11 17	1 12 39	1 14 05	1 15 36	1 17 12		−57	−68
3 15	1 08 29	1 09 32	1 10 39	1 11 49	1 13 03	1 14 21	1 15 43	1 17 10	1 18 41	1 20 18	1 22 00		−60	−72
3 30	1 12 12	1 13 19	1 14 29	1 15 43	1 17 00	1 18 22	1 19 49	1 21 20	1 22 56	1 24 37	1 26 25		−63	−76
3 45	1 15 36	1 16 46	1 17 59	1 19 16	1 20 37	1 22 03	1 23 33	1 25 08	1 26 49	1 28 35	1 30 27		−66	−80
4 00	1 18 40	1 19 52	1 21 08	1 22 28	1 23 53	1 25 21	1 26 54	1 28 34	1 30 18	1 32 08	1 34 05		−69	−83
4 15	1 21 23	1 22 38	1 23 56	1 25 19	1 26 46	1 28 17	1 29 54	1 31 33	1 33 24	1 35 17	1 37 17		−72	−86
4 30	1 23 45	1 25 02	1 26 22	1 27 47	1 29 16	1 30 50	1 32 30	1 34 14	1 36 06	1 38 01	1 40 04		−74	−88
4 45	1 25 45	1 27 03	1 28 26	1 29 52	1 31 23	1 32 59	1 34 41	1 36 28	1 38 20	1 40 19	1 42 25		−75	−90
5 00	1 27 23	1 28 42	1 30 06	1 31 34	1 33 06	1 34 44	1 36 27	1 38 16	1 40 10	1 42 11	1 44 19		−76	−91
5 15	1 28 38	1 29 58	1 31 23	1 32 52	1 34 25	1 36 04	1 37 48	1 39 38	1 41 34	1 43 36	1 45 45		−77	−92
5 30	1 29 30	1 30 50	1 32 16	1 33 45	1 35 20	1 36 59	1 38 44	1 40 34	1 42 31	1 44 34	1 46 44		−78	−93
5 45	1 29 58	1 31 20	1 32 45	1 34 15	1 35 50	1 37 29	1 39 14	1 41 05	1 43 02	1 45 05	1 47 16		−78	−94
6 00	1 30 04	1 31 25	1 32 50	1 34 20	1 35 55	1 37 34	1 39 19	1 41 09	1 43 06	1 45 09	1 47 19		−78	−93
6 15	1 29 46	1 31 07	1 32 32	1 34 01	1 35 35	1 37 14	1 38 58	1 40 48	1 42 44	1 44 46	1 46 56		−78	−93
6 30	1 29 06	1 30 26	1 31 50	1 33 18	1 34 51	1 36 29	1 38 12	1 40 01	1 41 56	1 43 57	1 46 04		−77	−92
6 45	1 28 03	1 29 20	1 30 44	1 32 11	1 33 43	1 35 19	1 37 01	1 38 48	1 40 41	1 42 40	1 44 46		−76	−91

* From Appendix No. 9, Coast and Geodetic Survey Report of 1895.

TABLE VII.*—AZIMUTH OF POLARIS AT ANY HOUR ANGLE.—*Concluded*

Hour angle before or after upper culmination	Azimuth of Polaris computed for declination 88° 51′												Correction for 1′ increase in declination of polaris	
	Latitude 40°	Latitude 41°	Latitude 42°	Latitude 43°	Latitude 44°	Latitude 45°	Latitude 46°	Latitude 47°	Latitude 48°	Latitude 49°	Latitude 50°		Latitude 40°	Latitude 50°
h m	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″		″	″
7 00	1 26 37	1 27 54	1 29 15	1 30 41	1 32 11	1 33 45	1 35 25	1 37 10	1 39 01	1 40 58	1 43 02		−75	−89
7 15	1 24 50	1 26 05	1 27 24	1 28 48	1 30 16	1 31 48	1 33 25	1 35 08	1 36 56	1 38 51	1 40 51		−73	−87
7 30	1 22 41	1 23 54	1 25 11	1 26 32	1 27 58	1 29 27	1 31 02	1 32 42	1 34 27	1 36 18	1 38 16		−72	−85
7 45	1 20 11	1 21 22	1 22 36	1 23 55	1 25 17	1 26 44	1 28 16	1 29 52	1 31 34	1 33 22	1 35 15		−69	−82
8 00	1 17 21	1 18 29	1 19 41	1 20 57	1 22 16	1 23 40	1 25 08	1 26 41	1 28 19	1 30 02	1 31 51		−66	−79
8 15	1 14 12	1 15 17	1 16 26	1 17 38	1 18 54	1 20 14	1 21 39	1 23 07	1 24 41	1 26 20	1 28 05		−64	−76
8 30	1 10 44	1 11 46	1 12 52	1 14 00	1 15 12	1 16 29	1 17 49	1 19 14	1 20 43	1 22 17	1 23 56		−61	−72
8 45	1 06 59	1 07 57	1 08 59	1 10 04	1 11 12	1 12 23	1 13 40	1 15 00	1 16 24	1 17 53	1 19 27		−58	−68
9 00	1 02 57	1 03 52	1 04 50	1 05 50	1 06 55	1 08 02	1 09 13	1 10 28	1 11 47	1 13 10	1 14 38		−54	−64
9 15	0 58 39	0 59 30	1 00 24	1 01 21	1 02 20	1 03 23	1 04 29	1 05 39	1 06 52	1 08 10	1 09 32		−50	−59
9 30	0 54 07	0 54 54	0 55 44	0 56 36	0 57 31	0 58 28	0 59 29	1 00 34	1 01 41	1 02 53	1 04 08		−46	−55
9 45	0 49 21	0 50 04	0 50 49	0 51 37	0 52 27	0 53 20	0 54 15	0 55 13	0 56 15	0 57 20	0 58 29		−42	−50
10 00	0 44 24	0 45 02	0 45 43	0 46 25	0 47 10	0 47 58	0 48 47	0 49 40	0 50 35	0 51 34	0 52 35		−38	−45
10 15	0 39 15	0 39 49	0 40 25	0 41 03	0 41 42	0 42 24	0 43 08	0 43 54	0 44 43	0 45 35	0 46 29		−34	−40
10 30	0 33 57	0 34 26	0 34 57	0 35 30	0 36 04	0 36 40	0 37 18	0 37 58	0 38 40	0 39 24	0 40 12		−29	−34
10 45	0 28 30	0 28 55	0 29 21	0 29 48	0 30 17	0 30 47	0 31 19	0 31 53	0 32 28	0 33 05	0 33 45		−24	−29
11 00	0 22 57	0 23 16	0 23 37	0 23 59	0 24 22	0 24 47	0 25 12	0 25 39	0 26 08	0 26 38	0 27 09		−20	−23
11 15	0 17 17	0 17 32	0 17 48	0 18 05	0 18 22	0 18 40	0 19 00	0 19 20	0 19 42	0 20 04	0 20 28		−15	−18
11 30	0 11 34	0 11 44	0 11 54	0 12 06	0 12 17	0 12 29	0 12 42	0 12 56	0 13 10	0 13 25	0 13 43		−10	−12
11 45	0 05 47	0 05 52	0 05 58	0 06 04	0 06 09	0 06 15	0 06 22	0 06 29	0 06 36	0 06 44	0 06 51		− 5	− 6
Elongation: Azimuth.	1 30 05	1 31 26	1 32 51	1 34 21	1 35 56	1 37 35	1 39 20	1 41 11	1 43 08	1 45 11	1 47 21		−78	−93
	h m s	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*	*h m s*		*s*	*h m s*
Hour angle.	5 56 08	5 56 00	5 55 51	5 55 43	5 55 33	5 55 24	5 55 14	5 55 04	5 54 53	5 54 42	5 54 31		+ 3	+ 3

* From Appendix No. 9, Coast and Geodetic Survey Report of 1895.

TABLE VIII.*—POLAR DISTANCE OF POLARIS FOR THE BEGINNING OF EACH YEAR, 1914–1928

Year	Mean polar distance	Year	Mean polar distance
	° ′ ″		° ′ ″
1914	1 09 12.1	1921	1 07 02.5
1915	1 08 53.5	1922	1 06 44.1
1916	1 08 35.0	1923	1 06 25.7
1917	1 08 16.5	1924	1 06 07.3
1918	1 07 57.9	1925	1 05 48.9
1919	1 07 39.5	1926	1 05 30.5
1920	1 07 21.0	1927	1 05 12.2
		1928	1 04 53.8

Mean Polar Distances of Polaris are found by subtracting from 90° the corresponding *Mean* declinations. For precise observations the *Apparent* declination (to be found in the Nautical Almanac) should be used since the variation between Mean and Apparent Declinations is constantly changing, and has as a maximum value about ½′.

* From data furnished by the Superintendent of the U. S. Coast and Geodetic Survey for this book.

TABLE IX.[1] HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

	0°		1°		2°		3°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
c = 0.75	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
c = 1.00	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
c = 1.25	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

[1] From "Theory of Practice of Surveying," by Prof. J. B. Johnson, New York; John Wiley & Sons.

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS.—*Continued*

	4°		5°		6°		7°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
c = 0.75	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
c = 1.00	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
c = 1.25	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	8°		9°		10°		11°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
c = 0.75	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
c = 1.00	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
c = 1.25	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	12°		13°		14°		15°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
c = 0.75	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
c = 1.00	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
c = 1.25	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	16°		17°		18°		19°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
c = 0.75.	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
c = 1.00.	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
c = 1.25.	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	20°		21°		22°		23°	
Minutes.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
c = 0.75.	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
c = 1.00.	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
c = 1.25.	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Continued*

	24°		25°		26°		27°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
c = 0.75	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
c = 1.00	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
c = 1.25	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE IX. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS. — *Concluded*

	28°		29°		30°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
c = 0.75	0.66	0.36	0.65	0.37	0.65	0.38
c = 1.00	0.88	0.48	0.87	0.49	0.86	0.51
c = 1.25	1.10	0.60	1.09	0.62	1.08	0.64

TABLE X.*—MINUTES IN DECIMALS OF A DEGREE

′	0″	10″	15″	20″	30″	40″	45″	50″	′
0	.00000	.00278	.00417	.00556	.00833	.01111	.01250	.01389	0
1	.01667	.01944	.02083	.02222	.02500	.02778	.02917	.03055	1
2	.03333	.03611	.03750	.03889	.04167	.04444	.04583	.04722	2
3	.05000	.05278	.05417	.05556	.05833	.06111	.06250	.06389	3
4	.06667	.06944	.07083	.07222	.07500	.07778	.07917	.08056	4
5	.08333	.08611	.08750	.08889	.09167	.09444	.09583	.09722	5
6	.10000	.10278	.10417	.10556	.10833	.11111	.11250	.11389	6
7	.11667	.11944	.12083	.12222	.12500	.12778	.12917	.13056	7
8	.13333	.13611	.13750	.13889	.14167	.14444	.14583	.14722	8
9	.15000	.15278	.15417	.15556	.15833	.16111	.16250	.16389	9
10	.16667	.16944	.17083	.17222	.17500	.17778	.17917	.18056	10
11	.18333	.18611	.18750	.18889	.19167	.19444	.19583	.19722	11
12	.20000	.20278	.20417	.20556	.20833	.21111	.21250	.21389	12
13	.21667	.21944	.22083	.22222	.22500	.22778	.22917	.23056	13
14	.23333	.23611	.23750	.23889	.24167	.24444	.24583	.24722	14
15	.25000	.25278	.25417	.25556	.25833	.26111	.26250	.26389	15
16	.26667	.26944	.27083	.27222	.27500	.27778	.27917	.28056	16
17	.28333	.28611	.28750	.28889	.29167	.29444	.29583	.29722	17
18	.30000	.30278	.30417	.30556	.30833	.31111	.31250	.31389	18
19	.31667	.31944	.32083	.32222	.32500	.32778	.32917	.33056	19
20	.33333	.33611	.33750	.33889	.34167	.34444	.34583	.34722	20
21	.35000	.35278	.35417	.35556	.35833	.36111	.36250	.36389	21
22	.36667	.36944	.37083	.37222	.37500	.37778	.37917	.38056	22
23	.38333	.38611	.38750	.38889	.39167	.39444	.39583	.39722	23
24	.40000	.40278	.40417	.40556	.40833	.41111	.41250	.41389	24
25	.41667	.41944	.42083	.42222	.42500	.42778	.42917	.43056	25
26	.43333	.43611	.43750	.43889	.44167	.44444	.44583	.44722	26
27	.45000	.45278	.45417	.45556	.45833	.46111	.46250	.46389	27
28	.46667	.46944	.47083	.47222	.47500	.47778	.47917	.48056	28
29	.48333	.48611	.48750	.48889	.49167	.49444	.49583	.49722	29
30	.50000	.50278	.50417	.50556	.50833	.51111	.51250	.51389	30
31	.51667	.51944	.52083	.52222	.52500	.52778	.52917	.53056	31
32	.53333	.53611	.53750	.53889	.54167	.54444	.54583	.54722	32
33	.55000	.55278	.55417	.55556	.55833	.56111	.56250	.56389	33
34	.56667	.56944	.57083	.57222	.57500	.57778	.57917	.58056	34
35	.58333	.58611	.58750	.58889	.59167	.59444	.59583	.59722	35
36	.60000	.60278	.60417	.60556	.60833	.61111	.61250	.61389	36
37	.61667	.61944	.62083	.62222	.62500	.62778	.62917	.63056	37
38	.63333	.63611	.63750	.63889	.64167	.64444	.64583	.64722	38
39	.65000	.65278	.65417	.65556	.65833	.66111	.66250	.66389	39
40	.66667	.66944	.67083	.67222	.67500	.67778	.67917	.68056	40
41	.68333	.68611	.68750	.68889	.69167	.69444	.69583	.69722	41
42	.70000	.70278	.70417	.70556	.70833	.71111	.71250	.71389	42
43	.71667	.71944	.72083	.72222	.72500	.72778	.72917	.73056	43
44	.73333	.73611	.73750	.73889	.74167	.74444	.74583	.74722	44
45	.75000	.75278	.75417	.75556	.75833	.76111	.76250	.76389	45
46	.76667	.76944	.77083	.77222	.77500	.77778	.77917	.78056	46
47	.78333	.78611	.78750	.78889	.79167	.79444	.79583	.79722	47
48	.80000	.80278	.80417	.80556	.80833	.81111	.81250	.81389	48
49	.81667	.81944	.82083	.82222	.82500	.82778	.82917	.83056	49
50	.83333	.83611	.83750	.83889	.84167	.84444	.84583	.84722	50
51	.85000	.85278	.85417	.85556	.85833	.86111	.86250	.86389	51
52	.86667	.86944	.87083	.87222	.87500	.87778	.87917	.88056	52
53	.88333	.88611	.88750	.88889	.89167	.89444	.89583	.89722	53
54	.90000	.90278	.90417	.90556	.90833	.91111	.91250	.91389	54
55	.91667	.91944	.92083	.92222	.92500	.92778	.92917	.93056	55
56	.93333	.93611	.93750	.93889	.94167	.94444	.94583	.94722	56
57	.95000	.95278	.95417	.95556	.95833	.96111	.96250	.96389	57
58	.96667	.96944	.97083	.97222	.97500	.97778	.97917	.98056	58
59	.98333	.98611	.98750	.98889	.99167	.99444	.99583	.99722	59
′	0″	10″	15″	20″	30″	40″	45″	50″	′

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No. 100 Log. 000

Table XI.

No. 109 Log. 040

N.	0	1	2	3	4	5	6	7	8	9	Diff.
100	00 0000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
2	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415	424
3	01 2837	3259	3680	4100	4521	4940	5360	5779	6197	6616	420
4	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775	416
105	02 1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
7	9384	9789	0195	0600	1004	1408	1812	2216	2619	3021	404
8	03 3424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
9	7426	7825	8223	8620	9017	9414	9811	0207	0602	0998	397
	04										

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
433	43.3	86.6	129.9	173.2	216.5	259.8	303.1	346.4	389.7
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
431	43.1	86.2	129.3	172.4	215.5	258.6	301.7	344.8	387.9
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
429	42.9	85.8	128.7	171.6	214.5	257.4	300.3	343.2	386.1
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
427	42.7	85.4	128.1	170.8	213.5	256.2	298.9	341.6	384.3
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
425	42.5	85.0	127.5	170.0	212.5	255.0	297.5	340.0	382.5
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
423	42.3	84.6	126.9	169.2	211.5	253.8	296.1	338.4	380.7
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
421	42.1	84.2	126.3	168.4	210.5	252.6	294.7	336.8	378.9
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
419	41.9	83.8	125.7	167.6	209.5	251.4	293.3	335.2	377.1
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
417	41.7	83.4	125.1	166.8	208.5	250.2	291.9	333.6	375.3
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
415	41.5	83.0	124.5	166.0	207.5	249.0	290.5	332.0	373.5
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
413	41.3	82.6	123.9	165.2	206.5	247.8	289.1	330.4	371.7
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
411	41.1	82.2	123.3	164.4	205.5	246.6	287.7	328.8	369.9
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
409	40.9	81.8	122.7	163.6	204.5	245.4	286.3	327.2	368.1
408	40.8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
407	40.7	81.4	122.1	162.8	203.5	244.2	284.9	325.6	366.3
406	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4
405	40.5	81.0	121.5	162.0	202.5	243.0	283.5	324.0	364.5
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
403	40.3	80.6	120.9	161.2	201.5	241.8	282.1	322.4	362.7
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
401	40.1	80.2	120.3	160.4	200.5	240.6	280.7	320.8	360.9
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
399	39.9	79.8	119.7	159.6	199.5	239.4	279.3	319.2	359.1
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
397	39.7	79.4	119.1	158.8	198.5	238.2	277.9	317.6	357.3
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	356.4

No. 110 — TABLE XI.—*Continued.* — No. 1
Log. 041 — Log. 0

N.	0	1	2	3	4	5	6	7	8	9	Diff.
110	04 1393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
2	9218	9606	9993	0380	0766	1153	1538	1924	2309	2694	386
3	05 3078	3463	3846	4230	4613	4996	5378	5760	6142	6524	383
4	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320	379
115	06 0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
7	8186	8557	8928	9298	9668	0038	0407	0776	1145	1514	370
8	07 1882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
395	39.5	79.0	118.5	158.0	197.5	237.0	276.5	316.0	355.5
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
393	39.3	78.6	117.9	157.2	196.5	235.8	275.1	314.4	353.7
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
391	39.1	78.2	117.3	156.4	195.5	234.6	273.7	312.8	351.9
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
389	38.9	77.8	116.7	155.6	194.5	233.4	272.3	311.2	350.1
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
387	38.7	77.4	116.1	154.8	193.5	232.2	270.9	309.6	348.3
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
385	38.5	77.0	115.5	154.0	192.5	231.0	269.5	308.0	346.5
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
383	38.3	76.6	114.9	153.2	191.5	229.8	268.1	306.4	344.7
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
381	38.1	76.2	114.3	152.4	190.5	228.6	266.7	304.8	342.9
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
379	37.9	75.8	113.7	151.6	189.5	227.4	265.3	303.2	341.1
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
377	37.7	75.4	113.1	150.8	188.5	226.2	263.9	301.6	339.3
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
375	37.5	75.0	112.5	150.0	187.5	225.0	262.5	300.0	337.5
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
373	37.3	74.6	111.9	149.2	186.5	223.8	261.1	298.4	335.7
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
371	37.1	74.2	111.3	148.4	185.5	222.6	259.7	296.8	333.9
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
369	36.9	73.8	110.7	147.6	184.5	221.4	258.3	295.2	332.1
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
367	36.7	73.4	110.1	146.8	183.5	220.2	256.9	293.6	330.3
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
365	36.5	73.0	109.5	146.0	182.5	219.0	255.7	292.0	328.5
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
363	36.3	72.6	108.9	145.2	181.5	217.8	254.1	290.4	326.7
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
361	36.1	72.2	108.3	144.4	180.5	216.6	252.7	288.8	324.9
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0
359	35.9	71.8	107.7	143.6	179.5	215.4	251.3	287.2	323.1
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
357	35.7	71.4	107.1	142.8	178.5	214.2	249.9	285.6	321.3
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4

N.	0	1	2	3	4	5	6	7	8	9	Diff.
120	07 9181	9543	9904	0266	0626	0987	1347	1707	2067	2426	360
1	08 2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
3	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071	352
4	09 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	0026	346
6	10 0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
8	7210	7549	7888	8227	8565	8903	9241	9579	9916	0253	338
9	11 0590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
1	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245	330
2	12 0574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
4	7105	7429	7753	8076	8399	8722	9045	9368	9690	0012	323
	13										

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
355	35.5	71.0	106.5	142.0	177.5	213.0	248.5	284.0	319.5
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
353	35.3	70.6	105.9	141.2	176.5	211.8	247.1	282.4	317.7
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
351	35.1	70.2	105.3	140.4	175.5	210.6	245.7	280.8	315.9
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
349	34.9	69.8	104.7	139.6	174.5	209.4	244.3	279.2	314.1
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
347	34.7	69.4	104.1	138.8	173.5	208.2	242.9	277.6	312.3
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
345	34.5	69.0	103.5	138.0	172.5	207.0	241.5	276.0	310.5
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
343	34.3	68.6	102.9	137.2	171.5	205.8	240.1	274.4	308.7
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
341	34.1	68.2	102.3	136.4	170.5	204.6	238.7	272.8	306.9
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
339	33.9	67.8	101.7	135.6	169.5	203.4	237.3	271.2	305.1
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
337	33.7	67.4	101.1	134.8	168.5	202.2	235.9	269.6	303.3
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
335	33.5	67.0	100.5	134.0	167.5	201.0	234.5	268.0	301.5
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
333	33.3	66.6	99.9	133.2	166.5	199.8	233.1	266.4	299.7
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
331	33.1	66.2	99.3	132.4	165.5	198.6	231.7	264.8	297.9
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
329	32.9	65.8	98.7	131.6	164.5	197.4	230.3	263.2	296.1
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
327	32.7	65.4	98.1	130.8	163.5	196.2	228.9	261.6	294.3
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
325	32.5	65.0	97.5	130.0	162.5	195.0	227.5	260.0	292.5
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
323	32.3	64.6	96.9	129.2	161.5	193.8	226.1	258.4	290.7
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8

No. 135
Log. 130

TABLE XI.—*Continued.*

No. 142
Log. 175

N.	0	1	2	3	4	5	6	7	8	9	Diff.
135	13 0334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
8	9879	0194	0508	0822	1136	1450	1763	2076	2389	2702	314
9	14 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
1	9219	9527	9835	0142	0449	0756	1063	1370	1676	1982	307
2	15 2288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
4	8362	8664	8965	9266	9567	9868	0168	0469	0769	1068	301
145	16 1368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
8	17 0262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
321	32.1	64.2	96.3	128.4	160.5	192.6	224.7	256.8	288.9
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
319	31.9	63.8	95.7	127.6	159.5	191.4	223.3	255.2	287.1
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	254.4	286.2
317	31.7	63.4	95.1	126.8	158.5	190.2	221.9	253.6	285.3
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
315	31.5	63.0	94.5	126.0	157.5	189.0	220.5	252.0	283.5
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
313	31.3	62.6	93.9	125.2	156.5	187.8	219.1	250.4	281.7
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
311	31.1	62.2	93.3	124.4	155.5	186.6	217.7	248.8	279.9
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
309	30.9	61.8	92.7	123.6	154.5	185.4	216.3	247.2	278.1
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
307	30.7	61.4	92.1	122.8	153.5	184.2	214.9	245.6	276.3
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
305	30.5	61.0	91.5	122.0	152.5	183.0	213.5	244.0	274.5
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
303	30.3	60.6	90.9	121.2	151.5	181.8	212.1	242.4	272.7
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
301	30.1	60.2	90.3	120.4	150.5	180.6	210.7	240.8	270.9
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
299	29.9	59.8	89.7	119.6	149.5	179.4	209.3	239.2	269.1
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
297	29.7	59.4	89.1	118.8	148.5	178.2	207.9	237.6	267.3
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
295	29.5	59.0	88.5	118.0	147.5	177.0	206.5	236.0	265.5
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
293	29.3	58.6	87.9	117.2	146.5	175.8	205.1	234.4	263.7
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
291	29.1	58.2	87.3	116.4	145.5	174.6	203.7	232.8	261.9
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
289	28.9	57.8	86.7	115.6	144.5	173.4	202.3	231.2	260.1
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
287	28.7	57.4	86.1	114.8	143.5	172.2	200.9	229.6	258.3
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4

No. 150 — Log. 176 — TABLE XI.—*Continued.* — No. 169 — Log. 230

N.	0	1	2	3	4	5	6	7	8	9	Diff.
150	17 6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
1	8977	9264	9552	9839	0126	0413	0699	0986	1272	1558	287
2	18 1844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
4	7521	7803	8084	8366	8647	8928	9209	9490	9771	0051	281
155	19 0332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
8	8657	8932	9206	9481	9755	0029	0303	0577	0850	1124	274
9	20 1397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
160	4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
2	9515	9783	0051	0319	0586	0853	1121	1388	1654	1921	267
3	21 2188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
6	22 0108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
9	7887	8144	8400	8657	8913	9170	9426	9682	9938	0193	256
	23										

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
285	28.5	57.0	85.5	114.0	142.5	171.0	199.5	228.0	256.5
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
283	28.3	56.6	84.9	113.2	141.5	169.8	198.1	226.4	254.7
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
281	28.1	56.2	84.3	112.4	140.5	168.6	196.7	224.8	252.9
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0
279	27.9	55.8	83.7	111.6	139.5	167.4	195.3	223.2	251.1
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
277	27.7	55.4	83.1	110.8	138.5	166.2	193.9	221.6	249.3
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
275	27.5	55.0	82.5	110.0	137.5	165.0	192.5	220.0	247.5
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
273	27.3	54.6	81.9	109.2	136.5	163.8	191.1	218.4	245.7
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
271	27.1	54.2	81.3	108.4	135.5	162.6	189.7	216.8	243.9
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
269	26.9	53.8	80.7	107.6	134.5	161.4	188.3	215.2	242.1
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
267	26.7	53.4	80.1	106.8	133.5	160.2	186.9	213.6	240.3
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
265	26.5	53.0	79.5	106.0	132.5	159.0	185.5	212.0	238.5
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
263	26.3	52.6	78.9	105.2	131.5	157.8	184.1	210.4	236.7
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
261	26.1	52.2	78.3	104.4	130.5	156.6	182.7	208.8	234.9
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
259	25.9	51.8	77.7	103.6	129.5	155.4	181.3	207.2	233.1
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
257	25.7	51.4	77.1	102.8	128.5	154.2	179.9	205.6	231.3
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
255	25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5

0	1	2	3	4	5	6	7	8	9	Diff.
23 0449	0704	0960	1215	1470	1724	1979	2234	2488	2742	255
2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
8046	8297	8548	8799	9049	9299	9550	9800	0050	0300	250
24 0549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
7973	8219	8464	8709	8954	9198	9443	9687	9932	0176	245
25 0420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
26 0071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
9513	9746	9980	0213	0446	0679	0912	1144	1377	1609	233
27 1842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229

PROPORTIONAL PARTS.

1	2	3	4	5	6	7	8	9
25.5	51.0	76.5	102.0	127.5	153.0	178.5	204.0	229.5
25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
25.3	50.6	75.9	101.2	126.5	151.8	177.1	202.4	227.7
25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
25.1	50.2	75.3	100.4	125.5	150.6	175.7	200.8	225.9
25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
24.9	49.8	74.7	99.6	124.5	149.4	174.3	199.2	224.1
24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
24.7	49.4	74.1	98.8	123.5	148.2	172.9	197.6	222.3
24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
24.5	49.0	73.5	98.0	122.5	147.0	171.5	196.0	220.5
24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
24.3	48.6	72.9	97.2	121.5	145.8	170.1	194.4	218.7
24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
24.1	48.2	72.3	96.4	120.5	144.6	168.7	192.8	216.9
24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
23.9	47.8	71.7	95.6	119.5	143.4	167.3	191.2	215.1
23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
23.7	47.4	71.1	94.8	118.5	142.2	165.9	189.6	213.3
23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
23.5	47.0	70.5	94.0	117.5	141.0	164.5	188.0	211.5
23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
23.3	46.6	69.9	93.2	116.5	139.8	163.1	186.4	209.7
23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
23.1	46.2	69.3	92.4	115.5	138.6	161.7	184.8	207.9
23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0
22.9	45.8	68.7	91.6	114.5	137.4	160.3	183.2	206.1
22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205.2
22.7	45.4	68.1	90.8	113.5	136.2	158.9	181.6	204.3
22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4

No. 190 LOG. 278 — TABLE XI.—*Continued.* — No. 214 LOG. 332

N.	0	1	2	3	4	5	6	7	8	9	Diff.
190	27 8754	8982	9211	9439	9667	9895	0123	0351	0578	0806	228
1	28 1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29 0035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
9	8853	9071	9289	9507	9725	9943	0161	0378	0595	0813	218
200	30 1030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
4	9630	9843	0056	0268	0481	0693	0906	1118	1330	1542	212
205	31 1754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
9	32 0146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
3	8380	8583	8787	8991	9194	9398	9601	9805	0008	0211	203
4	33 0414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
225	22.5	45.0	67.5	90.0	112.5	135.0	157.5	180.0	202.5
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
223	22.3	44.6	66.9	89.2	111.5	133.8	156.1	178.4	200.7
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
221	22.1	44.2	66.3	88.4	110.5	132.6	154.7	176.8	198.9
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
219	21.9	43.8	65.7	87.6	109.5	131.4	153.3	175.2	197.1
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
217	21.7	43.4	65.1	86.8	108.5	130.2	151.9	173.6	195.3
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
215	21.5	43.0	64.5	86.0	107.5	129.0	150.5	172.0	193.5
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
213	21.3	42.6	63.9	85.2	106.5	127.8	149.1	170.4	191.7
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
211	21.1	42.2	63.3	84.4	105.5	126.6	147.7	168.8	189.9
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
209	20.9	41.8	62.7	83.6	104.5	125.4	146.3	167.2	188.1
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
207	20.7	41.4	62.1	82.8	103.5	124.2	144.9	165.6	186.3
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
205	20.5	41.0	61.5	82.0	102.5	123.0	143.5	164.0	184.5
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
203	20.3	40.6	60.9	81.2	101.5	121.8	142.1	162.4	182.7
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8

No. 215 — No. 23
Log. 332 — Log. 38

TABLE XI.—*Continued.*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
215	33 2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
8	8456	8656	8855	9054	9253	9451	9650	9849	0047	0246	199
9	34 0444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
220	2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
3	8305	8500	8694	8889	9083	9278	9472	9666	9860	0054	194
4	35 0248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
9	9835	0025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	36 1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
4	9216	9401	9587	9772	9958	0143	0328	0513	0698	0883	185
235	37 1068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
9	8398	8580	8761	8943	9124	9306	9487	9668	9849	0030	181
	38										

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
201	20.1	40.2	60.3	80.4	100.5	120.6	140.7	160.8	180.9
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
199	19.9	39.8	59.7	79.6	99.5	119.4	139.3	159.2	179.1
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
197	19.7	39.4	59.1	78.8	98.5	118.2	137.9	157.6	177.3
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
195	19.5	39.0	58.5	78.0	97.5	117.0	136.5	156.0	175.5
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
193	19.3	38.6	57.9	77.2	96.5	115.8	135.1	154.4	173.7
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
191	19.1	38.2	57.3	76.4	95.5	114.6	133.7	152.8	171.9
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
189	18.9	37.8	56.7	75.6	94.5	113.4	132.3	151.2	170.1
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
187	18.7	37.4	56.1	74.8	93.5	112.2	130.9	149.6	168.3
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
185	18.5	37.0	55.5	74.0	92.5	111.0	129.5	148.0	166.5
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
183	18.3	36.6	54.9	73.2	91.5	109.8	128.1	146.4	164.7
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
181	18.1	36.2	54.3	72.4	90.5	108.6	126.7	144.8	162.9
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
179	17.9	35.8	53.7	71.6	89.5	107.4	125.3	143.2	161.1

No. 240 — No. 269
Log. 380 — TABLE XI.—*Continued.* — Log. 431

N.	0	1	2	3	4	5	6	7	8	9	Diff.
240	38 0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
4	7390	7568	7746	7924	8101	8279	8456	8634	8811	8989	178
245	9166	9343	9520	9698	9875	0051	0228	0405	0582	0759	177
6	39 0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
1	9674	9847	0020	0192	0365	0538	0711	0883	1056	1228	173
2	40 1401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
7	9933	0102	0271	0440	0609	0777	0946	1114	1283	1451	169
8	41 1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
3	9956	0121	0286	0451	0616	0781	0945	1110	1275	1439	165
4	42 1604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
9	9752	9914	0075	0236	0398	0559	0720	0881	1042	1203	161
	43										

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
177	17.7	35.4	53.1	70.8	88.5	106.2	123.9	141.6	159.3
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
175	17.5	35.0	52.5	70.0	87.5	105.0	122.5	140.0	157.5
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
173	17.3	34.6	51.9	69.2	86.5	103.8	121.1	138.4	155.7
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
171	17.1	34.2	51.3	68.4	85.5	102.6	119.7	136.8	153.9
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
169	16.9	33.8	50.7	67.6	84.5	101.4	118.3	135.2	152.1
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
167	16.7	33.4	50.1	66.8	83.5	100.2	116.9	133.6	150.3
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
165	16.5	33.0	49.5	66.0	82.5	99.0	115.5	132.0	148.5
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
163	16.3	32.6	48.9	65.2	81.5	97.8	114.1	130.4	146.7
162	16.2	32.4	48.5	64.8	81.0	97.2	113.4	129.6	145.8
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9

No. 270 Log. 431 — TABLE XI.—*Continued.* — No. 2[illegible] Log. 4[illegible]

N.	0	1	2	3	4	5	6	7	8	9	Diff.
270	43 1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	9333	9491	9648	9806	9964	0122	0279	0437	0594	0752	158
6	44 0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155
280	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
1	8706	8861	9015	9170	9324	9478	9633	9787	9941	0095	154
2	45 0249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
7	7882	8033	8184	8336	8487	8638	8789	8940	9091	9242	151
8	9392	9543	9694	9845	9995	0146	0296	0447	0597	0748	151
9	46 0898	1048	1198	1348	1499	1649	1799	1948	2098	2248	150
290	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
295	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145	147
6	47 1292	1438	1585	1732	1878	2025	2171	2318	2464	2610	146
7	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
8	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
9	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
161	16.1	32.2	48.3	64.4	80.5	96.6	112.7	128.8	144.9
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
159	15.9	31.8	47.7	63.6	79.5	95.4	111.3	127.2	143.1
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
157	15.7	31.4	47.1	62.8	78.5	94.2	109.9	125.6	141.3
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4
155	15.5	31.0	46.5	62.0	77.5	93.0	108.5	124.0	139.5
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
153	15.3	30.6	45.9	61.2	76.5	91.8	107.1	122.4	137.7
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
151	15.1	30.2	45.3	60.4	75.5	90.6	105.7	120.8	135.9
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
149	14.9	29.8	44.7	59.6	74.5	89.4	104.3	119.2	134.1
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
147	14.7	29.4	44.1	58.8	73.5	88.2	102.9	117.6	132.3
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
145	14.5	29.0	43.5	58.0	72.5	87.0	101.5	116.0	130.5
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
143	14.3	28.6	42.9	57.2	71.5	85.8	100.1	114.4	128.7
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
141	14.1	28.2	42.3	56.4	70.5	84.6	98.7	112.8	126.9
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.0

No. 300 Log. 477 — TABLE XI.—*Continued.* — No. 339 Log. 531

N.	0	1	2	3	4	5	6	7	8	9	Diff.
300	47 7121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
1	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
2	48 0007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
6	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
7	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
8	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
9	9958	0099	0239	0380	0520	0661	0801	0941	1081	1222	140
310	49 1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
1	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
4	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
6	9687	9824	9962	0099	0236	0374	0511	0648	0785	0922	137
7	50 1059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
8	2427	2564	2700	2837	2973	3109	3246	3382	3518	3655	136
9	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	5150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
1	6505	6640	6776	6911	7046	7181	7316	7451	7586	7721	135
2	7856	7991	8126	8260	8395	8530	8664	8799	8934	9068	135
3	9203	9337	9471	9606	9740	9874	0009	0143	0277	0411	134
4	51 0545	0679	0813	0947	1081	1215	1349	1482	1616	1750	134
325	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
6	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415	133
7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
8	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
9	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382	132
330	8514	8646	8777	8909	9040	9171	9303	9434	9566	9697	131
1	9828	9959	0090	0221	0353	0484	0615	0745	0876	1007	131
2	52 1138	1269	1400	1530	1661	1792	1922	2053	2183	2314	131
3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
8	8917	9045	9174	9302	9430	9559	9687	9815	9943	0072	128
9	53 0200	0328	0456	0584	0712	0840	0968	1096	1223	1351	128

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
139	13.9	27.8	41.7	55.6	69.5	83.4	97.3	111.2	125.1
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
137	13.7	27.4	41.1	54.8	68.5	82.2	95.9	109.6	123.3
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
135	13.5	27.0	40.5	54.0	67.5	81.0	94.5	108.0	121.5
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
133	13.3	26.6	39.9	53.2	66.5	79.8	93.1	106.4	119.7
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
131	13.1	26.2	39.3	52.4	65.5	78.6	91.7	104.8	117.9
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
129	12.9	25.8	38.7	51.6	64.5	77.4	90.3	103.2	116.1
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3

No. 340 — No. 379
Log. 531 — Log. 578

TABLE XI.—*Continued.*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
340	53 1479	1607	1734	1862	1990	2117	2245	2372	2500	2627	128
1	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
4	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
6	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204	125
7	54 0329	0455	0580	0705	0830	0955	1080	1205	1330	1454	125
8	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183	124
1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
4	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106	123
355	55 0228	0351	0473	0595	0717	0840	0962	1084	1206	1328	122
6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
8	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
9	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
360	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
1	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
2	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
3	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
4	56 1101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
7	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
1	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426	117
2	57 0543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
127	12.7	25.4	38.1	50.8	63.5	76.2	88.9	101.6	114.3
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
125	12.5	25.0	37.5	50.0	62.5	75.0	87.5	100.0	112.5
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
123	12.3	24.6	36.9	49.2	61.5	73.8	86.1	98.4	110.7
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
121	12.1	24.2	36.3	48.4	60.5	72.6	84.7	96.8	108.9
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
119	11.9	23.8	35.7	47.6	59.5	71.4	83.3	95.2	107.1

N.	0	1	2	3	4	5	6	7	8	9	Diff.
380	57 9784	9898	0012	0126	0241	0355	0469	0583	0697	0811	114
1	58 0925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	
4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	
7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
9	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953	
390	59 1065	1176	1287	1399	1510	1621	1732	1843	1955	2066	
1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	
4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
6	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	
8	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864	109
9	60 0973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	
1	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	108
2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
7	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554	
8	61 0660	0767	0873	0979	1086	1192	1298	1405	1511	1617	
9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
410	2784	2890	2996	3102	3207	3313	3419	3525	3630	3736	
1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	
2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	
3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
117	11.7	23.4	35.1	46.8	58.5	70.2	81.9	93.6	105.3
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
115	11.5	23.0	34.5	46.0	57.5	69.0	80.5	92.0	103.5
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6
113	11.3	22.6	33.9	45.2	56.5	67.8	79.1	90.4	101.7
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
111	11.1	22.2	33.3	44.4	55.5	66.6	77.7	88.8	99.9
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
109	10.9	21.8	32.7	43.6	54.5	65.4	76.3	87.2	98.1
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
107	10.7	21.4	32.1	42.8	53.5	64.2	74.9	85.6	96.3
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6

No. 415
Log. 618

TABLE XI.—*Continued.*

No. 459
Log. 662

N.	0	1	2	3	4	5	6	7	8	9	Dif.
415	61 8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
6	9093	9198	9302	9406	9511	9615	9719	9824	9928	0032	
7	62 0136	0240	0344	0448	0552	0656	0760	0864	0968	1072	104
8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	
9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	
420	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	
1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	
3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	
4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308	102
6	9410	9512	9613	9715	9817	9919	0021	0123	0224	0326	
7	63 0428	0530	0631	0733	0835	0936	1038	1139	1241	1342	
8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	
9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376	101
1	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	
2	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	
3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	
4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	
6	9486	9586	9686	9785	9885	9984	0084	0183	0283	0382	
7	64 0481	0581	0680	0779	0879	0978	1077	1177	1276	1375	
8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	
9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
440	3453	3551	3650	3749	3847	3946	4044	4143	4242	4340	
1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	
2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	
3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	
445	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	
6	9335	9432	9530	9627	9724	9821	9919	0016	0113	0210	
7	65 0308	0405	0502	0599	0696	0793	0890	0987	1084	1181	
8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	
450	3213	3309	3405	3502	3598	3695	3791	3888	3984	4080	
1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	
2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	
4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	
455	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	
6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	
7	9916	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
8	66 0865	0960	1055	1150	1245	1339	1434	1529	1623	1718	
9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	

PROPORTIONAL PARTS.

Dif.	1	2	3	4	5	6	7	8	9
105	10.5	21.0	31.5	42.0	52.5	63.0	73.5	84.0	94.5
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6
103	10.3	20.6	30.9	41.2	51.5	61.8	72.1	82.4	92.7
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4	81.6	91.8
101	10.1	20.2	30.3	40.4	50.5	60.6	70.7	80.8	90.9
100	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
99	9.9	19.8	29.7	39.6	49.5	59.4	69.3	79.2	89.1

No. 460 Log. 662

TABLE XI.—*Continued.*

No. 499 Log. 698

N.	0	1	2	3	4	5	6	7	8	9	Diff.
460	66 2758	2852	2947	3041	3135	3230	3324	3418	3512	3607	
1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	
2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	
4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	
465	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	
6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	
7	9317	9410	9503	9596	9689	9782	9875	9967	0060	0153	93
8	67 0246	0339	0431	0524	0617	0710	0802	0895	0988	1080	
9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	
470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	
1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	
2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	
4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	
6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	
7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
8	9428	9519	9610	9700	9791	9882	9973	0063	0154	0245	
9	68 0336	0426	0517	0607	0698	0789	0879	0970	1060	1151	
480	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	
1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	
2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	
4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	
485	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	
6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	
7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	
9	9309	9398	9486	9575	9664	9753	9841	9930	0019	0107	
490	69 0196	0285	0373	0462	0550	0639	0728	0816	0905	0993	
1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	
2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	
3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	
6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	
7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	
8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	
9	8100	8188	8275	8362	8449	8535	8622	8709	8796	8883	87

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
97	9.7	19.4	29.1	38.8	48.5	58.2	67.9	77.6	87.3
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
95	9.5	19.0	28.5	38.0	47.5	57.0	66.5	76.0	85.5
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
93	9.3	18.6	27.9	37.2	46.5	55.8	65.1	74.4	83.7
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
91	9.1	18.2	27.3	36.4	45.5	54.6	63.7	72.8	81.9
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
89	8.9	17.8	26.7	35.6	44.5	53.4	62.3	71.2	80.1
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4

No. 500 Log. 698 — TABLE XI.—*Continued.* — No. 544 Log. 736

N.	0	1	2	3	4	5	6	7	8	9	Diff.
500	69 8970	9057	9144	9231	9317	9404	9491	9578	9664	9751	
1	9838	9924	0011	0098	0184	0271	0358	0444	0531	0617	
2	70 0704	0790	0877	0963	1050	1136	1222	1309	1395	1482	
3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	
4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	
505	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	
7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	
8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	
9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	
510	7570	7655	7740	7826	7911	7996	8081	8166	8251	8336	85
1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	
2	9270	9355	9440	9524	9609	9694	9779	9863	9948	0033	
3	71 0117	0202	0287	0371	0456	0540	0625	0710	0794	0879	
4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	
6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	
8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	
9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	
520	6003	6087	6170	6254	6337	6421	6504	6588	6671	6754	
1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	
2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	
3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
4	9331	9414	9497	9580	9663	9745	9828	9911	9994	0077	
525	72 0159	0242	0325	0407	0490	0573	0655	0738	0821	0903	
6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	
7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	
8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	
1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	
2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	
3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	
4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	
7	9974	0055	0136	0217	0298	0378	0459	0540	0621	0702	
8	73 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	
9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	
540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	
1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	
2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
3	4800	4880	4960	5040	5120	5200	5279	5359	5439	5519	
4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
87	8.7	17.4	26.1	34.8	43.5	52.2	60.9	69.6	78.3
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
85	8.5	17.0	25.5	34.0	42.5	51.0	59.5	68.0	76.5
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6

No. 545 Log. 736 — TABLE XI.—*Continued.* — No. 58[illegible] Log. 76[illegible]

N.	0	1	2	3	4	5	6	7	8	9	Diff.
545	73 6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	
6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	
7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	
8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	
9	9572	9651	9731	9810	9889	9968	0047	0126	0205	0284	79
550	74 0363	0442	0521	0600	0678	0757	0836	0915	0994	1073	
1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	
2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	
3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	
4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	
6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	
8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	
9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	
560	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	
1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	
2	9736	9814	9891	9968	0045	0123	0200	0277	0354	0431	
3	75 0508	0586	0663	0740	0817	0894	0971	1048	1125	1202	
4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	
7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	
8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	
9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	
570	5875	5951	6027	6103	6180	6256	6332	6408	6484	6560	
1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	
3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	
4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	
575	9668	9743	9819	9894	9970	0045	0121	0196	0272	0347	
6	76 0422	0498	0573	0649	0724	0799	0875	0950	1025	1101	
7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	
8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	
9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	
1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	
2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	
3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	
4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
83	8.3	16.6	24.9	33.2	41.5	49.8	58.1	66.4	74.7
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
81	8.1	16.2	24.3	32.4	40.5	48.6	56.7	64.8	72.9
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
79	7.9	15.8	23.7	31.6	39.5	47.4	55.3	63.2	71.1
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
77	7.7	15.4	23.1	30.8	38.5	46.2	53.9	61.6	69.3
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
[illegible]	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5

No. 585 Log. 767 — TABLE XI.—*Continued.* — No. 62[illegible] Log. 79[illegible]

N.	0	1	2	3	4	5	6	7	8	9	Diff.
585	76 7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	
6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	
8	9377	9451	9525	9599	9673	9746	9820	9894	9968	0042	
9	77 0115	0189	0263	0336	0410	0484	0557	0631	0705	0778	
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	
1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	
2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	
3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	
4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	
6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	
7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	
8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	
9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	
600	8151	8224	8296	8368	8441	8513	8585	8658	8730	8802	
1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	
2	9596	9669	9741	9813	9885	9957	0029	0101	0173	0245	72
3	78 0317	0389	0461	0533	0605	0677	0749	0821	0893	0965	
4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	
605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	
6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	
7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	
8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	
9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	
1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	
3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	
4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	
6	9581	9651	9722	9792	9863	9933	0004	0074	0144	0215	
7	79 0285	0356	0426	0496	0567	0637	0707	0778	0848	0918	
8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	
9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	
620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	
2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	
3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	
4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	
6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	
7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	
8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	
9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
75	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
73	7.3	14.6	21.9	29.2	36.5	43.8	51.1	58.4	65.7
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
71	7.1	14.2	21.3	28.4	35.5	42.6	49.7	56.8	63.9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
69	6.9	13.8	20.7	27.6	34.5	41.4	48.3	55.2	62.1

No. 630 Log. 799 — TABLE XI.—*Continued.* — No. 674 Log. 829

N.	0	1	2	3	4	5	6	7	8	9	Diff.
630	79 9341	9409	9478	9547	9616	9685	9754	9823	9892	9961	
1	80 0029	0098	0167	0236	0305	0373	0442	0511	0580	0648	
2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	
3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	
4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	
6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	
7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	
8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	
640	80 6180	6248	6316	6384	6451	6519	6587	6655	6723	6790	
1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	
2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	
3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	
4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	
645	9560	9627	9694	9762	9829	9896	9964	0031	0098	0165	
6	81 0233	0300	0367	0434	0501	0569	0636	0703	0770	0837	
7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	
9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	
1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	
2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	
3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	
4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	
6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	
7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	
8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	
9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	9544	9610	9676	9741	9807	9873	9939	0004	0070	0136	
1	82 0201	0267	0333	0399	0464	0530	0595	0661	0727	0792	
2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	
3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	
4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	
6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	
7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	
8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	
1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	
2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	
3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	
4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
67	6.7	13.4	20.1	26.8	33.5	40.2	46.9	53.6	60.3
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6

No. 675
Log. 829

TABLE XI.—*Continued.*

No. 719
Log. 857

N.	0	1	2	3	4	5	6	7	8	9	Dif.
675	82 9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	
6	9947	0011	0075	0139	0204	0268	0332	0396	0460	0525	
7	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	
8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	
680	2509	2573	2637	2700	2764	2828	2892	2956	3020	3083	
1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	
2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	
3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	
4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	
6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	
7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	
8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	
9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	
1	9478	9541	9604	9667	9729	9792	9855	9918	9981	0043	
2	84 0106	0169	0232	0294	0357	0420	0482	0545	0608	0671	
3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	
4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	
6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	
7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	
8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	
9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	
700	5098	5160	5222	5284	5346	5408	5470	5532	5594	5656	62
1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	
2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	
3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	
4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	
6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	
7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	
8	85 0033	0095	0156	0217	0279	0340	0401	0462	0524	0585	
9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	
1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	
2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	
4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	
6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	
7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	
8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	
9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	

PROPORTIONAL PARTS.

Dif.	1	2	3	4	5	6	7	8	9
65	6.5	13.0	19.5	26.0	32.5	39.0	45.5	52.0	58.5
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
63	6.3	12.6	18.9	25.2	31.5	37.8	44.1	50.4	56.7
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8
61	6.1	12.2	18.3	24.4	30.5	36.6	42.7	48.8	54.9
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0

No. 720 Log. 857 — TABLE XI.—*Continued.* — No. 764 Log. 883

N.	0	1	2	3	4	5	6	7	8	9	Diff.
720	85 7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	
1	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	
2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	
3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
4	9739	9799	9859	9918	9978	0038	0098	0158	0218	0278	
725	86 0338	0398	0458	0518	0578	0637	0697	0757	0817	0877	
6	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	
7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	
8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	
9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	
730	3323	3382	3442	3501	3561	3620	3680	3739	3799	3858	
1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	
2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	
3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	
4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	
6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	
8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	
9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	
740	9232	9290	9349	9408	9466	9525	9584	9642	9701	9760	
1	9818	9877	9935	9994	0053	0111	0170	0228	0287	0345	
2	87 0404	0462	0521	0579	0638	0696	0755	0813	0872	0930	
3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	
4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	
6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	
7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	
8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	
1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	
2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	
3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	
4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	
6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	
7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	
8	9669	9726	9784	9841	9898	9956	0013	0070	0127	0185	
9	88 0242	0299	0356	0413	0471	0528	0585	0642	0699	0756	
760	0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	
1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	
2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	
4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
59	5.9	11.8	17.7	23.6	29.5	35.4	41.3	47.2	53.1
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4

No. 765
Log. 883

TABLE XI.—*Continued.*

No. 80
Log. 90

N.	0	1	2	3	4	5	6	7	8	9	Diff.
765	88 3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	
6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	
7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	
8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	
9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	
770	6491	6547	6604	6660	6716	6773	6829	6885	6942	6998	
1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	
2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	
3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	
4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
6	9862	9918	9974	0030	0086	0141	0197	0253	0309	0365	
7	89 0421	0477	0533	0589	0645	0700	0756	0812	0868	0924	
8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	
9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	
780	2095	2150	2206	2262	2317	2373	2429	2484	2540	2595	
1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	
2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	
3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	
4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	
785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	
6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	
7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	
8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	
9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	
2	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	
3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	
4	9821	9875	9930	9985	0039	0094	0149	0203	0258	0312	
795	90 0367	0422	0476	0531	0586	0640	0695	0749	0804	0859	
6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	
7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	
8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	
9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	
800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	
1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	
2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	
3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	
4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	
6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	
7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	
8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	
9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
57	5.7	11.4	17.1	22.8	28.5	34.2	39.9	45.6	51.3
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
55	5.5	11.0	16.5	22.0	27.5	33.0	38.5	44.0	49.5
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

N.	0	1	2	3	4	5	6	7	8	9	Diff.
810	90 8485	8539	8592	8646	8699	8753	8807	8860	8914	8967	
1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	
2	9556	9610	9663	9716	9770	9823	9877	9930	9984	0037	
3	91 0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	
4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	
6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	
7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	
8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	
9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
820	3814	3867	3920	3973	4026	4079	4132	4184	4237	4290	
1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	
2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	
3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	
4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	
6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	
7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	
8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	
9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	
830	9078	9130	9183	9235	9287	9340	9392	9444	9496	9549	
1	9601	9653	9706	9758	9810	9862	9914	9967	0019	0071	
2	92 0123	0176	0228	0280	0332	0384	0436	0489	0541	0593	
3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	
4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	
6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	
7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	
8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	
9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	
840	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	
1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	
2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	
3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	
4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	
6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	
7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	
8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	
9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	
850	9419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
1	9930	9981	0032	0083	0134	0185	0236	0287	0338	0389	
2	93 0440	0491	0542	0592	0643	0694	0745	0796	0847	0898	
3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	
4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
53	5.3	10.6	15.9	21.2	26.5	31.8	37.1	42.4	47.7
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0

No. 855 Log. 931 — TABLE XI.—*Continued.* — No. 8[illegible] Log. [illegible]

N.	0	1	2	3	4	5	6	7	8	9	Diff.
855	93 1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	
6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	
7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	
8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	
9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	
860	4498	4549	4599	4650	4700	4751	4801	4852	4902	4953	
1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	
2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	
3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	
4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	
865	7016	7066	7116	7167	7217	7267	7317	7367	7418	7468	
6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	
7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	
9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	
870	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	
1	94 0018	0068	0118	0168	0218	0267	0317	0367	0417	0467	
2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	
3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	
4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	
6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	
7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	
8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	
9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	
880	4483	4532	4581	4631	4680	4729	4779	4828	4877	4927	
1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	
2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	
3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	
4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	
885	6943	6992	7041	7090	7139	7189	7238	7287	7336	7385	
6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	
8	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	
9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	
1	9878	9926	9975	0024	0073	0121	0170	0219	0267	0316	
2	95 0365	0414	0462	0511	0560	0608	0657	0706	0754	0803	
3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	
4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	
6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	
7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	
8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	
9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
51	5.1	10.2	15.3	20.4	25.5	30.6	35.7	40.8	45.9
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
49	4.9	9.8	14.7	19.6	24.5	29.4	34.3	39.2	44.1
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2

No. 900 Log. 954

TABLE XI.—*Continued.*

No. 944 Log. 975

N.	0	1	2	3	4	5	6	7	8	9	Diff.
900	95 4243	4291	4339	4387	4435	4484	4532	4580	4628	4677	
1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	
2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	
3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	
4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	
6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	
7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	
8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	
9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	
1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	
2	9995	0042	0090	0138	0185	0233	0280	0328	0376	0423	
3	96 0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	
4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	
6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	
7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	
8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	
9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	
1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	
2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	
3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	
4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	
6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	
7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	
8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	
9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	
1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	
2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	
3	9882	9928	9975	0021	0068	0114	0161	0207	0254	0300	
4	97 0347	0393	0440	0486	0533	0579	0626	0672	0719	0765	
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	
6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	
7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	
8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	
9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	
940	3128	3174	3220	3266	3313	3359	3405	3451	3497	3543	
1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	
2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	
3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	
4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
47	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4

No. 945 — No. 999
Log. 975 — Log. 995

TABLE XI.—*Continued.*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
945	97 5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
955	98 0003	0049	0094	0140	0185	0231	0276	0322	0367	0412	
6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	
6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	
7	9895	9939	9983	0028	0072	0117	0161	0206	0250	0294	
8	99 0339	0383	0428	0472	0516	0561	0605	0650	0694	0738	
9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	

PROPORTIONAL PARTS.

Diff.	1	2	3	4	5	6	7	8	9
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
45	4.5	9.0	13.5	18.0	22.5	27.0	31.5	36.0	40.5
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
43	4.3	8.6	12.9	17.2	21.5	25.8	30.1	34.4	38.7

No. 990 Log. 995 — TABLE XI.—*Concluded.* — No. 999 Log. 999

N.	0	1	2	3	4	5	6	7	8	9	Diff.
990	99 5635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

VALUES OF *S*, *T*, AND *C* IN TABLE XII, PAGES 299 AND 300

If we were to plot the values of the logarithmic functions given in Table XII as ordinates and corresponding minutes as abscissas it would be found that the points for each function were on a curve with variable radius. It would further be noted that the curves for sines, tangents, and cotangents were of comparatively small radii when the angles were small; that the curves for cosines, cotangents, and tangents of angles near 90° respectively had the same shape as the curves for sines, tangents, and cotangents of the complements of the angles; and that other than the portions of the curves just mentioned were nearly straight lines for short distances.

When seconds are involved it will be sufficiently accurate to interpolate in the ordinary manner between adjacent values in the tables—or in other words, to assume that the curve joining two adjacent points is a straight line—for all functions between 2° and 88°, and also for sines of angles between 88° and 90° and for cosines of angles between 0° and 2°. The values in the columns headed *S*, *T*, and *C* provide a means (1) of accurately determining for any given angle between 0° and 2° the logarithmic sine, tangent, or cotangent, and for any given angle between 88° and 90°, the logarithmic cosine, cotangent, or tangent; or (2) for a given value of the logarithmic sine, tangent, or cotangent of accurately determining the angle when it lies between 0° and 2°, and for any given value of the cosine, cotangent, or tangent, the angle when it lies between 88° and 90°.

Given: *angle.* *Required*: *logarithmic function.*

$$\left.\begin{array}{l}\log \sin \alpha = \log \alpha \text{ (in seconds)} + S \\ \log \tan \alpha = \log \alpha \text{ (in seconds)} + T \\ \log \cot \alpha = C - \log \alpha \text{ (in seconds)}\end{array}\right\} \text{In which } \alpha \text{ is less than } 2°.$$

$$\left.\begin{array}{l}\log \cos \beta = \log (90° - \beta) \text{ (in seconds)} + S \\ \log \tan \beta = C - \log (90° - \beta) \text{ (in seconds)} \\ \log \cot \beta = \log (90° - \beta) \text{ (in seconds)} + T\end{array}\right\} \begin{array}{l}\text{In which } \beta \text{ lies} \\ \text{between } 88° \text{ and} \\ 90°\end{array}$$

Given: *logarithmic function.* *Required*: *angle.*

$$\left.\begin{array}{l}\log \alpha \text{ (in seconds)} = \log \sin \alpha - S \\ \qquad\qquad\qquad = \log \tan \alpha - T \\ \qquad\qquad\qquad = C - \log \cot \alpha\end{array}\right\} \text{In which } \alpha \text{ is less than } 2°.$$

$$\left.\begin{array}{l}\log (90° - \beta) \text{ (in seconds)} = \log \cos \beta - S \\ \qquad\qquad\qquad\qquad = C - \log \tan \beta \\ \qquad\qquad\qquad\qquad = \log \cot \beta - T\end{array}\right\} \begin{array}{l}\text{In which } \beta \text{ lies} \\ \text{between } 88° \text{ and} \\ 90°.\end{array}$$

EXAMPLES

Given: *angle.*

Required: *logarithmic function.*

When $\left\{\begin{array}{c}\alpha \\ 90°-\beta\end{array}\right\} = \begin{array}{c}19'\ 22'' = \\ 1162''\end{array}$

$\log 1162'' = 3.065206$

S (for 19′) $= 4.685573$

$$\left.\begin{array}{l}\log \sin 19'\ 22'' \\ \log \cos 89°\ 40'\ 38''\end{array}\right\} = 7.750779$$

Given: *logarithmic function.*

Required: *angle.*

When $\left\{\begin{array}{l}\log \sin \alpha \\ \log \cos \beta\end{array}\right\} = 7.750779$

S (for 19′) $= 4.685573$

$\log \left\{\begin{array}{c}\alpha \\ 90°-\beta\end{array}\right\}$ (in seconds) $= 3.065206$

or

$\alpha = 1162'' = 19'\ 22''$

$\beta = 89°\ 40'\ 38''$

When $\left\{\begin{array}{c}\alpha \\ 90°-\beta\end{array}\right\} = \begin{array}{c}23'\ 21'' = \\ 1401''\end{array}$

$\log 1401'' = 3.146438$

C (for 23′) $= 15.314419$

$$\left.\begin{array}{l}\log \cot 23'\ 21'' \\ \log \tan 89°\ 36'\ 39''\end{array}\right\} = 12.167981$$

When $\left\{\begin{array}{l}\log \cot \alpha \\ \log \tan \beta\end{array}\right\} = 12.167981$

C (for 23′) $= 15.314419$

$\log \left\{\begin{array}{c}\alpha \\ 90°-\beta\end{array}\right\}$ (in seconds) $= 3.146438$

or

$\alpha = 1401'' = 23'\ 21''$

$\beta = 89°\ 36'\ 39''$

″	′	Sine.	S.°	T.°	Tang.	Cotang.	C.°	D. 1″.	Cosine.	′
			4.685				15.314			
0	0	Inf. neg.	575	575	Inf. neg.	Inf. pos.	425		10.00 0000	60
60	1	6.46 3726	575	575	6.46 3726	13.53 6274	425		0000	59
120	2	.76 4756	575	575	.76 4756	.23 5244	425		0000	58
180	3	6.94 0847	575	575	6.94 0847	13.05 9153	425		0000	57
240	4	7.06 5786	575	575	7.06 5786	12.93 4214	425		0000	56
300	5	7.16 2696	575	575	7.16 2696	12.83 7304	425	.02	10.00 0000	55
360	6	.24 1877	575	575	.24 1878	.75 8122	425	.00	9.99 9999	54
420	7	.30 8824	575	575	.30 8825	.69 1175	425	.00	9999	53
480	8	.36 6816	574	576	.36 6817	.63 3183	424	.00	9999	52
540	9	.41 7968	574	576	.41 7970	.58 2030	424	.00	9999	51
								.02		
600	10	7.46 3726	574	576	7.46 3727	12.53 6273	424	.00	9.99 9998	50
660	11	.50 5118	574	576	.50 5120	.49 4880	424	.02	9998	49
720	12	.54 2906	574	577	.54 2909	.45 7091	423	.00	9997	48
780	13	.57 7668	574	577	.57 7672	.42 2328	423	.00	9997	47
840	14	.60 9853	574	577	.60 9857	.39 0143	423	.02	9996	46
								.00		
900	15	7.63 9816	573	578	7.63 9820	12.36 0180	422	.02	9.99 9996	45
960	16	.66 7845	573	578	.66 7849	.33 2151	422	.00	9995	44
1020	17	.69 4173	573	578	.69 4179	.30 5821	422	.00	9995	43
1080	18	.71 8997	573	579	.71 9003	.28 0997	421	.02	9994	42
1140	19	.74 2478	573	579	.74 2484	.25 7516	421	.02	9993	41
								.00		
1200	20	7.76 4754	572	580	7.76 4761	12.23 5239	420	.02	9.99 9993	40
1260	21	.78 5943	572	580	.78 5951	.21 4049	420	.02	9992	39
1320	22	.80 6146	572	581	.80 6155	.19 3845	419	.02	9991	38
1380	23	.82 5461	572	581	.82 5460	.17 4540	419	.02	9990	37
1440	24	.84 3934	571	582	.84 3944	.15 6056	418	.00	9989	36
1500	25	7.86 1662	571	583	7.86 1674	12.13 8326	417	.02	9.99 9989	35
1560	26	.87 8695	571	583	.87 8708	.12 1292	417	.02	9988	34
1620	27	.89 5085	570	584	.89 5099	.10 4901	416	.02	9987	33
1680	28	.91 0879	570	584	.91 0894	.08 9106	416	.02	9986	32
1740	29	.92 6119	570	585	.92 6134	.07 3866	415	.03	9985	31
1800	30	7.94 0842	569	586	7.94 0858	12.05 9142	414	.02	9.99 9983	30
1860	31	.95 5082	569	587	.95 5100	.04 4900	413	.02	9982	29
1920	32	.96 8870	569	587	.96 8889	.03 1111	413	.02	9981	28
1980	33	.98 2233	568	588	.98 2253	.01 7747	412	.02	9980	27
2040	34	7.99 5198	568	589	7.99 5219	12.00 4781	411	.03	9979	26
2100	35	8.00 7787	567	590	8.00 7809	11.99 2191	410	.02	9.99 9977	25
2160	36	.02 0021	567	591	.02 0044	.97 9956	409	.02	9976	24
2220	37	.03 1919	566	592	.03 1945	.96 8055	408	.02	9975	23
2280	38	.04 3501	566	593	.04 3527	.95 6473	407	.03	9973	22
2340	39	.05 4781	566	593	.05 4809	.94 5191	407	.02	9972	21
2400	40	8.06 5576	565	594	8.06 5806	11.93 4194	406	.03	9.99 9971	20
2460	41	.07 6500	565	595	.07 6531	.92 3469	405	.02	9969	19
2520	42	.08 6965	564	596	.08 6997	.91 3003	404	.03	9968	18
2580	43	.09 7183	564	598	.09 7217	.90 2783	402	.03	9966	17
2640	44	.10 7167	563	599	.10 7203	.89 2797	401	.02	9964	16
2700	45	8.11 6926	562	600	8.11 6963	11.88 3037	400	.03	9.99 9963	15
2760	46	.12 6471	562	601	.12 6510	.87 3490	399	.03	9961	14
2820	47	.13 5810	561	602	.13 5851	.86 4149	398	.02	9959	13
2880	48	.14 4953	561	603	.14 4996	.85 5004	397	.03	9958	12
2940	49	.15 3907	560	604	.15 3952	.84 6048	396	.03	9956	11
3000	50	8.16 2681	560	605	8.16 2727	11.83 7273	395	.03	9.99 9954	10
3060	51	.17 1280	559	607	.17 1328	.82 8672	393	.03	9952	9
3120	52	.17 9713	558	608	.17 9763	.82 0237	392	.03	9950	8
3180	53	.18 7985	558	609	.18 8036	.81 1964	391	.03	9948	7
3240	54	.19 6102	557	611	.19 6156	.80 3844	389	.03	9946	6
3300	55	8.20 4070	556	612	8.20 4126	11.79 5874	388	.03	9.99 9944	5
3360	56	.21 1895	556	613	.21 1953	.78 8047	387	.03	9942	4
3420	57	.21 9581	555	615	.21 9641	.77 0359	385	.03	9940	3
3480	58	.22 7134	554	616	.22 7195	.77 2805	384	.03	9938	2
3540	59	.23 4557	554	618	.23 4621	.76 5379	382	.03	9936	1
3600	60	8.24 1855	553	619	8.24 1921	11.75 8079	381	.03	9.99 9934	0
			4.685				15.314			
	′									′

″	′	Sine.	S.°	T.°	Tang.	Cotang.	C.°	D. 1″.	Cosine.	′
			4.685				15.314			
3600	0	8.24 1855	553	619	8.24 1921	11.75 8079	381	.03	9.99 9934	60
3660	1	.24 9033	552	620	.24 9102	.75 0898	380	.05	9932	59
3720	2	.25 6094	551	622	.25 6165	.74 3835	378	.03	9929	58
3780	3	.26 3042	551	623	.26 3115	.73 6885	377	.03	9927	57
3840	4	.26 9881	550	625	.26 9956	.73 0044	375	.05	9925	56
3900	5	8.27 6614	549	627	8.27 6691	11.72 3309	373	.03	9.99 9922	55
3960	6	.28 3243	548	628	.28 3323	.71 6677	372	.03	9920	54
4020	7	.28 9773	547	630	.28 9856	.71 0144	370	.05	9918	53
4080	8	.29 6207	546	632	.29 6292	.70 3708	368	.03	9915	52
4140	9	.30 2546	546	633	.30 2634	.69 7366	367	.05	9913	51
4200	10	8.30 8794	545	635	8.30 8884	11.69 1116	365	.05	9.99 9910	50
4260	11	.31 4954	544	637	.31 5046	.68 4954	363	.03	9907	49
4320	12	.32 1027	543	638	.32 1122	.67 8878	362	.05	9905	48
4380	13	.32 7016	542	640	.32 7114	.67 2886	360	.05	9902	47
4440	14	.33 2924	541	642	.33 3025	.66 6975	358	.03	9899	46
4500	15	8.33 8753	540	644	8.33 8856	11.66 1144	356	.05	9.99 9897	45
4560	16	.34 4504	539	646	.34 4610	.65 5390	354	.05	9894	44
4620	17	.35 0181	539	648	.35 0289	.64 9711	352	.05	9891	43
4680	18	.35 5783	538	649	.35 5895	.64 4105	351	.05	9888	42
4740	19	.36 1315	537	651	.36 1430	.63 8570	349	.05	9885	41
4800	20	8.36 6777	536	653	8.36 6895	11.63 3105	347	.05	9.99 9882	40
4860	21	.37 2171	535	655	.37 2292	.62 7708	345	.05	9879	39
4920	22	.37 7499	534	657	.37 7622	.62 2378	343	.05	9876	38
4980	23	.38 2762	533	659	.38 2889	.61 7111	341	.05	9873	37
5040	24	.38 7962	532	661	.38 8092	.61 1908	339	.05	9870	36
5100	25	8.39 3101	531	663	8.39 3234	11.60 6766	337	.05	9.99 9867	35
5160	26	.39 8179	530	666	.39 8315	.60 1685	334	.05	9864	34
5220	27	.40 3199	529	668	.40 3338	.59 6662	332	.05	9861	33
5280	28	.40 8161	527	670	.40 8304	.59 1696	330	.07	9858	32
5340	29	.41 3068	526	672	.41 3213	.58 6787	328	.05	9854	31
5400	30	8.41 7919	525	674	8.41 8068	11.58 1932	326	.05	9.99 9851	30
5460	31	.42 2717	524	676	.42 2869	.57 7131	324	.07	9848	29
5520	32	.42 7462	523	679	.42 7618	.57 2382	321	.05	9844	28
5580	33	.43 2156	522	681	.43 2315	.56 7685	319	.05	9841	27
5640	34	.43 6800	521	683	.43 6962	.56 3038	317	.07	9838	26
5700	35	8.44 1394	520	685	8.44 1560	11.55 8440	315	.05	9.99 9834	25
5760	36	.44 5941	518	688	.44 6110	.55 3890	312	.07	9831	24
5820	37	.45 0440	517	690	.45 0613	.54 9387	310	.05	9827	23
5880	38	4893	516	693	5070	4930	307	.07	9824	22
5940	39	.45 9301	515	695	.45 9481	.54 0519	305	.07	9820	21
6000	40	8.46 3665	514	697	8.46 3849	11.53 6151	303	.05	9.99 9816	20
6060	41	.46 7985	512	700	.46 8172	.53 1828	300	.07	9813	19
6120	42	.47 2263	511	702	.47 2454	.52 7546	298	.07	9809	18
6180	43	.47 6498	510	705	.47 6693	.52 3307	295	.07	9805	17
6240	44	.48 0693	509	707	.48 0892	.51 9108	293	.07	9801	16
6300	45	8.48 4848	507	710	8.48 5050	11.51 4950	290	.05	9.99 9797	15
6360	46	.48 8963	506	713	.48 9170	.51 0830	287	.07	9794	14
6420	47	.49 3040	505	715	.49 3250	.50 0750	285	.07	9790	13
6480	48	.49 7078	503	718	.49 7293	.50 2707	282	.07	9786	12
6540	49	.50 1080	502	720	.50 1298	.49 8702	280	.07	9782	11
6600	50	8.50 5045	501	723	8.50 5267	11.49 4733	277	.07	9.99 9778	10
6660	51	.50 8974	499	726	.50 9200	.49 0800	274	.08	9774	9
6720	52	.51 2867	498	729	.51 3098	.48 6902	271	.07	9769	8
6780	53	.51 6726	497	731	.51 6961	.48 3039	269	.07	9765	7
6840	54	.52 0551	495	734	.52 0790	.47 9210	266	.07	9761	6
6900	55	8.52 4343	494	737	8.52 4586	11.47 5414	263	.07	9.99 9757	5
6960	56	.52 8102	492	740	.52 8349	.47 1651	260	.08	9753	4
7020	57	.53 1828	491	743	.53 2080	.46 7920	257	.07	9748	3
7080	58	5523	490	745	5779	4221	255	.07	9744	2
7140	59	.53 9186	488	748	.53 9447	.46 0553	252	.08	9740	1
7200	60	8.54 2819	487	751	8.54 3084	11.45 6916	249		9.99 9735	0
			4.685				15.314			
″	′	Cosine.	S.°	T.°	Cotang.	Tang.	C.°	D. 1″.	Sine.	′

′	Sine.	D. 1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	8.54 2819		9.99 9735		8.54 3084		11.45 6916	60
1	6422	60.05	9731	.07	.54 6691	60.12	.45 3309	59
2	.54 9995	59.55	9726	.08	.55 0268	59.62	.44 9732	58
3	.55 3539	59.07	9722	.07	3817	59.15	6183	57
4	.55 7054	58.58	9717	.08	.55 7336	58.65	.44 2664	56
5	8.56 0540	58.10	9.99 9713	.07	8.56 0828	58.20	11.43 9172	55
6	3999	57.65	9708	.08	4291	57.72	5709	54
7	.56 7431	57.20	9704	.07	.56 7727	57.27	.43 2273	53
8	.57 0836	56.75	9699	.08	.57 1137	56.83	.42 8863	52
9	4214	56.30	9694	.08	4520	56.38	5480	51
10	8.57 7566	55.87	9.99 9689	.08	8.57 7877	55.95	11.42 2123	50
11	.58 0892	55.43	9685	.07	.58 1208	55.52	.41 8792	49
12	4193	55.02	9680	.08	4514	55.10	5486	48
13	.58 7469	54.60	9675	.08	.58 7795	54.68	.41 2205	47
14	.59 0721	54.20	9670	.08	.59 1051	54.27	.40 8949	46
15	8.59 3948	53.78	9.99 9665	.08	8.59 4283	53.87	11.40 5717	45
16	.59 7152	53.40	9660	.08	.59 7492	53.48	.40 2508	44
17	.60 0332	53.00	9655	.08	.60 0677	53.08	.39 9323	43
18	3489	52.62	9650	.08	3839	52.70	6161	42
19	6623	52.23	9645	.08	.60 6978	52.32	.39 3022	41
20	8.60 9734	51.85	9.99 9640	.08	8.61 0094	51.93	11.38 9906	40
21	.61 2823	51.48	9635	.10	3189	51.58	6811	39
22	5891	51.13	9629	.08	6262	51.22	3738	38
23	.61 8937	50.77	9624	.08	.61 9313	50.85	.38 0687	37
24	.62 1962	50.42	9619	.08	.62 2343	50.50	.37 7657	36
25	8.62 4965	50.05	9.99 9614	.10	8.62 5352	50.15	11.37 4648	35
26	.62 7948	49.72	9608	.08	.62 8340	49.80	.37 1660	34
27	.63 0911	49.38	9603	.10	.63 1308	49.47	.36 8692	33
28	3854	49.05	9597	.08	4256	49.13	5744	32
29	6776	48.70	9592	.10	.63 7184	48.80	.36 2816	31
30	8.63 9680	48.40	9.99 9586	.08	8.64 0093	48.48	11.35 9907	30
31	.64 2563	48.05	9581	.10	2982	48.15	7018	29
32	5428	47.75	9575	.08	5853	47.85	4147	28
33	.64 8274	47.43	9570	.10	.64 8704	47.52	.35 1296	27
34	.65 1102	47.13	9564	.10	.65 1537	47.22	.34 8463	26
35	8.65 3911	46.82	9.99 9558	.08	8.65 4352	46.92	11.34 5648	25
36	6702	46.52	9553	.10	7149	46.62	2851	24
37	.65 9475	46.22	9547	.10	.65 9928	46.32	.34 0072	23
38	.66 2230	45.92	9541	.10	.66 2689	46.02	.33 7311	22
39	4968	45.63	9535	.10	5433	45.73	4567	21
40	8.66 7689	45.35	9.99 9529	.08	8.66 8160	45.45	11.33 1840	20
41	.67 0393	45.07	9524	.10	.67 0870	45.17	.32 9130	19
42	3080	44.78	9518	.10	3563	44.88	6437	18
43	5751	44.52	9512	.10	6239	44.60	3761	17
44	.67 8405	44.23	9506	.10	.67 8900	44.35	.32 1100	16
45	8.68 1043	43.97	9.99 9500	.12	8.68 1544	44.07	11.31 8456	15
46	3665	43.70	9493	.10	4172	43.80	5828	14
47	6272	43.45	9487	.10	6784	43.53	3216	13
48	.68 8863	43.18	9481	.10	.68 9381	43.28	.31 0619	12
49	.69 1438	42.92	9475	.10	.69 1963	43.03	.30 8037	11
50	8.69 3998	42.67	9.99 9469	.10	8.69 4529	42.77	11.30 5471	10
51	6543	42.42	9463	.12	7081	42.53	2919	9
52	.69 9073	42.17	9456	.10	.69 9617	42.27	.30 0383	8
53	.70 1589	41.93	9450	.12	.70 2139	42.03	.29 7861	7
54	4090	41.68	9443	.10	4646	41.78	5354	6
55	8.70 6577	41.45	9.99 9437	.10	8.70 7140	41.57	11.29 2860	5
56	.70 9049	41.20	9431	.12	.70 9618	41.30	.29 0382	4
57	.71 1507	40.97	9424	.10	.71 2083	41.08	.28 7917	3
58	3952	40.75	9418	.12	4534	40.85	5466	2
59	6383	40.52	9411	.12	6972	40.63	3028	1
60	8.71 8800	40.28	9.99 9404	.12	8.71 9396	40.40	11.28 0604	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	
0	8.71 8800	40.07	9.99 9404	.10	8.71 9396	40.17	11.28 0604	60
1	.72 1204	39.85	9398	.12	.72 1806	39.97	.27 8194	59
2	3595	39.62	9391	.12	4204	39.73	5796	58
3	5972	39.42	9384	.10	6588	39.52	3412	57
4	.72 8337	39.18	9378	.12	.72 8959	39.30	.27 1041	56
5	8.73 0683	38.98	9.99 9371	.12	8.73 1317	39.10	11.26 8683	55
6	3027	38.78	9364	.12	3663	38.88	6337	54
7	5354	38.55	9357	.12	5996	38.68	4004	53
8	7667	38.37	9350	.12	.73 8317	38.48	.26 1683	52
9	.73 9969	38.17	9343	.12	.74 0626	38.27	.25 9374	51
10	8.74 2259	37.95	9.99 9336	.12	8.74 2922	38.08	11.25 7078	50
11	4536	37.77	9329	.12	5207	37.87	4793	49
12	6802	37.55	9322	.12	7479	37.68	2521	48
13	.74 9055	37.37	9315	.12	.74 9740	37.48	.25 0260	47
14	.75 1297	37.18	9308	.12	.75 1989	37.30	.24 8011	46
15	8.75 3528	36.98	9.99 9301	.12	8.75 4227	37.10	11.24 5773	45
16	5747	36.80	9294	.12	6453	36.92	3547	44
17	.75 7955	36.60	9287	.13	.75 8668	36.73	.24 1332	43
18	.76 0151	36.43	9279	.12	.76 0872	36.55	.23 9128	42
19	2337	36.23	9272	.12	3065	36.35	6935	41
20	8.76 4511	36.07	9.99 9265	.13	8.76 5246	36.18	11.23 4754	40
21	6675	35.88	9257	.12	7417	36.02	2583	39
22	.76 8828	35.70	9250	.13	.76 9578	35.82	.23 0422	38
23	.77 0970	35.52	9242	.12	.77 1727	35.65	.22 8273	37
24	3101	35.37	9235	.13	3866	35.48	6134	36
25	8.77 5223	35.17	9.99 9227	.12	8.77 5995	35.32	11.22 4005	35
26	7333	35.02	9220	.13	.77 8114	35.13	.22 1886	34
27	.77 9434	34.83	9212	.13	.78 0222	34.97	.21 9778	33
28	.78 1524	34.68	9205	.12	2320	34.80	7680	32
29	3605	34.50	9197	.13	4408	34.63	5592	31
30	8.78 5675	34.35	9.99 9189	.13	8.78 6486	34.47	11.21 3514	30
31	7736	34.18	9181	.12	.78 8554	34.32	.21 1446	29
32	.78 9787	34.02	9174	.13	.79 0613	34.15	.20 9387	28
33	.79 1828	33.85	9166	.13	2662	33.98	7338	27
34	3859	33.70	9158	.13	4701	33.83	5299	26
35	8.79 5881	33.55	9.99 9150	.13	8.79 6731	33.68	11.20 3269	25
36	7894	33.38	9142	.13	.79 8752	33.52	.20 1248	24
37	.79 9897	33.25	9134	.13	.80 0763	33.37	.19 9237	23
38	.80 1892	33.07	9126	.13	2765	33.22	7235	22
39	3876	32.93	9118	.13	4758	33.07	5242	21
40	8.80 5852	32.78	9.99 9110	.13	8.80 6742	32.92	11.19 3258	20
41	7819	32.63	9102	.13	.80 8717	32.77	.19 1283	19
42	.80 9777	32.48	9094	.13	.81 0683	32.63	.18 9317	18
43	.81 1726	32.48	9086	.13	2641	32.63	7359	17
44	3667	32.35	9077	.15	4589	32.47	5411	16
45	8.81 5599	32.20	9.99 9069	.13	8.81 6529	32.33	11.18 3471	15
46	7522	32.05	9061	.13	.81 8461	32.20	.18 1539	14
47	.81 9436	31.90	9053	.13	.82 0384	32.05	.17 9616	13
48	.82 1343	31.78	9044	.15	2298	31.90	7702	12
49	3240	31.62	9036	.13	4205	31.78	5795	11
50	8.82 5130	31.50	9.99 9027	.15	8.82 6103	31.63	11.17 3897	10
51	7011	31.35	9019	.13	7992	31.48	2008	9
52	.82 8884	31.22	9010	.15	.82 9874	31.37	.17 0126	8
53	.83 0749	31.08	9002	.13	.83 1748	31.23	.16 8252	7
54	2607	30.97	8993	.15	3613	31.08	6387	6
55	8.83 4456	30.82	9.99 8984	.15	8.83 5471	30.97	11.16 4529	5
56	6297	30.68	8976	.13	7321	30.83	2679	4
57	8130	30.55	8967	.15	.83 9163	30.70	.16 0837	3
58	.83 9956	30.43	8958	.15	.84 0998	30.58	.15 9002	2
59	.84 1774	30.30	8950	.13	2825	30.45	7175	1
60	8.84 3585	30.18	9.99 8941	.15	8.84 4644	30.32	11.15 5356	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	8.84 3585		9.99 8941	.15	8.84 4644	30.18	11.15 5356	60
1	5387	30.03	8932	.15	6455	30.08	3545	59
2	7183	29.93	8923	.15	.84 8260	29.95	.15 1740	58
3	.84 8971	29.80	8914	.15	.85 0057	29.82	.14 9943	57
4	.85 0751	29.67	8905	.15	1846	29.70	8154	56
5	8.85 2528	29.57	9.99 8896	.15	8.85 3628	29.58	11.14 6372	55
6	4291	29.43	8887	.15	5403	29.47	4597	54
7	6049	29.30	8878	.15	7171	29.35	2829	53
8	7801	29.20	8869	.15	.85 8932	29.23	.14 1068	52
9	.85 9546	29.08	8860	.15	.86 0686	29.12	.13 9314	51
10	8.86 1283	28.95	9.99 8851	.17	8.86 2433	29.00	11.13 7567	50
11	3014	28.85	8841	.15	4173	28.88	5827	49
12	4738	28.73	8832	.15	5906	28.77	4094	48
13	6455	28.62	8823	.15	7632	28.65	2368	47
14	8165	28.50	8813	.17	.86 9351	28.55	.13 0649	46
15	8.86 9868	28.38	9.99 8804	.15	8.87 1064	28.43	11.12 8936	45
16	.87 1565	28.28	8795	.17	2770	28.32	7230	44
17	3255	28.17	8785	.15	4469	28.22	5531	43
18	4938	28.05	8776	.17	6162	28.12	3838	42
19	6615	27.95	8766	.15	7849	28.12	2151	41
20	8.87 8285	27.83	9.99 8757	.17	8.87 9529	28.00	11.12 0471	40
21	.87 9949	27.73	8747	.15	.88 1202	27.88	.11 8798	39
22	.88 1607	27.63	8738	.17	2869	27.78	7131	38
23	3258	27.52	8728	.17	4530	27.68	5470	37
24	4903	27.42	8718	.17	6185	27.58	3815	36
25	8.88 6542	27.32	9.99 8708	.15	8.88 7833	27.47	11.11 2167	35
26	8174	27.20	8699	.17	.88 9476	27.38	.11 0524	34
27	.88 9801	27.12	8689	.17	.89 1112	27.27	.10 8888	33
28	.89 1421	27.00	8679	.17	2742	27.17	7258	32
29	3035	26.90	8669	.17	4366	27.07	5634	31
30	8.89 4643	26.80	9.99 8659	.17	8.89 5984	26.97	11.10 4016	30
31	6246	26.72	8649	.17	7596	26.87	2404	29
32	7842	26.60	8639	.17	.89 9203	26.78	.10 0797	28
33	.89 9432	26.50	8629	.17	.90 0803	26.67	.09 9197	27
34	.90 1017	26.42	8619	.17	2398	26.58	7602	26
35	8.90 2596	26.32	9.99 8609	.17	8.90 3987	26.48	11.09 6013	25
36	4169	26.22	8599	.17	5570	26.38	4430	24
37	5736	26.12	8589	.17	7147	26.28	2853	23
38	7297	26.02	8578	.18	.90 8719	26.20	.09 1281	22
39	.90 8853	25.93	8568	.17	.91 0285	26.10	.08 9715	21
40	8.91 0404	25.85	9.99 8558	.17	8.91 1846	26.02	11.08 8154	20
41	1949	25.75	8548	.17	3401	25.92	6599	19
42	3488	25.65	8537	.18	4951	25.83	5049	18
43	5022	25.57	8527	.17	6495	25.73	3505	17
44	6550	25.47	8516	.18	8034	25.63	1966	16
45	8.91 8073	25.38	9.99 8506	.17	8.91 9568	25.57	11.08 0432	15
46	.91 9591	25.30	8495	.18	.92 1096	25.47	.07 8904	14
47	.92 1103	25.20	8485	.17	2619	25.38	7381	13
48	2610	25.12	8474	.18	4136	25.28	5864	12
49	4112	25.03	8464	.18	5649	25.22	4351	11
50	8.92 5609	24.95	9.99 8453	.18	8.92 7156	25.12	11.07 2844	10
51	7100	24.85	8442	.18	.92 8658	25.03	.07 1342	9
52	.92 8587	24.78	8431	.18	.93 0155	24.95	.06 9845	8
53	.93 0068	24.68	8421	.17	1647	24.87	8353	7
54	1544	24.60	8410	.18	3134	24.78	6866	6
55	8.93 3015	24.52	9.99 8399	.18	8.93 4616	24.70	11.06 5384	5
56	4481	24.43	8388	.18	6093	24.62	3907	4
57	5942	24.35	8377	.18	7565	24.53	2435	3
58	7398	24.27	8366	.18	.93 9032	24.45	.06 0968	2
59	.93 8850	24.20	8355	.18	.94 0494	24.37	.05 9506	1
60	8.94 0296	24.10	9.99 8344	.18	8.94 1952	24.30	11.05 8048	0

TABLE XII.—*Continued.* 174°

Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
8.94 0296		**9.99 8344**		**8.94 1952**		**11.05 8048**	**60**
1738	24.03	8333	.18	3404	24.20	6596	59
3174	23.93	8322	.18	4852	24.13	5148	58
4606	23.87	8311	.18	6295	24.05	3705	57
6034	23.80	8300	.18	7734	23.98	2266	56
8.94 7456	23.70	**9.99 8289**		**8.94 9166**	23.90	**11.05 0834**	**55**
.94 8874	23.63	8277	.20	.95 0597	23.82	.04 9403	54
.95 0287	23.55	8266	.18	2021	23.73	7979	53
1696	23.48	8255	.18	3441	23.67	6559	52
3100	23.40	8243	.20	4856	23.58	5144	51
8.95 4499	23.32	**9.99 8232**	.18	**8.95 6267**	23.52	**11.04 3733**	**50**
5894	23.25	8220	.20	7674	23.45	2326	49
7284	23.17	8209	.18	.95 9075	23.35	.04 0925	48
.95 8670	23.10	8197	.30	.96 0473	23.30	.03 9527	47
.96 0052	23.03	8186	.18	1866	23.22	8134	46
8.96 1429	22.95	**9.99 8174**	.20	**8.96 3255**	23.15	**11.03 6745**	**45**
2801	22.87	8163	.18	4639	23.07	5361	44
4170	22.82	8151	.20	6019	23.00	3981	43
5534	22.73	8139	.20	7394	22.92	2606	42
6893	22.65	8128	.18	.96 8766	22.87	.03 1234	41
8.96 8249	22.60	**9.99 8116**	.20	**8.97 0133**	22.78	**11.02 9867**	**40**
.96 9600	22.52	8104	.20	1496	22.72	8504	39
.97 0947	22.45	8092	.20	2855	22.65	7145	38
2289	22.37	8080	.20	4209	22.57	5791	37
3628	22.32	8068	.20	5560	22.52	4440	36
8.97 4962	22.23	**9.99 8056**	.20	**8.97 6906**	22.43	**11.02 3094**	**35**
6293	22.18	8044	.20	8248	22.37	1752	34
7619	22.10	8032	.20	.97 9586	22.30	.02 0414	33
.97 8941	22.03	8020	.20	.98 0921	22.25	.01 9079	32
.98 0259	21.97	8008	.20	2251	22.17	7749	31
8.98 1573	21.90	**9.99 7996**	.20	**8.98 3577**	22.10	**11.01 6423**	**30**
2883	21.83	7984	.20	4899	22.03	5101	29
4189	21.77	7972	.22	6217	21.97	3783	28
5491	21.72	7959	.20	7532	21.92	2468	27
6789	21.63	7947	.20	.98 8842	21.83	.01 1158	26
8.98 8083	21.57	**9.99 7935**	.22	**8.99 0149**	21.78	**11.00 9851**	**25**
.98 9374	21.52	7922	.20	1451	21.70	8549	24
.99 0660	21.43	7910	.22	2750	21.65	7250	23
1943	21.38	7897	.20	4045	21.58	5955	22
3222	21.32	7885	.22	5337	21.53	4663	21
8.99 4497	21.25	**9.99 7872**	.20	**8.99 6624**	21.45	**11.00 3376**	**20**
5768	21.18	7860	.22	7908	21.40	2092	19
7036	21.13	7847	.20	8.99 9188	21.33	11.00 0812	18
8299	21.05	7835	.22	9.00 0465	21.28	10.99 9535	17
8.99 9560	21.02	7822	.22	1738	21.22	8262	16
9.00 0816	20.93	**9.99 7809**	.20	**9.00 3007**	21.15	**10.99 6993**	**15**
2069	20.88	7797	.22	4272	21.08	5728	14
3318	20.82	7784	.22	5534	21.03	4466	13
4563	20.75	7771	.22	6792	20.97	3208	12
5805	20.70	7758	.22	8047	20.92	1953	11
9.00 7044	20.65	**9.99 7745**	.22	**9.00 9298**	20.85	**10.99 0702**	**10**
8278	20.57	7732	.22	.01 0546	20.80	.98 9454	9
.00 9510	20.53	7719	.22	1790	20.73	8210	8
.01 0737	20.45	7706	.22	3031	20.68	6969	7
1962	20.42	7693	.22	4268	20.62	5732	6
9.01 3182	20.33	**9.99 7680**	.22	**9.01 5502**	20.57	**10.98 4498**	**5**
4400	20.30	7667	.22	6732	20.50	3268	4
5613	20.22	7654	.22	7959	20.45	2041	3
6824	20.18	7641	.22	.01 9183	20.40	.98 0817	2
8031	20.12	7628	.22	.02 0403	20.33	.97 9597	1
9.01 9235	20.07	**9.99 7614**	.23	**9.02 1620**	20.28	**10.97 8380**	**0**
Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.01 9235	20.00	9.99 7614	.22	9.02 1620	20.23	10.97 8380	60
1	.02 0435	19.95	7601	.22	2834	20.17	7166	59
2	1632	19.88	7588	.23	4044	20.12	5956	58
3	2825	19.85	7574	.22	5251	20.07	4749	57
4	4016	19.78	7561	.23	6455	20.00	3545	56
5	9.02 5203	19.72	9.99 7547	.22	9.02 7655	19.95	10.97 2345	55
6	6386	19.68	7534	.23	.02 8852	19.90	.97 1148	54
7	7567	19.62	7520	.22	.03 0046	19.85	.96 9954	53
8	8744	19.57	7507	.23	1237	19.80	8763	52
9	.02 9918	19.52	7493	.22	2425	19.73	7575	51
10	9.03 1089	19.47	9.99 7480	.23	9.03 3609	19.70	10.96 6391	50
11	2257	19.40	7466	.23	4791	19.63	5209	49
12	3421	19.35	7452	.22	5969	19.58	4031	48
13	4582	19.32	7439	.23	7144	19.53	2856	47
14	5741	19.25	7425	.23	8316	19.48	1684	46
15	9.03 6896	19.20	9.99 7411	.23	9.03 9485	19.43	10.96 0515	45
16	8048	19.15	7397	.23	.04 0651	19.37	.95 9349	44
17	.03 9197	19.08	7383	.23	1813	19.33	8187	43
18	.04 0342	19.05	7369	.23	2973	19.28	7027	42
19	1485	19.00	7355	.23	4130	19.23	5870	41
20	9.04 2625	18.95	9.99 7341	.23	9.04 5284	19.17	10.95 4716	40
21	3762	18.88	7327	.23	6434	19.13	3566	39
22	4895	18.85	7313	.23	7582	19.08	2418	38
23	6026	18.80	7299	.23	8727	19.03	1273	37
24	7154	18.75	7285	.23	.04 9869	18.98	.95 0131	36
25	9.04 8279	18.68	9.99 7271	.23	9.05 1008	18.93	10.94 8992	35
26	.04 9400	18.65	7257	.25	2144	18.88	7856	34
27	.05 0519	18.60	7242	.23	3277	18.83	6723	33
28	1635	18.57	7228	.23	4407	18.80	5593	32
29	2749	18.50	7214	.25	5535	18.73	4465	31
30	9.05 3859	18.45	9.99 7199	.23	9.05 6659	18.70	10.94 3341	30
31	4966	18.42	7185	.25	7781	18.65	2219	29
32	6071	18.35	7170	.23	.05 8900	18.60	.94 1100	28
33	7172	18.32	7156	.23	.06 0016	18.57	.93 9984	27
34	8271	18.27	7141	.25	1130	18.50	8870	26
35	9.05 9367	18.22	9.99 7127	.25	9.06 2240	18.47	10.93 7760	25
36	.06 0460	18.18	7112	.23	3348	18.42	6652	24
37	1551	18.13	7098	.25	4453	18.38	5547	23
38	2639	18.08	7083	.25	5556	18.32	4444	22
39	3724	18.03	7068	.25	6655	18.28	3345	21
40	9.06 4806	17.98	9.99 7053	.23	9.06 7752	18.25	10.93 2248	20
41	5885	17.95	7039	.25	8846	18.20	1154	19
42	6962	17.90	7024	.25	.06 9938	18.15	.93 0062	18
43	8036	17.85	7009	.25	.07 1027	18.10	.92 8973	17
44	.06 9107	17.82	6994	.25	2113	18.07	7887	16
45	9.07 0176	17.77	9.99 6979	.25	9.07 3197	18.02	10.92 6803	15
46	1242	17.73	6964	.25	4278	17.97	5722	14
47	2306	17.67	6949	.25	5356	17.93	4644	13
48	3366	17.63	6934	.25	6432	17.88	3568	12
49	4424	17.60	6919	.25	7505	17.85	2495	11
50	9.07 5480	17.55	9.99 6904	.25	9.07 8576	17.80	10.92 1424	10
51	6533	17.50	6889	.25	.07 9644	17.77	.92 0356	9
52	7583	17.47	6874	.25	.08 0710	17.72	.91 9290	8
53	8631	17.42	6858	.27	1773	17.67	8227	7
54	.07 9676	17.38	6843	.25	2833	17.63	7167	6
55	9.08 0719	17.33	9.99 6828	.27	9.08 3891	17.60	10.91 6109	5
56	1759	17.30	6812	.27	4947	17.55	5053	4
57	2797	17.25	6797	.25	6000	17.50	4000	3
58	3832	17.20	6782	.25	7050	17.47	2950	2
59	4864	17.17	6766	.27	8098	17.43	1902	1
60	9.08 5894		9.99 6751	.25	9.08 9144		10.91 0856	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.08 5894	17.13	9.99 6751	.27	9.08 9144	17.38	10.91 0856	60
1	6922	17.08	6735	.25	.09 0187	17.35	.90 9813	59
2	7947	17.05	6720	.27	1228	17.30	8772	58
3	8970	17.00	6704	.27	2266	17.27	7734	57
4	.08 9990	16.97	6688	.25	3302	17.23	6698	56
5	9.09 1008	16.93	9.99 6673	.27	9.09 4336	17.18	10.90 5664	55
6	2024	16.88	6657	.27	5367	17.13	4633	54
7	3037	16.83	6641	.27	6395	17.12	3605	53
8	4047	16.82	6625	.25	7422	17.07	2578	52
9	5056	16.77	6610	.27	8446	17.03	1554	51
10	9.09 6062	16.72	9.99 6594	.27	9.09 9468	16.98	10.90 0532	50
11	7065	16.68	6578	.27	.10 0487	16.95	.89 9513	49
12	8066	16.65	6562	.27	1504	16.92	8496	48
13	.09 9065	16.62	6546	.27	2519	16.88	7481	47
14	.10 0062	16.57	6530	.27	3532	16.83	6468	46
15	9.10 1056	16.53	9.99 6514	.27	9.10 4542	16.80	10.89 5458	45
16	2048	16.48	6498	.27	5550	16.77	4450	44
17	3037	16.47	6482	.28	6556	16.72	3444	43
18	4025	16.42	6465	.27	7559	16.68	2441	42
19	5010	16.37	6449	.27	8560	16.65	1440	41
20	9.10 5992	16.35	9.99 6433	.27	9.10 9559	16.62	10.89 0441	40
21	6973	16.30	6417	.28	.11 0556	16.58	.88 9444	39
22	7951	16.27	6400	.27	1551	16.53	8449	38
23	8927	16.23	6384	.27	2543	16.50	7457	37
24	.10 9901	16.20	6368	.28	3533	16.47	6467	36
25	9.11 0873	16.15	9.99 6351	.27	9.11 4521	16.43	10.88 5479	35
26	1842	16.12	6335	.28	5507	16.40	4493	34
27	2809	16.12	6318	.28	6491	16.35	3509	33
28	3774	16.08	6302	.27	7472	16.33	2528	32
29	4737	16.05	6285	.28	8452	16.28	1548	31
30	9.11 5698	16.02	9.99 6269	.27	9.11 9429	16.25	10.88 0571	30
31	6656	15.97	6252	.28	.12 0404	16.22	.87 9596	29
32	7613	15.95	6235	.28	1377	16.18	8623	28
33	8567	15.90	6219	.27	2348	16.15	7652	27
34	.11 9519	15.87	6202	.28	3317	16.12	6683	26
35	9.12 0469	15.83	9.99 6185	.28	9.12 4284	16.08	10.87 5716	25
36	1417	15.80	6168	.28	5249	16.03	4751	24
37	2362	15.75	6151	.28	6211	16.02	3789	23
38	3306	15.73	6134	.28	7172	15.97	2828	22
39	4248	15.70	6117	.28	8130	15.95	1870	21
40	9.12 5187	15.65	9.99 6100	.28	9.12 9087	15.90	10.87 0913	20
41	6125	15.63	6083	.28	.13 0041	15.88	.86 9959	19
42	7060	15.58	6066	.28	0994	15.83	9006	18
43	7993	15.55	6049	.28	1944	15.82	8056	17
44	8925	15.53	6032	.28	2893	15.77	7107	16
45	9.12 9854	15.48	9.99 6015	.28	9.13 3839	15.75	10.86 6161	15
46	.13 0781	15.45	5998	.28	4784	15.70	5216	14
47	1706	15.42	5980	.30	5726	15.68	4274	13
48	2630	15.40	5963	.28	6667	15.63	3333	12
49	3551	15.35	5946	.28	7605	15.62	2395	11
50	9.13 4470	15.32	9.99 5928	.30	9.13 8542	15.57	10.86 1458	10
51	5387	15.28	5911	.28	.13 9476	15.55	.86 0524	9
52	6303	15.27	5894	.28	.14 0409	15.52	.85 9591	8
53	7216	15.22	5876	.30	1340	15.48	8660	7
54	8128	15.20	5859	.28	2269	15.45	7731	6
55	9.13 9037	15.15	9.99 5841	.30	9.14 3196	15.42	10.85 6804	5
56	.13 9944	15.12	5823	.30	4121	15.38	5879	4
57	.14 0850	15.10	5806	.28	5044	15.37	4956	3
58	1754	15.07	5788	.30	5966	15.32	4034	2
59	2655	15.02	5771	.28	6885	15.30	3115	1
60	9.14 3555	15.00	9.99 5753	.30	9.14 7803		10.85 2197	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.14 3565	14.97	9.99 5753	.30	9.14 7803	15.25	10.85 2197	60
1	4453	14.93	5735	.30	8718	15.23	1282	59
2	5349	14.90	5717	.30	.14 9632	15.20	.85 0368	58
3	6243	14.88	5699	.30	.15 0544	15.17	.84 9456	57
4	7136	14.83	5681	.28	1454	15.15	8546	56
5	9.14 8026	14.82	9.99 5664	.30	9.15 2363	15.10	10.84 7637	55
6	8915	14.78	5646	.30	3269	15.08	6731	54
7	.14 9802	14.73	5628	.30	4174	15.05	5826	53
8	.15 0686	14.72	5610	.32	5077	15.02	4923	52
9	1569	14.70	5591	.30	5978	14.98	4022	51
10	9.15 2451	14.65	9.99 5573	.30	9.15 6877	14.97	10.84 3123	50
11	3330	14.63	5555	.30	7775	14.93	2225	49
12	4208	14.58	5537	.30	8671	14.90	1329	48
13	5083	14.57	5519	.30	..5 9565	14.87	.84 0435	47
14	5957	14.55	5501	.32	.16 0457	14.83	.83 9543	46
15	9.15 6830	14.50	9.99 5482	.30	9.16 1347	14.82	10.83 8653	45
16	7700	14.48	5464	.30	2236	14.78	7764	44
17	8569	14.43	5446	.32	3123	14.75	6877	43
18	.15 9435	14.43	5427	.30	4008	14.73	5992	42
19	.16 0301	14.38	5409	.32	4892	14.70	5108	41
20	9.16 1164	14.35	9.99 5390	.30	9.16 5774	14.67	10.83 4226	40
21	2025	14.33	5372	.32	6654	14.63	3346	39
22	2885	14.30	5353	.32	7532	14.62	2468	38
23	3743	14.28	5334	.30	8409	14.58	1591	37
24	4600	14.23	5316	.32	.16 9284	14.55	.83 0716	36
25	9.16 5454	14.22	9.99 5297	.32	9.17 0157	14.53	10.82 9843	35
26	6307	14.20	5278	.30	1029	14.50	8971	34
27	7159	14.15	5260	.32	1899	14.47	8101	33
28	8008	14.13	5241	.32	2767	14.45	7233	32
29	8856	14.10	5222	.32	3634	14.42	6366	31
30	9.16 9702	14.08	9.99 5203	.32	9.17 4499	14.38	10.82 5501	30
31	.17 0547	14.03	5184	.32	5362	14.37	4638	29
32	1389	14.02	5165	.32	6224	14.33	3776	28
33	2230	14.00	5146	.32	7084	14.30	2916	27
34	3070	13.97	5127	.32	7942	14.28	2058	26
35	9.17 3908	13.93	9.99 5108	.32	9.17 8799	14.27	10.82 1201	25
36	4744	13.90	5089	.32	.17 9655	14.22	.82 0345	24
37	5578	13.88	5070	.32	.18 0508	14.20	.81 9492	23
38	6411	13.85	5051	.32	1360	14.18	8640	22
39	7242	13.83	5032	.32	2211	14.13	7789	21
40	9.17 8072	13.80	9.99 5013	.33	9.18 3059	14.13	10.81 6941	20
41	8900	13.77	4993	.32	3907	14.08	6093	19
42	.17 9726	13.75	4974	.32	4752	14.08	5248	18
43	.18 0551	13.72	4955	.32	5597	14.03	4403	17
44	1374	13.70	4935	.32	6439	14.02	3561	16
45	9.18 2196	13.67	9.99 4916	.33	9.18 7280	14.00	10.81 2720	15
46	3016	13.63	4896	.32	8120	13.97	1880	14
47	3834	13.62	4877	.32	8958	13.93	1042	13
48	4651	13.58	4857	.33	.18 9794	13.92	.81 0206	12
49	5466	13.57	4838	.33	.19 0629	13.88	.80 9371	11
50	9.18 6280	13.53	9.99 4818	.33	9.19 1462	13.87	10.80 8538	10
51	7092	13.52	4798	.32	2294	13.83	7706	9
52	7903	13.48	4779	.33	3124	13.82	6876	8
53	8712	13.45	4759	.33	3953	13.78	6047	7
54	.18 9519	13.43	4739	.32	4780	13.77	5220	6
55	9.19 0325	13.42	9.99 4720	.33	9.19 5606	13.73	10.80 4394	5
56	1130	13.38	4700	.33	6430	13.72	3570	4
57	1933	13.35	4680	.33	7253	13.68	2747	3
58	2734	13.33	4660	.33	8074	13.67	1926	2
59	3534	13.30	4640	.33	8894	13.65	1106	1
60	9.19 4332		9.99 4620	.33	9.19 9713		10.80 0287	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.19 4332		9.99 4620		9.19 9713		10.80 0287	60
1	5129	13.28	4600	.33	.20 0529	13.60	.79 9471	59
2	5925	13.27	4580	.33	1345	13.60	8655	58
3	6719	13.23	4560	.33	2159	13.57	7841	57
4	7511	13.20	4540	.33	2971	13.53	7029	56
5	9.19 8302	13.18	9.99 4519	.35	9.20 3782	13.52	10.79 6218	55
6	9091	13.15	4499	.33	4592	13.50	5408	54
7	.19 9879	13.13	4479	.33	5400	13.47	4600	53
8	.20 0666	13.12	4459	.33	6207	13.45	3793	52
9	1451	13.08	4438	.35	7013	13.43	2987	51
10	9.20 2234	13.05	9.99 4418	.33	9.20 7817	13.40	10.79 2183	50
11	3017	13.05	4398	.33	8619	13.37	1381	49
12	3797	13.00	4377	.35	.20 9420	13.35	.79 0580	48
13	4577	13.00	4357	.33	.21 0220	13.33	.78 9780	47
14	5354	12.95	4336	.35	1018	13.30	8982	46
15	9.20 1631	12.95	9.99 4316	.33	9.21 1815	13.28	10.78 8185	45
16	6906	12.92	4295	.35	2611	13.27	7389	44
17	7679	12.88	4274	.35	3405	13.23	6595	43
18	8452	12.88	4254	.33	4198	13.22	5802	42
19	9222	12.83	4233	.35	4989	13.18	5011	41
20	9.20 9992	12.83	9.99 4212	.35	9.21 5780	13.18	10.78 4220	40
21	.21 0760	12.80	4191	.33	6568	13.13	3432	39
22	1526	12.77	4171	.35	7356	13.13	2644	38
23	2291	12.75	4150	.35	8142	13.10	1858	37
24	3055	12.73	4129	.35	8926	13.07	1074	36
25	9.21 3818	12.72	9.99 4108	.35	9.21 9710	13.07	10.78 0290	35
26	4579	12.68	4087	.35	.22 0492	13.03	.77 9508	34
27	5338	12.65	4066	.35	1272	13.00	8728	33
28	6097	12.65	4045	.35	2052	13.00	7948	32
29	6854	12.62	4024	.35	2830	12.97	7170	31
30	9.21 7609	12.58	9.99 4003	.35	9.22 3607	12.95	10.77 6393	30
31	8363	12.57	3982	.37	4382	12.92	5618	29
32	9116	12.55	3960	.35	5156	12.90	4844	28
33	.21 9868	12.53	3939	.35	5929	12.88	4071	27
34	.22 0618	12.50	3918	.35	6700	12.85	3300	26
35	9.22 1367	12.48	9.99 3897	.37	9.22 7471	12.85	10.77 2529	25
36	2115	12.47	3875	.35	8239	12.80	1761	24
37	2861	12.43	3854	.37	9007	12.80	0993	23
38	3606	12.42	3832	.35	.22 9773	12.77	.77 0227	22
39	4349	12.38	3811	.37	.23 0539	12.77	.76 9461	21
40	9.22 5092	12.38	9.99 3789	.35	9.23 1302	12.72	10.76 8698	20
41	5833	12.35	3768	.37	2065	12.72	7935	19
42	6573	12.33	3746	.35	2826	12.68	7174	18
43	7311	12.30	3725	.37	3586	12.67	6414	17
44	8048	12.28	3703	.37	4345	12.65	5655	16
45	9.22 8784	12.27	9.99 3681	.35	9.23 5103	12.63	10.76 4897	15
46	.22 9518	12.23	3660	.37	5859	12.60	4141	14
47	.23 0252	12.23	3638	.37	6614	12.58	3386	13
48	0984	12.20	3616	.37	7368	12.57	2632	12
49	1715	12.18	3594	.37	8120	12.53	1880	11
50	9.23 2444	12.15	9.99 3572	.37	9.23 8872	12.53	10.76 1128	10
51	3172	12.13	3550	.37	.23 9622	12.50	.76 0378	9
52	3899	12.12	3528	.37	.24 0371	12.48	.75 9629	8
53	4625	12.10	3506	.37	1118	12.45	8882	7
54	5349	12.07	3484	.37	1865	12.45	8135	6
55	9.23 6073	12.07	9.99 3462	.37	9.24 2610	12.42	10.75 7390	5
56	6795	12.03	3440	.37	3354	12.40	6646	4
57	7515	12.00	3418	.37	4097	12.38	5903	3
58	8235	12.00	3396	.37	4839	12.37	5161	2
59	8953	11.97	3374	.37	5579	12.33	4421	1
60	9.23 9670	11.95	9.99 3351	.38	9.24 6319	12.33	10.75 3681	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.23 9670	11.93	9.99 3351	.37	9.24 6319	12.30	10.75 3681	60
1	.24 0386	11.92	3329	.37	7057	12.28	2943	59
2	1101	11.92	3307	.37	7794	12.27	2206	58
3	1814	11.88	3284	.38	8530	12.27	1470	57
4	2526	11.87	3262	.37	9264	12.23	0736	56
5	9.24 3237	11.85	9.99 3240	.37	9.24 9998	12.23	10.75 0002	55
6	3947	11.83	3217	.38	.25 0730	12.20	.74 9270	54
7	4656	11.82	3195	.37	1461	12.18	8539	53
8	5363	11.78	3172	.38	2191	12.17	7809	52
9	6069	11.77	3149	.38	2920	12.15	7080	51
10	9.24 6775	11.77	9.99 3127	.37	9.25 3648	12.13	10.74 6352	50
11	7478	11.72	3104	.38	4374	12.10	5626	49
12	8181	11.72	3081	.38	5100	12.10	4900	48
13	8883	11.70	3059	.37	5824	12.07	4176	47
14	.24 9583	11.67	3036	.38	6547	12.05	3453	46
15	9.25 0282	11.65	9.99 3013	.38	9.25 7269	12.03	10.74 2731	45
16	0980	11.63	2990	.38	7990	12.02	2010	44
17	1677	11.62	2967	.38	8710	12.00	1290	43
18	2373	11.60	2944	.38	.25 9429	11.98	.74 0571	42
19	3067	11.57	2921	.38	.26 0146	11.95	.73 9854	41
20	9.25 3761	11.57	9.99 2898	.38	9.26 0863	11.95	10.73 9137	40
21	4453	11.53	2875	.38	1578	11.92	8422	39
22	5144	11.52	2852	.38	2292	11.90	7708	38
23	5834	11.50	2829	.38	3005	11.88	6995	37
24	6523	11.48	2806	.38	3717	11.87	6283	36
25	9.25 7211	11.47	9.99 2783	.38	9.26 4428	11.85	10.73 5572	35
26	7898	11.45	2759	.40	5138	11.83	4862	34
27	8583	11.42	2736	.38	5847	11.82	4153	33
28	9268	11.42	2713	.38	6555	11.80	3445	32
29	.25 9951	11.38	2690	.38	7261	11.77	2739	31
30	9.26 0633	11.37	9.99 2666	.40	9.26 7967	11.77	10.73 2033	30
31	1314	11.35	2643	.38	8671	11.73	1329	29
32	1994	11.33	2619	.40	.26 9375	11.73	.73 0625	28
33	2673	11.32	2596	.38	.27 0077	11.70	.72 9923	27
34	3351	11.30	2572	.40	0779	11.70	9221	26
35	9.26 4027	11.27	9.99 2549	.38	9.27 1479	11.67	10.72 8521	25
36	4703	11.27	2525	.40	2178	11.65	7822	24
37	5377	11.23	2501	.40	2876	11.63	7124	23
38	6051	11.23	2478	.38	3573	11.62	6427	22
39	6723	11.20	2454	.40	4269	11.60	5731	21
40	9.26 7395	11.20	9.99 2430	.40	9.27 4964	11.58	10.72 5036	20
41	8065	11.17	2406	.40	5658	11.57	4342	19
42	8734	11.15	2382	.38	6351	11.55	3649	18
43	.26 9402	11.13	2359	.40	7043	11.53	2957	17
44	.27 0069	11.12	2335	.40	7734	11.52	2266	16
45	9.27 0735	11.10	9.99 2311	.40	9.27 8424	11.50	10.72 1576	15
46	1400	11.08	2287	.40	9113	11.48	0887	14
47	2064	11.07	2263	.40	.27 9801	11.47	.72 0199	13
48	2726	11.03	2239	.40	.28 0488	11.45	.71 9512	12
49	3388	11.03	2214	.42	1174	11.43	8826	11
50	9.27 4049	11.02	9.99 2190	.40	9.28 1858	11.40	10.71 8142	10
51	4708	10.98	2166	.40	2542	11.40	7458	9
52	5367	10.98	2142	.40	3225	11.38	6775	8
53	6025	10.97	2118	.40	3907	11.37	6093	7
54	6681	10.93	2093	.42	4588	11.35	5412	6
55	9.27 7337	10.93	9.99 2069	.40	9.28 5268	11.33	10.71 4732	5
56	7991	10.90	2044	.42	5947	11.32	4053	4
57	8645	10.90	2020	.40	6624	11.28	3376	3
58	9297	10.87	1996	.40	7301	11.28	2699	2
59	.27 9948	10.85	1971	.42	7977	11.27	2023	1
60	9.28 0599	10.85	9.99 1947	.40	9.28 8652	11.25	10.71 1348	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.28 0599	10.82	9.99 1947	.42	9.28 8652	11.23	10.71 1348	60
1	1248	10.82	1922	.42	9326	11.22	0674	59
2	1897	10.78	1897	.40	.28 9999	11.22	.71 0001	58
3	2544	10.77	1873	.42	.29 0671	11.20	.70 9329	57
4	3190	10.77	1848	.42	1342	11.18	8658	56
5	9.28 3836	10.73	9.99 1823	.40	9.29 2013	11.18	10.70 7987	55
6	4480	10.73	1799	.42	2682	11.15	7318	54
7	5124	10.70	1774	.42	3350	11.13	6650	53
8	5766	10.70	1749	.42	4017	11.12	5983	52
9	6408	10.67	1724	.42	4684	11.12	5316	51
10	9.28 7048	10.67	9.99 1699	.42	9.29 5349	11.08	10.70 4651	50
11	7688	10.63	1674	.42	6013	11.07	3987	49
12	8326	10.63	1649	.42	6677	11.07	3323	48
13	8964	10.60	1624	.42	7339	11.03	2661	47
14	.28 9600	10.60	1599	.42	8001	11.02	1999	46
15	9.29 0236	10.57	9.99 1574	.42	9.29 8662	11.00	10.70 1338	45
16	0870	10.57	1549	.42	9322	10.97	0678	44
17	1504	10.55	1524	.43	.29 9980	10.97	.70 0020	43
18	2137	10.52	1498	.42	.30 0638	10.95	.69 9362	42
19	2768	10.52	1473	.42	1295	10.93	8705	41
20	9.29 3399	10.50	9.99 1448	.43	9.30 1951	10.93	10.69 8049	40
21	4029	10.48	1422	.42	2607	10.90	7393	39
22	4658	10.47	1397	.42	3261	10.88	6739	38
23	5286	10.45	1372	.43	3914	10.88	6086	37
24	5913	10.43	1346	.42	4567	10.86	5433	36
25	9.29 6539	10.42	9.99 1321	.43	9.30 5218	10.85	10.69 4782	35
26	7164	10.40	1295	.42	5869	10.83	4131	34
27	7788	10.40	1270	.43	6519	10.82	3481	33
28	8412	10.37	1244	.43	7168	10.80	2832	32
29	9034	10.35	1218	.42	7816	10.78	2184	31
30	9.29 9655	10.35	9.99 1193	.43	9.30 8463	10.77	10.69 1537	30
31	.30 0276	10.32	1167	.43	9109	10.75	0891	29
32	0895	10.32	1141	.43	.30 9754	10.75	.69 0246	28
33	1514	10.30	1115	.43	.31 0399	10.72	.68 9601	27
34	2132	10.27	1090	.42	1042	10.72	8958	26
35	9.30 2748	10.27	9.99 1064	.43	9.31 1685	10.70	10.68 8315	25
36	3364	10.25	1038	.43	2327	10.68	7673	24
37	3979	10.23	1012	.43	2968	10.67	7032	23
38	4593	10.23	0986	.43	3608	10.65	6392	22
39	5207	10.20	0960	.43	4247	10.63	5753	21
40	9.30 5819	10.18	9.99 0934	.43	9.31 4885	10.63	10.68 5115	20
41	6430	10.18	0908	.43	5523	10.60	4477	19
42	7041	10.18	0882	.43	6159	10.60	3841	18
43	7650	10.15	0855	.45	6795	10.60	3205	17
44	8259	10.15	0829	.43	7430	10.58	2570	16
45	9.30 8867	10.13	9.99 0803	.43	9.31 8064	10.57	10.68 1936	15
46	.30 9474	10.12	0777	.43	8697	10.55	1303	14
47	.31 0080	10.10	0750	.45	9330	10.55	0670	13
48	0685	10.08	0724	.43	.31 9961	10.52	.68 0039	12
49	1289	10.07	0697	.45	.32 0592	10.52	.67 9408	11
50	9.31 1893	10.07	9.99 0671	.43	9.32 1222	10.50	10.67 8778	10
51	2495	10.03	0645	.45	1851	10.48	8149	9
52	3097	10.03	0618	.45	2479	10.47	7521	8
53	3698	10.02	0591	.45	3106	10.45	6894	7
54	4297	9.98	0565	.43	3733	10.45	6267	6
55	9.31 4897	10.00	9.99 0538	.45	9.32 4358	10.42	10.67 5642	5
56	5495	9.97	0511	.45	4983	10.42	5017	4
57	6092	9.95	0485	.43	5607	10.40	4393	3
58	6689	9.95	0458	.45	6231	10.40	3769	2
59	7284	9.92	0431	.45	6853	10.37	3147	1
60	9.31 7879	9.92	9.99 0404	.45	9.32 7475	10.37	10.67 2525	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.31 7879	9.90	9.99 0404	.43	9.32 7475	10.33	10.67 2525	60
1	8473	9.88	0378	.45	8095	10.33	1905	59
2	9066	9.87	0351	.45	8715	10.32	1285	58
3	.31 9658	9.85	0324	.45	9334	10.32	0666	57
4	.32 0249	9.85	0297	.45	.32 9953	10.28	.67 0047	56
5	9.32 0840	9.83	9.99 0270	.45	9.33 0570	10.28	10.66 9430	55
6	1430	9.82	0243	.47	1187	10.27	8813	54
7	2019	9.80	0215	.45	1803	10.25	8197	53
8	2607	9.78	0188	.45	2418	10.25	7582	52
9	3194	9.77	0161	.45	3083	10.22	6967	51
10	9.32 3780	9.77	9.99 0134	.45	9.33 3646	10.22	10.66 6354	50
11	4366	9.73	0107	.47	4259	10.20	5741	49
12	4950	9.73	0079	.45	4871	10.18	5129	48
13	5534	9.72	0052	.45	5482	10.18	4518	47
14	6117	9.72	.99 0025	.47	6093	10.15	3907	46
15	9.32 6700	9.68	9.98 9997	.45	9.33 6702	10.15	10.66 3298	45
16	7281	9.68	9970	.47	7311	10.13	2689	44
17	7862	9.67	9942	.45	7919	10.13	2081	43
18	8442	9.67	9915	.47	8527	10.13	1473	42
19	9021	9.65	9887	.47	9133	10.10	0867	41
20	9.32 9599	9.63	9.98 9860	.45	9.33 9739	10.08	10.66 0261	40
21	.33 0176	9.62	9832	.47	.34 0344	10.07	.65 9656	39
22	0753	9.62	9804	.47	0948	10.07	9052	38
23	1329	9.60	9777	.45	1552	10.07	8448	37
24	1903	9.57	9749	.47	2155	10.05	7845	36
25	9.33 2478	9.58	9.98 9721	.47	9.34 2757	10.03	10.65 7243	35
26	3051	9.55	9693	.47	3358	10.02	6642	34
27	3624	9.55	9665	.47	3958	10.00	6042	33
28	4195	9.52	9637	.47	4558	10.00	5442	32
29	4767	9.53	9610	.45	5157	9.98	4843	31
30	9.33 5337	9.50	9.98 9582	.47	9.34 5755	9.97	10.65 4245	30
31	5906	9.48	9553	.48	6353	9.97	3647	29
32	6475	9.48	9525	.47	6949	9.93	3051	28
33	7043	9.47	9497	.47	7545	9.93	2455	27
34	7610	9.45	9469	.47	8141	9.93	1859	26
35	9.33 8176	9.43	9.98 9441	.47	9.34 8735	9.90	10.65 1265	25
36	8742	9.42	9413	.47	9329	9.88	0671	24
37	9307	9.40	9385	.48	.34 9922	9.87	.65 0078	23
38	.33 9871	9.38	9356	.47	.35 0514	9.87	.64 9486	22
39	.34 0434	9.37	9328	.47	1106	9.85	8894	21
40	9.34 0996	9.37	9.98 9300	.48	9.35 1697	9.83	10.64 8303	20
41	1558	9.35	9271	.47	2287	9.82	7713	19
42	2119	9.33	9243	.48	2876	9.82	7124	18
43	2679	9.33	9214	.47	3465	9.80	6535	17
44	3239	9.30	9186	.48	4053	9.78	5947	16
45	9.34 3797	9.30	9.98 9157	.48	9.35 4640	9.78	10.64 5360	15
46	4355	9.28	9128	.47	5227	9.77	4773	14
47	4912	9.28	9100	.48	5813	9.75	4187	13
48	5469	9.25	9071	.48	6398	9.73	3602	12
49	6024	9.25	9042	.47	6982	9.73	3018	11
50	9.34 6579	9.25	9.98 9014	.48	9.35 7566	9.72	10.64 2434	10
51	7134	9.22	8985	.48	8149	9.70	1851	9
52	7687	9 22	8956	.48	8731	9.70	1269	8
53	8240	9.20	8927	.48	9313	9.67	0687	7
54	8792	9.18	8898	.48	.35 9893	9.68	.64 0107	6
55	9.34 9343	9.17	9.98 8869	.48	9.36 0474	9.65	10.63 9526	5
56	.34 9893	9.17	8840	.48	1053	9.65	8947	4
57	.35 0443	9.15	8811	.48	1632	9.63	8368	3
58	0992	9.13	8782	.48	2210	9.63	7790	2
59	1540	9.13	8753	.48	2787	9.62	7213	1
60	9.35 2088		9.98 8724	.48	9.36 3364	9.62	10.63 6636	0

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.35 2088	9.12	9.98 8724	.48	9.36 3364	9.60	10.63 6636	60
1	2635	9.10	8695	.48	3940	9.58	6060	59
2	3181	9.08	8666	.50	4515	9.58	5485	58
3	3726	9.08	8636	.48	5090	9.57	4910	57
4	4271	9.07	8607	.48	5664	9.55	4336	56
5	9.35 4815	9.05	9.98 8578	.50	9.36 6237	9.55	10.63 3763	55
6	5358	9.05	8548	.48	6810	9.53	3190	54
7	5901	9.03	8519	.50	7382	9.52	2618	53
8	6443	9.02	8489	.48	7953	9.52	2047	52
9	6984	9.00	8460	.50	8524	9.50	1476	51
10	9.35 7524	9.00	9.98 8430	.48	9.36 9094	9.48	10.63 0906	50
11	8064	8.98	8401	.50	.36 9663	9.48	.63 0337	49
12	8603	8.97	8371	.48	.37 0232	9.45	.62 9768	48
13	9141	8.95	8342	.50	0799	9.47	9201	47
14	.35 9678	8.95	8312	.50	1367	9.43	8633	46
15	9.36 0215	8.95	9.98 8282	.50	9.37 1933	9.43	10.62 8067	45
16	0752	8.92	8252	.48	2499	9.42	7501	44
17	1287	8.92	8223	.50	3064	9.42	6936	43
18	1822	8.90	8193	.50	3629	9.40	6371	42
19	2356	8.88	8163	.50	4193	9.38	5807	41
20	9.36 2889	8.88	9.98 8133	.50	9.37 4756	9.38	10.62 5244	40
21	3422	8.87	8103	.50	5319	9.37	4681	39
22	3954	8.85	8073	.50	5881	9.35	4119	38
23	4485	8.85	8043	.50	6442	9.35	3558	37
24	5016	8.83	8013	.50	7003	9.33	2997	36
25	9.36 5546	8.82	9.98 7983	.50	9.37 7563	9.32	10.62 2437	35
26	6075	8.82	7953	.52	8122	9.32	1878	34
27	6604	8.78	7922	.50	8681	9.30	1319	33
28	7131	8.80	7892	.50	9239	9.30	0761	32
29	7659	8.77	7862	.50	.37 9797	9.28	.62 0203	31
30	9.36 8185	8.77	9.98 7832	.52	9.38 0354	9.27	10.61 9646	30
31	8711	8.75	7801	.50	0910	9.27	9090	29
32	9236	8.75	7771	.52	1466	9.23	8534	28
33	.36 9761	8.72	7740	.50	2020	9.25	7980	27
34	.37 0285	8.72	7710	.52	2575	9.23	7425	26
35	9.37 0808	8.70	9.98 7679	.50	9.38 3129	9.22	10.61 6871	25
36	1330	8.70	7649	.52	3682	9.20	6318	24
37	1852	8.68	7618	.50	4234	9.20	5766	23
38	2373	8.68	7588	.52	4786	9.18	5214	22
39	2894	8.67	7557	.52	5337	9.18	4663	21
40	9.37 3414	8.65	9.98 7526	.50	9.38 5888	9.17	10.61 4112	20
41	3933	8.65	7496	.52	6438	9.15	3562	19
42	4452	8.63	7465	.52	6987	9.15	3013	18
43	4970	8.62	7434	.52	7536	9.13	2464	17
44	5487	8.62	7403	.52	8084	9.13	1916	16
45	9.37 6003	8.60	9.98 7372	.52	9.38 8631	9.12	10.61 1369	15
46	6519	8.60	7341	.52	9178	9.10	0822	14
47	7035	8.57	7310	.52	.38 9724	9.10	.61 0276	13
48	7549	8.57	7279	.52	.39 0270	9.08	.60 9730	12
49	8063	8.57	7248	.52	0815	9.08	9185	11
50	9.37 8577	8.53	9.98 7217	.52	9.39 1360	9.05	10.60 8640	10
51	9089	8.53	7186	.52	1903	9.07	8097	9
52	.37 9601	8.53	7155	.52	2447	9.03	7553	8
53	.38 0113	8.53	7124	.52	2989	9.03	7011	7
54	0624	8.52	7092	.53	3531	9.03	6469	6
55	9.38 1134	8.50	9.98 7061	.52	9.39 4073	9.02	10.60 5927	5
56	1643	8.48	7030	.53	4614	9.00	5386	4
57	2152	8.48	6998	.53	5154	9.00	4846	3
58	2661	8.48	6967	.52	5694	9.00	4306	2
59	3168	8.45	6936	.52	6233	8.98	3767	1
60	9.38 3675	8.45	9.98 6904	.53	9.39 6771	8.97	10.60 3229	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.38 3675	8.45	9.98 6904	.52	9.39 6771	8.97	10.60 3229	60
1	4182	8.42	6873	.53	7309	8.95	2691	59
2	4687	8.42	6841	.53	7846	8.95	2154	58
3	5192	8.42	6809	.53	8383	8.95	1617	57
4	5697	8.40	6778	.53	8919	8.93	1081	56
5	9.38 6201	8.38	9.98 6746	.53	9.39 9455	8.92	10.60 0545	55
6	6704	8.38	6714	.52	.39 9990	8.90	.60 0010	54
7	7207	8.38	6683	.53	.40 0524	8.90	.59 9476	53
8	7709	8.37	6651	.53	1058	8.90	8942	52
9	8210	8.35	6619	.53	1591	8.88	8409	51
10	9.38 8711	8.33	9.98 6587	.53	9.40 2124	8.87	10.59 7876	50
11	9211	8.33	6555	.53	2656	8.85	7344	49
12	.38 9711	8.32	6523	.53	3187	8.85	6813	48
13	.39 0210	8.30	6491	.53	3718	8.85	6282	47
14	0708	8.30	6459	.53	4249	8.82	5751	46
15	9.39 1206	8.28	9.98 6427	.53	9.40 4778	8.83	10.59 5222	45
16	1703	8.27	6395	.53	5308	8.80	4692	44
17	2199	8.27	6363	.53	5836	8.80	4164	43
18	2695	8.27	6331	.53	6364	8.80	3636	42
19	3191	8.23	6299	.55	6892	8.78	3108	41
20	9.39 3685	8.23	9.98 6266	.53	9.40 7419	8.77	10.59 2581	40
21	4179	8.23	6234	.53	7945	8.77	2055	39
22	4673	8.22	6202	.55	8471	8.75	1529	38
23	5166	8.20	6169	.53	8996	8.75	1004	37
24	5658	8.20	6137	.55	.40 9521	8.73	.59 0479	36
25	9.39 6150	8.18	9.98 6104	.53	9.41 0045	8.73	10.58 9955	35
26	6641	8.18	6072	.55	0569	8.72	9431	34
27	7132	8.15	6039	.53	1092	8.72	8908	33
28	7621	8.17	6007	.55	1615	8.70	8385	32
29	8111	8.15	5974	.53	2137	8.68	7863	31
30	9.39 8600	8.13	9.98 5942	.55	9.41 2658	8.68	10.58 7342	30
31	9088	8.12	5909	.55	3179	8.67	6821	29
32	.39 9575	8.12	5876	.55	3699	8.67	6301	28
33	.40 0062	8.12	5843	.53	4219	8.65	5781	27
34	0549	8.10	5811	.55	4738	8.65	5262	26
35	9.40 1035	8.08	9.98 5778	.55	9.41 5257	8.63	10.58 4743	25
36	1520	8.08	5745	.55	5775	8.63	4225	24
37	2005	8.07	5712	.55	6293	8.62	3707	23
38	2489	8.05	5679	.55	6810	8.60	3190	22
39	2972	8.05	5646	.55	7326	8.60	2674	21
40	9.40 3455	8.05	9.98 5613	.55	9.41 7843	8.60	10.58 2158	20
41	3938	8.03	5580	.55	8358	8.58	1642	19
42	4420	8.02	5547	.55	8873	8.57	1127	18
43	4901	8.02	5514	.57	9387	8.57	0613	17
44	5382	8.00	5480	.55	.41 9901	8.57	.58 0099	16
45	9.40 5862	7.98	9.98 5447	.55	9.42 0415	8.55	10.57 9585	15
46	6341	7.98	5414	.55	0927	8.55	9073	14
47	6820	7.98	5381	.57	1440	8.53	8560	13
48	7299	7.97	5347	.55	1952	8.52	8048	12
49	7777	7.95	5314	.57	2463	8.52	7537	11
50	9.40 8254	7.95	9.98 5280	.55	9.42 2974	8.50	10.57 7026	10
51	8731	7.93	5247	.57	3484	8.48	6516	9
52	9207	7.93	5213	.55	3993	8.50	6007	8
53	.40 9682	7.92	5180	.57	4503	8.47	5497	7
54	.41 0157	7.92	5146	.55	5011	8.47	4989	6
55	9.41 0632	7.90	9.98 5113	.57	9.42 5519	8.47	10.57 4481	5
56	1106	7.88	5079	.57	6027	8.45	3973	4
57	1579	7.88	5045	.57	6534	8.45	3466	3
58	2052	7.87	5011	.57	7041	8.43	2959	2
59	2524	7.87	4978	.55	7547	8.43	2453	1
60	9.41 2996	7.87	9.98 4944	.57	9.42 8052	8.42	10.57 1948	0

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.41 2996	7.85	9.98 4944	.57	9.42 3052	8.43	10.57 1948	60
1	3467	7.85	4910	.57	8558	8.40	1442	59
2	3938	7.83	4876	.57	9062	8.40	0938	58
3	4408	7.83	4842	.57	.42 9566	8.40	.57 0434	57
4	4878	7.82	4808	.57	.43 0070	8.38	.56 9930	56
5	9.41 5347	7.80	9.98 4774	.57	9.43 0573	8.37	10.56 9427	55
6	5815	7.80	4740	.57	1075	8.37	8925	54
7	6283	7.80	4706	.57	1577	8.37	8423	53
8	6751	7.77	4672	.57	2079	8.35	7921	52
9	7217	7.78	4638	.58	2580	8.33	7420	51
10	9.41 7684	7.77	9.98 4603	.57	9.43 3080	8.33	10.56 6920	50
11	8150	7.75	4569	.57	3580	8.33	6420	49
12	8615	7.73	4535	.58	4080	8.32	5920	48
13	9079	7.75	4500	.57	4579	8.32	5421	47
14	.41 9544	7.72	4466	.57	5078	8.30	4922	46
15	9.42 0007	7.72	9.98 4432	.58	9.43 5576	8.28	10.56 4424	45
16	0470	7.72	4397	.57	6073	8.28	3927	44
17	0933	7.70	4363	.58	6570	8.28	3430	43
18	1395	7.70	4328	.57	7067	8.27	2933	42
19	1857	7.68	4294	.58	7563	8.27	2437	41
20	9.42 2318	7.67	9.98 4259	.58	9.43 8059	8.25	10.56 1941	40
21	2778	7.67	4224	.57	8554	8.23	1446	39
22	3238	7.65	4190	.58	9048	8.25	0952	38
23	3697	7.65	4155	.58	.43 9543	8.22	.56 0457	37
24	4156	7.65	4120	.58	.44 0036	8.22	.55 9964	36
25	9.42 4615	7.63	9.98 4085	.58	9.44 0529	8.22	10.55 9471	35
26	5073	7.62	4050	.58	1022	8.20	8978	34
27	5530	7.62	4015	.57	1514	8.20	8486	33
28	5987	7.62	3981	.57	2006	8.20	7994	32
29	6443	7.60	3946	.58	2497	8.18	7503	31
30	9.42 6899	7.60	9.98 3911	.58	9.44 2988	8.18	10.55 7012	30
31	7354	7.58	3875	.60	3479	8.18	6521	29
32	7809	7.58	3840	.58	3968	8.15	6032	28
33	8263	7.57	3805	.58	4458	8.17	5542	27
34	8717	7.57	3770	.58	4947	8.15	5053	26
35	9.42 9170	7.55	9.98 3735	.58	9.44 5435	8.13	10.55 4565	25
36	.42 9623	7.53	3700	.60	5923	8.13	4077	24
37	.43 0075	7.53	3664	.60	6411	8.13	3589	23
38	0527	7.53	3629	.58	6898	8.12	3102	22
39	0978	7.52	3594	.58	7384	8.10	2616	21
40	9.43 1429	7.50	9.98 3558	.58	9.44 7870	8.10	10.55 2130	20
41	1879	7.50	3523	.60	8356	8.08	1644	19
42	2329	7.50	3487	.60	8841	8.08	1159	18
43	2778	7.48	3452	.58	9326	8.08	0674	17
44	3226	7.47	3416	.60	.44 9810	8.07	.55 0190	16
45	9.43 3675	7.48	9.98 3381	.58	9.45 0294	8.07	10.54 9706	15
46	4122	7.45	3345	.60	0777	8.05	9223	14
47	4569	7.45	3309	.60	1260	8.05	8740	13
48	5016	7.45	3273	.60	1743	8.05	8257	12
49	5462	7.43	3238	.58	2225	8.03	7775	11
50	9.43 5908	7.43	9.98 3202	.60	9.45 2706	8.02	10.54 7294	10
51	6353	7.42	3166	.60	3187	8.02	6813	9
52	6798	7.42	3130	.60	3668	8.02	6332	8
53	7242	7.40	3094	.60	4148	8.00	5852	7
54	7686	7.40	3058	.60	4628	8.00	5372	6
55	9.43 8129	7.38	9.98 3022	.60	9.45 5107	7.98	10.54 4893	5
56	8572	7.38	2986	.60	5586	7.98	4414	4
57	9014	7.37	2950	.60	6064	7.97	3936	3
58	9456	7.37	2914	.60	6542	7.97	3458	2
59	.43 9897	7.35	2878	.60	7019	7.95	2981	1
60	9.44 0338	7.35	9.98 2842	.60	9.45 7496	7.95	10.54 2464	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.44 0338	7.33	9.98 2842	.62	9.45 7496	7.95	10.54 2504	60
1	0778	7.33	2805	.60	7973	7.93	2027	59
2	1218	7.33	2769	.60	8449	7.93	1551	58
3	1658	7.30	2733	.62	8925	7.92	1075	57
4	2096	7.32	2696	.60	9400	7.92	0600	56
5	9.44 2535	7.30	9.98 2660	.60	9.45 9875	7.90	10.54 0125	55
6	2973	7.28	2624	.62	.46 0349	7.90	.53 9651	54
7	3410	7.28	2587	.60	0823	7.90	9177	53
8	3847	7.28	2551	.62	1297	7.88	8703	52
9	4284	7.27	2514	.62	1770	7.87	8230	51
10	9.44 4720	7.25	9.98 2477	.60	9.46 2242	7.88	10.53 7758	50
11	5155	7.25	2441	.62	2715	7.85	7285	49
12	5590	7.25	2404	.62	3186	7.87	6814	48
13	6025	7.23	2367	.60	3658	7.83	6342	47
14	6459	7.23	2331	.62	4128	7.85	5872	46
15	9.44 6893	7.22	9.98 2294	.62	9.46 4599	7.83	10.53 5401	45
16	7326	7.22	2257	.62	5069	7.83	4931	44
17	7759	7.20	2220	.62	5539	7.82	4461	43
18	8191	7.20	2183	.62	6008	7.82	3992	42
19	8623	7.18	2146	.62	6477	7.80	3523	41
20	9.44 9054	7.18	9.98 2109	.62	9.46 6945	7.80	10.53 3055	40
21	9485	7.17	2072	.62	7413	7.78	2587	39
22	.44 9915	7.17	2035	.62	7880	7.78	2120	38
23	.45 0345	7.17	1998	.62	8347	7.78	1653	37
24	0775	7.15	1961	.62	8814	7.77	1186	36
25	9.45 1204	7.13	9.98 1924	.63	9.46 9280	7.77	10.53 0720	35
26	1632	7.13	1886	.62	.46 9746	7.75	.53 0254	34
27	2060	7.13	1849	.62	.47 0211	7.75	.52 9789	33
28	2488	7.13	1812	.62	0676	7.75	9324	32
29	2915	7.12	1774	.63	1141	7.73	8859	31
30	9.45 3342	7.12	9.98 1737	.62	9.47 1605	7.73	10.52 8395	30
31	3768	7.10	1700	.63	2069	7.72	7931	29
32	4194	7.10	1662	.63	2532	7.72	7468	28
33	4619	7.08	1625	.62	2995	7.72	7005	27
34	5044	7.08	1587	.63	3457	7.70	6543	26
35	9.45 5469	7.08	9.98 1549	.63	9.47 3919	7.70	10.52 6081	25
36	5893	7.07	1512	.62	4381	7.70	5619	24
37	6316	7.05	1474	.63	4842	7.68	5158	23
38	6739	7.05	1436	.63	5303	7.68	4697	22
39	7162	7.05	1399	.62	5763	7.67	4237	21
40	9.45 7584	7.03	9.98 1361	.63	9.47 6223	7.67	10.52 3777	20
41	8006	7.02	1323	.63	6683	7.67	3317	19
42	8427	7.02	1285	.63	7142	7.65	2858	18
43	8848	7.02	1247	.63	7601	7.65	2399	17
44	9268	7.00	1209	.63	8059	7.63	1941	16
45	9.45 9688	7.00	9.98 1171	.63	9.47 8517	7.63	10.52 1483	15
46	.46 0108	6.98	1133	.63	8975	7.62	1025	14
47	0527	6.98	1095	.63	9432	7.62	0568	13
48	0946	6.97	1057	.63	.47 9889	7.60	.52 0111	12
49	1364	6.97	1019	.63	.48 0345	7.60	.51 9655	11
50	9.46 1782	6.95	9.98 0981	.65	9.48 0801	7.60	10.51 9199	10
51	2199	6.95	0942	.63	1257	7.58	8743	9
52	2616	6.93	0904	.63	1712	7.58	8288	8
53	3032	6.93	0866	.65	2167	7.57	7833	7
54	3448	6.93	0827	.63	2621	7.57	7379	6
55	9.46 3864	6.92	9.98 0789	.65	9.48 3075	7.57	10.51 6925	5
56	4279	6.92	0750	.63	3529	7.55	6471	4
57	4694	6.90	0712	.65	3982	7.55	6018	3
58	5108	6.90	0673	.63	4435	7.53	5565	2
59	5522	6.88	0635	.65	4887	7.53	5113	1
60	9.46 5935		9.98 0596		9.48 5339		10.51 4661	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.46 5935		9.98 0596		9.48 5339		10.51 4661	60
1	6348	6.88	0558	.63	5791	7.53	4209	59
2	6761	6.88	0519	.65	6242	7.52	3758	58
3	7173	6.87	0480	.65	6693	7.52	3307	57
4	7585	6.87	0442	.63	7143	7.50	2857	56
5	9.46 7996	6.85	9.98 0403	.65	9.48 7593	7.50	10.51 2407	55
6	8407	6.83	0364	.65	8043	7.48	1957	54
7	8817	6.83	0325	.65	8492	7.48	1508	53
8	9227	6.83	0286	.65	8941	7.48	1059	52
9	.46 9637	6.82	0247	.65	9390	7.47	0610	51
10	9.47 0046	6.82	9.98 0208	.65	9.48 9838	7.47	10.51 0162	50
11	0455	6.80	0169	.65	.49 0286	7.45	.50 9714	49
12	0863	6.80	0130	.65	0733	7.45	9267	48
13	1271	6.80	0091	.65	1180	7.45	8820	47
14	1679	6.78	0052	.67	1627	7.43	8373	46
15	9.47 2086	6.77	9.98 0012	.65	9.49 2073	7.43	10.50 7927	45
16	2492	6.77	.97 9973	.65	2519	7.43	7481	44
17	2898	6.77	9934	.65	2965	7.42	7035	43
18	3304	6.77	9895	.67	3410	7.40	6590	42
19	3710	6.75	9855	.65	3854	7.42	6146	41
20	9.47 4115	6.73	9.97 9816	.67	9.49 4299	7.40	10.50 5701	40
21	4519	6.73	9776	.65	4743	7.38	5257	39
22	4923	6.73	9737	.67	5186	7.40	4814	38
23	5327	6.72	9697	.65	5630	7.38	4370	37
24	5730	6.72	9658	.67	6073	7.37	3927	36
25	9.47 6133	6.72	9.97 9618	.65	9.49 6515	7.37	10.50 3485	35
26	6536	6.70	9579	.67	6957	7.37	3043	34
27	6938	6.70	9539	.67	7399	7.37	2601	33
28	7340	6.68	9499	.67	7841	7.35	2159	32
29	7741	6.68	9459	.65	8282	7.33	1718	31
30	9.47 8142	6.67	9.97 9420	.67	9.49 8722	7.35	10.50 1278	30
31	8542	6.67	9380	.67	9163	7.33	0837	29
32	8942	6.67	9340	.67	.49 9603	7.32	.50 0397	28
33	9342	6.65	9300	.67	.50 0042	7.32	.49 9958	27
34	.47 9741	6.65	9260	.67	0481	7.32	9519	26
35	9.48 0140	6.65	9.97 9220	.67	9.50 0920	7.32	10.49 9080	25
36	0539	6.63	9180	.67	1359	7.30	8641	24
37	0937	6.62	9140	.67	1797	7.30	8203	23
38	1334	6.62	9100	.67	2235	7.30	7765	22
39	1731	6.62	9059	.68	2672	7.28	7328	21
40	9.48 2128	6.62	9.97 9019	.67	9.50 3109	7.28	10.49 6891	20
41	2525	6.60	8979	.67	3546	7.27	6454	19
42	2921	6.58	8939	.68	3982	7.27	6018	18
43	3316	6.60	8898	.67	4418	7.27	5582	17
44	3712	6.58	8858	.68	4854	7.25	5146	16
45	9.48 4107	6.57	9.97 8817	.67	9.50 5289	7.25	10.49 4711	15
46	4501	6.57	8777	.67	5724	7.25	4276	14
47	4895	6.57	8737	.68	6159	7.23	3841	13
48	5289	6.57	8696	.68	6593	7.23	3407	12
49	5682	6.55	8655	.68	7027	7.23	2973	11
50	9.48 6075	6.55	9.97 8615	.67	9.50 7460	7.22	10.49 2540	10
51	6467	6.53	8574	.68	7893	7.22	2107	9
52	6860	6.55	8533	.68	8326	7.22	1674	8
53	7251	6.52	8493	.67	8759	7.22	1241	7
54	7643	6.53	8452	.68	9191	7.20	0809	6
55	9.48 8034	6.52	9.97 8411	.68	9.50 9622	7.18	10.49 0378	5
56	8424	6.50	8370	.68	.51 0054	7.20	.48 9946	4
57	8814	6.50	8329	.68	0485	7.18	9515	3
58	9204	6.50	8288	.68	0916	7.18	9084	2
59	9593	6.48	8247	.68	1346	7.17	8654	1
60	9.48 9982	6.48	9.97 8206	.68	9.51 1776	7.17	10.48 8224	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.48 9982	6.48	9.97 8206	.68	9.51 1776	7.17	10.48 8224	60
1	.49 0371	6.47	8165	.68	2206	7.15	7794	59
2	0759	6.47	8124	.68	2635	7.15	7365	58
3	1147	6.47	8083	.68	3064	7.15	6936	57
4	1535	6.45	8042	.68	3493	7.13	6507	56
5	9.49 1922	6.43	9.97 8001	.70	9.51 3921	7.13	10.48 6079	55
6	2308	6.45	7959	.68	4349	7.13	5651	54
7	2695	6.43	7918	.68	4777	7.12	5223	53
8	3081	6.42	7877	.70	5204	7.12	4796	52
9	3466	6.42	7835	.68	5631	7.10	4369	51
10	9.49 3851	6.42	9.97 7794	.70	9.51 6057	7.12	10.48 3934	50
11	4236	6.42	7752	.68	6484	7.10	3516	49
12	4621	6.40	7711	.70	6910	7.08	3090	48
13	5005	6.38	7669	.68	7335	7.10	2665	47
14	5388	6.40	7628	.70	7761	7.08	2239	46
15	9.49 5772	6.37	9.97 7586	.70	9.51 8186	7.07	10.48 1814	45
16	6154	6.38	7544	.68	8610	7.07	1390	44
17	6537	6.37	7503	.70	9034	7.07	0966	43
18	6919	6.37	7461	.70	9458	7.07	0542	42
19	7301	6.35	7419	.70	.51 9882	7.05	.48 0118	41
20	9.49 7682	6.35	9.97 7377	.70	9.52 0305	7.05	10.47 9695	40
21	8064	6.33	7335	.70	0728	7.05	9272	39
22	8444	6.35	7293	.70	1151	7.03	8849	38
23	8825	6.32	7251	.70	1573	7.03	8427	37
24	9204	6.33	7209	.70	1995	7.03	8005	36
25	9.49 9584	6.32	9.97 7167	.70	9.52 2417	7.02	10.47 7583	35
26	.49 9963	6.32	7125	.70	2838	7.02	7162	34
27	.50 0342	6.32	7083	.70	3259	7.02	6741	33
28	0721	6.30	7041	.70	3680	7.00	6320	32
29	1099	6.28	6999	.70	4100	7.00	5900	31
30	9.50 1476	6.30	9.97 6957	.72	9.52 4520	7.00	10.47 5480	30
31	1854	6.28	6914	.70	4940	6.98	5060	29
32	2231	6.27	6872	.70	5359	6.98	4641	28
33	2607	6.28	6830	.72	5778	6.98	4222	27
34	2984	6.27	6787	.70	6197	6.97	3803	26
35	9.50 3360	6.25	9.97 6745	.72	9.52 6615	6.97	10.47 3385	25
36	3735	6.25	6702	.70	7033	6.97	2967	24
37	4110	6.25	6660	.72	7451	6.95	2549	23
38	4485	6.25	6617	.72	7868	6.95	2132	22
39	4860	6.23	6574	.70	8285	6.95	1715	21
40	9.50 5234	6.23	9.97 6532	.72	9.52 8702	6.95	10.47 1298	20
41	5608	6.22	6489	.72	9119	6.93	0881	19
42	5981	6.22	6446	.70	9535	6.93	0465	18
43	6354	6.22	6404	.72	.52 9951	6.92	.47 0049	17
44	6727	6.20	6361	.72	.53 0366	6.92	.46 9634	16
45	9.50 7099	6.20	9.97 6318	.72	9.53 0781	6.92	10.46 9219	15
46	7471	6.20	6275	.72	1196	6.92	8804	14
47	7843	6.18	6232	.72	1611	6.90	8389	13
48	8214	6.18	6189	.72	2025	6.90	7975	12
49	8585	6.18	6146	.72	2439	6.90	7561	11
50	9.50 8956	6.17	9.97 6103	.72	9.53 2853	6.88	10.46 7147	10
51	9326	6.17	6060	.72	3266	6.88	6734	9
52	.50 9696	6.15	6017	.72	3679	6.88	6321	8
53	.51 0065	6.15	5974	.73	4092	6.87	5908	7
54	0434	6.15	5930	.72	4504	6.87	5496	6
55	9.51 0803	6.15	9.97 5887	.72	9.53 4916	6.87	10.46 5084	5
56	1172	6.13	5844	.73	5328	6.85	4672	4
57	1540	6.12	5800	.72	5739	6.85	4261	3
58	1907	6.13	5757	.72	6150	6.85	3850	2
59	2275	6.12	5714	.73	6561	6.85	3439	1
60	9.51 2642		9.97 5670		9.53 6972		10.46 3028	0

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.51 2642	6.12	9.97 5670	.72	9.53 6972	6.83	10.46 3028	60
1	3009	6.10	5627	.73	7382	6.83	2618	59
2	3375	6.10	5583	.73	7792	6.83	2208	58
3	3741	6.10	5539	.72	8202	6.82	1798	57
4	4107	6.08	5496	.73	8611	6.82	1389	56
5	9.51 4472	6.08	9.97 5452	.73	9.53 9020	6.82	10.46 0980	55
6	4837	6.08	5408	.72	9429	6.80	0571	54
7	5202	6.07	5365	.73	.53 9837	6.80	.46 0163	53
8	5566	6.07	5321	.73	.54 0245	6.80	.45 9755	52
9	5930	6.07	5277	.73	0653	6.80	9347	51
10	9.51 6294	6.05	9.97 5233	.73	9.54 1061	6.78	10 45 8939	50
11	6657	6.05	5189	.73	1468	6.78	8532	49
12	7020	6.03	5145	.73	1875	6.77	8125	48
13	7382	6.05	5101	.73	2281	6.78	7719	47
14	7745	6.03	5057	.73	2688	6.77	7312	46
15	9.51 8107	6.02	9.97 5013	.73	9.54 3094	6.75	10.45 6906	45
16	8468	6.02	4969	.73	3499	6.77	6501	44
17	8829	6.02	4925	.75	3905	6.75	6095	43
18	9190	6.02	4880	.73	4310	6.75	5690	42
19	9551	6.00	4836	.73	4715	6.73	5285	41
20	9.51 9911	6.00	9.97 4792	.73	9.54 5119	6.75	10.45 4881	40
21	.52 0271	6.00	4748	.75	5524	6.73	4476	39
22	0631	5.98	4703	.73	5928	6.72	4072	38
23	0990	5.98	4659	.75	6331	6.73	3669	37
24	1349	5.97	4614	.73	6735	6.72	3265	36
25	9.52 1707	5.98	9.97 4570	.75	9.54 7138	6.70	10.45 2862	35
26	2066	5.97	4525	.73	7540	6.72	2460	34
27	2424	5.95	4481	.75	7943	6.70	2057	33
28	2781	5.95	4436	.75	8345	6.70	1655	32
29	3138	5.95	4391	.73	8747	6.70	1253	31
30	9.52 3495	5.95	9.97 4347	.75	9.54 9149	6.68	10.45 0851	30
31	3852	5.93	4302	.75	9550	6.68	0450	29
32	4208	5.93	4257	.75	.54 9951	6.68	.45 0049	28
33	4564	5.93	4212	.75	.55 0352	6.67	.44 9648	27
34	4920	5.92	4167	.75	0752	6.68	9248	26
35	9.52 5275	5.92	9.97 4122	.75	9.55 1153	6.65	10.44 8847	25
36	5630	5.90	4077	.75	1552	6.67	8448	24
37	5984	5.92	4032	.75	1952	6.65	8048	23
38	6339	5.90	3987	.75	2351	6.65	7649	22
39	6693	5.88	3942	.75	2750	6.65	7250	21
40	9.52 7046	5.90	9.97 3897	.75	9.55 3149	6.65	10.44 6851	20
41	7400	5.88	3852	.75	3548	6.63	6452	19
42	7753	5.87	3807	.77	3946	6.63	6054	18
43	8105	5.88	3761	.75	4344	6.62	5656	17
44	8458	5.87	3716	.75	4741	6.63	5259	16
45	9.52 8810	5.85	9.97 3671	.77	9.55 5139	6.62	10.44 4861	15
46	9161	5.87	3625	.75	5536	6.62	4464	14
47	9513	5.85	3580	.75	5933	6.60	4067	13
48	.52 9864	5.85	3535	.77	6329	6.60	3671	12
49	.53 0215	5.83	3489	.75	6725	6.60	3275	11
50	9.53 0565	5.83	9.97 3444	.77	9.55 7121	6.60	10.44 2879	10
51	0915	5.83	3398	.77	7517	6.60	2483	9
52	1265	5.82	3352	.75	7913	6.58	2087	8
53	1614	5.82	3307	.77	8308	6.58	1692	7
54	1963	5.82	3261	.77	8703	6.57	1297	6
55	9.53 2312	5.82	9.97 3215	.77	9.55 9097	6.57	10.44 0903	5
56	2661	5.80	3169	.75	9491	6.57	0509	4
57	3009	5.80	3124	.77	.55 9885	6.57	.44 0115	3
58	3357	5.78	3078	.77	.56 0279	6.57	.43 9721	2
59	3704	5.80	3032	.77	0673	6.55	9327	1
60	9.53 4052		9.97 2986		9.56 1066		10.43 8934	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.53 4052	5.78	9.97 2986	.77	9.56 1066	6.55	10.43 8934	60
1	4399	5.77	2940	.77	1459	6.53	8541	59
2	4745	5.78	2894	.77	1851	6.55	8149	58
3	5092	5.77	2848	.77	2244	6.53	7756	57
4	5438	5.75	2802	.78	2636	6.53	7364	56
5	9.53 5783	5.77	9.97 2755	.77	9.56 3028	6.52	10.43 6972	55
6	6129	5.75	2709	.77	3419	6.53	6581	54
7	6474	5.73	2663	.77	3811	6.52	6189	53
8	6818	5.75	2617	.78	4202	6.52	5798	52
9	7163	5.73	2570	.77	4593	6.50	5407	51
10	9.53 7507	5.73	9.97 2524	.77	9.56 4983	6.50	10.43 5017	50
11	7851	5.72	2478	.78	5373	6.50	4627	49
12	8194	5.73	2431	.77	5763	6.50	4237	48
13	8538	5.70	2385	.78	6153	6.48	3847	47
14	8880	5.72	2338	.78	6542	6.50	3458	46
15	9.53 9223	5.70	9.97 2291	.77	9.56 6932	6.47	10.43 3068	45
16	9565	5.70	2245	.78	7320	6.48	2680	44
17	.53 9907	5.68	2198	.78	7709	6.48	2291	43
18	.54 0249	5.68	2151	.77	8098	6.47	1902	42
19	0590	5.68	2105	.78	8486	6.45	1514	41
20	9.54 0931	5.68	9.97 2058	.78	9.56 8873	6.47	10.43 1127	40
21	1272	5.68	2011	.78	9261	6.45	0739	39
22	1613	5.67	1964	.78	.56 9648	6.45	.43 0352	38
23	1953	5.67	1917	.78	.57 0035	6.45	.42 9965	37
24	2293	5.65	1870	.78	0422	6.45	9578	36
25	9.54 2632	5.65	9.97 1823	.78	9.57 0809	6.43	10.42 9191	35
26	2971	5.65	1776	.78	1195	6.43	8805	34
27	3310	5.65	1729	.78	1581	6.43	8419	33
28	3649	5.63	1682	.78	1967	6.42	8033	32
29	3987	5.63	1635	.78	2352	6.43	7648	31
30	9.54 4325	5.63	9.97 1588	.80	9.57 2738	6.42	10.42 7262	30
31	4663	5.62	1540	.78	3123	6.40	6877	29
32	5000	5.63	1493	.78	3507	6.42	6493	28
33	5338	5.60	1446	.80	3892	6.40	6108	27
34	5674	5.62	1398	.78	4276	6.40	5724	26
35	9.54 6011	5.60	9.97 1351	.80	9.57 4660	6.40	10.42 5340	25
36	6347	5.60	1303	.78	5044	6.38	4956	24
37	6683	5.60	1256	.80	5427	6.38	4573	23
38	7019	5.58	1208	.78	5810	6.38	4190	22
39	7354	5.58	1161	.80	6193	6.38	3807	21
40	9.54 7689	5.58	9.97 1113	.78	9.57 6576	6.38	10.42 3424	20
41	8024	5.58	1066	.80	6959	6.37	3041	19
42	8359	5.57	1018	.80	7341	6.37	2659	18
43	8693	5.57	0970	.80	7723	6.35	2277	17
44	9027	5.55	0922	.80	8104	6.37	1896	16
45	9.54 9360	5.55	9.97 0874	.78	9.57 8486	6.35	10.42 1514	15
46	.54 9693	5.55	0827	.80	8867	6.35	1133	14
47	.55 0026	5.55	0779	.80	9248	6.35	0752	13
48	0359	5.55	0731	.80	.57 9629	6.33	.42 0371	12
49	0692	5.53	0683	.80	.58 0009	6.33	.41 9991	11
50	9.55 1024	5.53	9.97 0635	.82	9.58 0389	6.33	10.41 9611	10
51	1356	5.52	0586	.80	0769	6.33	9231	9
52	1687	5.52	0538	.80	1149	6.32	8851	8
53	2018	5.52	0490	.80	1528	6.32	8472	7
54	2349	5.52	0442	.80	1907	6.32	8093	6
55	9.55 2680	5.50	9.97 0394	.82	9.58 2286	6.32	10.41 7714	5
56	3010	5.52	0345	.80	2665	6.32	7335	4
57	3341	5.48	0297	.80	3044	6.30	6956	3
58	3670	5.50	0249	.82	3422	6.30	6578	2
59	4000	5.48	0200	.80	3800	6.28	6200	1
60	9.55 4329		9.97 0152		9.58 4177		10.41 5823	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.55 4329	5.48	9.97 0152	.82	9.58 4177	6.30	10.41 5823	60
1	4658	5.48	0103	.80	4555	6.28	5445	59
2	4987	5.47	0055	.82	4932	6.28	5068	58
3	5315	5.47	.97 0006	.82	5309	6.28	4691	57
4	5643	5.47	.96 9957	.80	5686	6.27	4314	56
5	9.55 5971	5.47	9.96 9909	.82	9.58 6062	6.28	10.41 3938	55
6	6299	5.45	9860	.82	6439	6.27	3561	54
7	6626	5.45	9811	.82	6815	6.25	3185	53
8	6953	5.45	9762	.80	7190	6.27	2810	52
9	7280	5.43	9714	.82	7566	6.25	2434	51
10	9.55 7606	5.43	9.96 9665	.82	9.58 7941	6.25	10.41 2059	50
11	7932	5.43	9616	.82	8316	6.25	1684	49
12	8258	5.42	9567	.82	8691	6.25	1309	48
13	8583	5.43	9518	.82	9066	6.23	0934	47
14	8909	5.42	9469	.82	9440	6.23	0560	46
15	9.55 9234	5.40	9.96 9420	.83	9.58 9814	6.23	10.41 0186	45
16	9558	5.42	9370	.82	.59 0188	6.23	.40 9812	44
17	.55 9883	5.40	9321	.82	0562	6.22	9438	43
18	.56 0207	5.40	9272	.82	0935	6.22	9065	42
19	0531	5.40	9223	.83	1308	6.22	8692	41
20	9.56 0855	5.38	9.96 9173	.82	9.59 1681	6.22	10.40 8319	40
21	1178	5.38	9124	.82	2054	6.20	7946	39
22	1501	5.38	9075	.83	2426	6.22	7574	38
23	1824	5.37	9025	.82	2799	6.20	7201	37
24	2146	5.37	8976	.83	3171	6.18	6829	36
25	9.56 2468	5.37	9.96 8926	.82	9.59 3542	6.20	10.40 6458	35
26	2790	5.37	8877	.83	3914	6.18	6086	34
27	3112	5.35	8827	.83	4285	6.18	5715	33
28	3433	5.37	8777	.82	4656	6.18	5344	32
29	3755	5.33	8728	.83	5027	6.18	4973	31
30	9.56 4075	5.35	9.96 8678	.83	9.59 5398	6.17	10.40 4602	30
31	4396	5.33	8628	.83	5768	6.17	4232	29
32	4716	5.33	8578	.83	6138	6.17	3862	28
33	5036	5.33	8528	.82	6508	6.17	3492	27
34	5356	5.33	8479	.83	6878	6.15	3122	26
35	9.56 5676	5.32	9.96 8429	.83	9.59 7247	6.15	10.40 2753	25
36	5995	5.32	8379	.83	7616	6.15	2384	24
37	6314	5.30	8329	.85	7985	6.15	2015	23
38	6632	5.32	8278	.83	8354	6.13	1646	22
39	6951	5.30	8228	.83	8722	6.15	1278	21
40	9.56 7269	5.30	9.96 8178	.83	9.59 9091	6.13	10.40 0909	20
41	7587	5.28	8128	.83	9459	6.13	0541	19
42	7904	5.30	8078	.85	.59 9827	6.12	.40 0173	18
43	8222	5.28	8027	.83	.60 0194	6.13	.39 9806	17
44	8539	5.28	7977	.83	0562	6.12	9438	16
45	9.56 8856	5.27	9.96 7927	.85	9.60 0929	6.12	10.39 9071	15
46	9172	5.27	7876	.83	1296	6.12	8704	14
47	9488	5.27	7826	.85	1663	6.10	8337	13
48	.56 9804	5.27	7775	.83	2029	6.10	7971	12
49	.57 0120	5.25	7725	.85	2395	6.10	7605	11
50	9.57 0435	5.27	9.96 7674	.83	9.60 2761	6.10	10.39 7239	10
51	0751	5.25	7624	.85	3127	6.10	6873	9
52	1066	5.23	7573	.85	3493	6.08	6507	8
53	1380	5.25	7522	.85	3858	6.08	6142	7
54	1695	5.23	7471	.83	4223	6.08	5777	6
55	9.57 2009	5.23	9.96 7421	.85	9.60 4588	6.08	10.39 5412	5
56	2323	5.22	7370	.85	4953	6.07	5047	4
57	2636	5.23	7319	.85	5317	6.08	4683	3
58	2950	5.22	7268	.85	5682	6.07	4318	2
59	3263	5.22	7217	.85	6046	6.07	3954	1
60	9.57 3575	5.20	9.96 7166	.85	9.60 6410	6.07	10.39 3590	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.57 3575	5.22	9.96 7166	.85	9.60 6410	6.05	10.39 3590	60
1	3888	5.20	7115	.85	6773	6.07	3227	59
2	4200	5.20	7064	.85	7137	6.05	2863	58
3	4512	5.20	7013	.87	7500	6.05	2500	57
4	4824	5.20	6961	.85	7863	6.03	2137	56
5	9.57 5136	5.18	9.96 6910	.85	9.60 8225	6.05	10.39 1775	55
6	5447	5.18	6859	.85	8588	6.03	1412	54
7	5758	5.18	6808	.87	8950	6.03	1050	53
8	6069	5.17	6756	.85	9312	6.03	0688	52
9	6379	5.17	6705	.87	.60 9674	6.03	.39 0326	51
10	9.57 6689	5.17	9.96 6653	.85	9.61 0036	6.02	10.38 9964	50
11	6999	5.17	6602	.87	0397	6.03	9603	49
12	7309	5.15	6550	.85	0759	6.02	9241	48
13	7618	5.15	6499	.87	1120	6.00	8880	47
14	7927	5.15	6447	.87	1480	6.02	8520	46
15	9.57 8236	5.15	9.96 6395	.85	9.61 1841	6.00	10.38 8159	45
16	8545	5.13	6344	.87	2201	6.00	7799	44
17	8853	5.15	6292	.87	2561	6.00	7439	43
18	9162	5.13	6240	.87	2921	6.00	7079	42
19	9470	5.12	6188	.87	3281	6.00	6719	41
20	9.57 9777	5.13	9.96 6136	.85	9.61 3641	5.98	10.38 6359	40
21	.58 0085	5.12	6085	.87	4000	5.98	6000	39
22	0392	5.12	6033	.87	4359	5.98	5641	38
23	0699	5.10	5981	.87	4718	5.98	5282	37
24	1005	5.12	5929	.88	5077	5.97	4923	36
25	9.58 1312	5.10	9.96 5876	.87	9.61 5435	5.97	10.38 4565	35
26	1618	5.10	5824	.87	5793	5.97	4207	34
27	1924	5.08	5772	.87	6151	5.97	3849	33
28	2229	5.10	5720	.87	6509	5.97	3491	32
29	2535	5.08	5668	.88	6867	5.95	3133	31
30	9.58 2840	5.08	9.96 5615	.87	9.61 7224	5.97	10.38 2776	30
31	3145	5.07	5563	.87	7582	5.95	2418	29
32	3449	5.08	5511	.88	7939	5.93	2061	28
33	3754	5.07	5458	.87	8295	5.95	1705	27
34	4058	5.05	5406	.88	8652	5.93	1348	26
35	9.58 4361	5.07	9.96 5353	.87	9.61 9008	5.93	10.38 0992	25
36	4665	5.05	5301	.88	9364	5.93	0636	24
37	4968	5.07	5248	.88	.61 9720	5.93	.38 0280	23
38	5272	5.03	5195	.87	.62 0076	5.93	.37 9924	22
39	5574	5.05	5143	.88	0432	5.92	9568	21
40	9.58 5877	5.03	9.96 5090	.88	9.62 0787	5.92	10.37 9213	20
41	6179	5.05	5037	.88	1142	5.92	8858	19
42	6482	5.02	4984	.88	1497	5.92	8503	18
43	6783	5.03	4931	.87	1852	5.92	8148	17
44	7085	5.02	4879	.88	2207	5.90	7793	16
45	9.58 7386	5.03	9.96 4826	.88	9.62 2561	5.90	10.37 7439	15
46	7688	5.02	4773	.88	2915	5.90	7085	14
47	7989	5.00	4720	.90	3269	5.90	6731	13
48	8289	5.02	4666	.88	3623	5.88	6377	12
49	8590	5.00	4613	.88	3976	5.90	6024	11
50	9.58 8890	5.00	9.96 4560	.88	9.62 4330	5.88	10.37 5670	10
51	9190	4.98	4507	.88	4683	5.88	5317	9
52	9489	5.00	4454	.90	5036	5.87	4964	8
53	.58 9780	4.98	4400	.88	5388	5.88	4612	7
54	.59 0088	4.98	4347	.88	5741	5.87	4259	6
55	9.59 0387	4.98	9.96 4294	.90	9.62 6093	5.87	10.37 3907	5
56	0686	4.97	4240	.88	6445	5.87	3555	4
57	0984	4.97	4187	.90	6797	5 87	3203	3
58	1282	4.97	4133	.88	7149	5 87	2851	2
59	1580	4.97	4080	.90	7501	5.85	2499	1
60	9.59 1878		9.96 4026		9.62 7852		10.37 2148	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.59 1878	4.97	9.96 4026	.90	9.62 7825	5.85	10.37 2148	60
1	2176	4.95	3972	.88	8203	5.85	1797	59
2	2473	4.95	3919	.90	8554	5.85	1446	58
3	2770	4.95	3865	.90	8905	5.83	1095	57
4	3067	4.93	3811	.90	9255	5.85	0745	56
5	9.59 3363	4.93	9.96 3757	.88	9.62 9606	5.83	10.37 0394	55
6	3659	4.93	3704	.90	.62 9956	5.83	.37 0044	54
7	3955	4.93	3650	.90	.63 0306	5.83	.36 9694	53
8	4251	4.93	3596	.90	0656	5.82	9344	52
9	4547	4.92	3542	.90	1005	5.83	8995	51
10	9.59 4842	4.92	9.96 3488	.90	9.63 1355	5.82	10.36 8645	50
11	5137	4.92	3434	.92	1704	5.82	8296	49
12	5432	4.92	3379	.90	2053	5.82	7947	48
13	5727	4.90	3325	.90	2402	5.80	7598	47
14	6021	4.90	3271	.90	2750	5.82	7250	46
15	9.59 6315	4.90	9.96 3217	.90	9.63 3099	5.80	10.36 6901	45
16	6609	4.90	3163	.92	3447	5.80	6553	44
17	6903	4.88	3108	.90	3795	5.80	6205	43
18	7196	4.90	3054	.92	4143	5.78	5857	42
19	7490	4.88	2999	.90	4490	5.80	5510	41
20	9.59 7783	4.87	9.96 2945	.92	9.63 4838	5.78	10.36 5162	40
21	8075	4.88	2890	.90	5185	5.78	4815	39
22	8368	4.87	2836	.92	5532	5.78	4468	38
23	8660	4.87	2781	.90	5879	5.78	4121	37
24	8952	4.87	2727	.92	6226	5.77	3774	36
25	9.59 9244	4.87	9.96 2672	.92	9.63 6572	5.78	10.36 3428	35
26	9536	4.85	2617	.92	6919	5.77	3081	34
27	.59 9827	4.85	2562	.90	7265	5.77	2735	33
28	.60 0118	4.85	2508	.92	7611	5.75	2389	32
29	0409	4.85	2453	.92	7956	5.77	2044	31
30	9.60 0700	4.83	9.96 2398	.92	9.63 8302	5.75	10.36 1698	30
31	0990	4.83	2343	.92	8647	5.75	1353	29
32	1280	4.83	2288	.92	8992	5.75	1008	28
33	1570	4.83	2233	.92	9337	5.75	0663	27
34	1860	4.83	2178	.92	.63 9682	5.75	.36 0318	26
35	9.60 2150	4.82	9.96 2123	.93	9.64 0027	5.73	10.35 9973	25
36	2439	4.82	2067	.92	0371	5.75	9629	24
37	2728	4.82	2012	.92	0716	5.73	9284	23
38	3017	4.80	1957	.92	1060	5.73	8940	22
39	3305	4.82	1902	.93	1404	5.72	8596	21
40	9.60 3594	4.80	9.96 1846	.92	9.64 1747	5.73	10.35 8253	20
41	3882	4.80	1791	.93	2091	5.72	7909	19
42	4170	4.78	1735	.92	2434	5.72	7566	18
43	4457	4.80	1680	.93	2777	5.72	7223	17
44	4745	4.78	1624	.92	3120	5.72	6880	16
45	9.60 5032	4.78	9.96 1569	.93	9.64 3463	5.72	10.35 6537	15
46	5319	4.78	1513	.92	3806	5.70	6194	14
47	5606	4.77	1458	.93	4148	5.70	5852	13
48	5892	4.78	1402	.93	4490	5.70	5510	12
49	6179	4.77	1346	.93	4832	5.70	5168	11
50	9.60 6465	4.77	9.96 1290	.92	9.64 5174	5.70	10.35 4826	10
51	6751	4.75	1235	.93	5516	5.68	4484	9
52	7036	4.77	1179	.93	5857	5.70	4143	8
53	7322	4.75	1123	.93	6199	5.68	3801	7
54	7607	4.75	1067	.93	6540	5.68	3460	6
55	9.60 7892	4.75	9.96 1011	.93	9.64 6881	5.68	10.35 3119	5
56	8177	4.73	0955	.93	7222	5.67	2778	4
57	8461	4.73	0899	.93	7562	5.68	2438	3
58	8745	4.73	0843	.95	7903	5.67	2097	2
59	9029	4.73	0786	.93	8243	5.67	1757	1
60	9.60 9313		9.96 0730		9.64 8583		10.35 1417	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

TABLE XII.—*Continued.*

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.
0	9.60 9313	4.73	9.96 0730	.93	9.64 8583	5.67	10.35 1417
1	9597	4.72	0674	.93	8923	5.67	1077
2	.60 9880	4.73	0618	.95	9263	5.65	0737
3	.61 0164	4.72	0561	.93	9602	5.67	0398
4	0447	4.70	0505	.95	.64 9942	5.65	.35 0058
5	9.61 0729	4.72	9.96 0448	.93	9.65 0281	5.65	10.34 9719
6	1012	4.70	0392	.95	0620	5.65	9380
7	1294	4.70	0335	.93	0959	5.63	9041
8	1576	4.70	0279	.95	1297	5.65	8703
9	1858	4.70	0222	.95	1636	5.63	8364
10	9.61 2140	4.68	9.96 0165	.93	9.65 1974	5.63	10.34 8026
11	2421	4.68	0109	.95	2312	5.63	7688
12	2702	4.68	.96 0052	.95	2650	5.63	7350
13	2983	4.68	.95 9995	.95	2988	5.63	7012
14	3264	4.68	9938	.93	3326	5.62	6674
15	9.61 3545	4.67	9.95 9882	.95	9.65 3663	5.62	10.34 6337
16	3825	4.67	9825	.95	4000	5.62	6000
17	4105	4.67	9768	.95	4337	5.62	5663
18	4385	4.67	9711	.95	4674	5.62	5326
19	4665	4.65	9654	.97	5011	5.62	4989
20	9.61 4944	4.65	9.95 9596	.95	9.65 5348	5.60	10.34 4652
21	5223	4.65	9539	.95	5684	5.60	4316
22	5502	4.65	9482	.95	6020	5.60	3980
23	5781	4.65	9425	.95	6356	5.60	3644
24	6060	4.63	9368	.97	6692	5.60	3308
25	9.61 6338	4.63	9.95 9310	.95	9.65 7028	5.60	10.34 2972
26	6616	4.63	9253	.97	7364	5.58	2636
27	6894	4.63	9195	.95	7699	5.58	2301
28	7172	4.63	9138	.97	8034	5.58	1966
29	7450	4.62	9080	.95	8369	5.58	1631
30	9.61 7727	4.62	9.95 9023	.97	9.65 8704	5.58	10.34 1296
31	8004	4.62	8965	.95	9039	5.57	0961
32	8281	4.62	8908	.97	9373	5.58	0627
33	8558	4.60	8850	.97	.65 9708	5.57	.34 0292
34	8834	4.60	8792	.97	.66 0042	5.57	.33 9958
35	9.61 9110	4.60	9.95 8734	.95	9.66 0376	5.57	10.33 9624
36	9386	4.60	8677	.97	0710	5.55	9290
37	9662	4.60	8619	.97	1043	5.57	8957
38	.61 9938	4.58	8561	.97	1377	5.55	8623
39	.62 0213	4.58	8503	.97	1710	5.55	8290
40	9.62 0488	4.58	9.95 8445	.97	9.66 2043	5.55	10.33 7957
41	0763	4.58	8387	.97	2376	5.55	7624
42	1038	4.58	8329	.97	2709	5.55	7291
43	1313	4.57	8271	.97	3042	5.55	6958
44	1587	4.57	8213	.98	3375	5.53	6625
45	9.62 1861	4.57	9.95 8154	.97	9.66 3707	5.53	10.33 6293
46	2135	4.57	8096	.97	4039	5.53	5961
47	2409	4.55	8038	.98	4371	5.53	5629
48	2682	4.57	7979	.97	4703	5.53	5297
49	2956	4.55	7921	.97	5035	5.52	4965
50	9.62 3229	4.55	9.95 7863	.98	9.66 5366	5.53	10.33 4634
51	3502	4.53	7804	.97	5698	5.52	4302
52	3774	4.55	7746	.98	6029	5.52	3971
53	4047	4.53	7687	.98	6360	5.52	3640
54	4319	4.53	7628	.97	6691	5.50	3309
55	9.62 4591	4.53	9.95 7570	.98	9.66 7021	5.52	10.33 2979
56	4863	4.53	7511	.98	7352	5.50	2648
57	5135	4.52	7452	.98	7682	5.52	2318
58	5406	4.52	7393	.97	8013	5.50	1987
59	5677	4.52	7335	.98	8343	5.50	1657
60	9.62 5948		9.95 7276		9.66 8673		10.33 1327
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.62 5948	4.52	9.95 7276	.98	9.66 8673	5.48	10.33 1327	60
1	6219	4.52	7217	.98	9002	5.50	0998	59
2	6490	4.50	7158	.98	9332	5.48	0668	58
3	6760	4.50	7099	.98	9661	5.50	0339	57
4	7030	4.50	7040	.98	.66 9991	5.48	.33 0009	56
5	9.62 7300	4.50	9.95 6981	1.00	9.67 0320	5.48	10.32 9680	55
6	7570	4.50	6921	.98	0649	5.47	9351	54
7	7840	4.48	6862	.98	0977	5.48	9023	53
8	8109	4.48	6803	.98	1306	5.48	8694	52
9	8378	4.48	6744	1.00	1635	5.47	8365	51
10	9.62 8647	4.48	9.95 6684	.98	9.67 1963	5.47	10.32 8037	50
11	8916	4.48	6625	.98	2291	5.47	7709	49
12	9185	4.47	6566	1.00	2619	5.47	7381	48
13	9453	4.47	6506	.98	2947	5.45	7053	47
14	9721	4.47	6447	1.00	3274	5.47	6726	46
15	9.62 9989	4.47	9.95 6387	1.00	9.67 3602	5.45	10.32 6398	45
16	.63 0257	4.45	6327	.98	3929	5.47	6071	44
17	0524	4.47	6268	1.00	4257	5.45	5743	43
18	0792	4.45	6208	1.00	4584	5.45	5416	42
19	1059	4.45	6148	.98	4911	5.43	5089	41
20	9.63 1326	4.45	9.95 6089	1.00	9.67 5237	5.45	10.32 4763	40
21	1593	4.43	6029	1.00	5564	5.43	4436	39
22	1859	4.43	5969	1.00	5890	5.45	4110	38
23	2125	4.45	5909	1.00	6217	5.43	3783	37
24	2392	4.43	5849	1.00	6543	5.43	3457	36
25	9.63 2658	4.42	9.95 5789	1.00	9.67 6869	5.42	10.32 3131	35
26	2923	4.43	5729	1.00	7194	5.43	2806	34
27	3189	4.42	5669	1.00	7520	5.43	2480	33
28	3454	4.42	5609	.98	7846	5.42	2154	32
29	3719	4.42	5548	1.00	8171	5.42	1829	31
30	9.63 3984	4.42	9.95 5488	1.00	9.67 8496	5.42	10.32 1504	30
31	4249	4.42	5428	1.00	8821	5.42	1179	29
32	4514	4.40	5368	1.02	9146	5.42	0854	28
33	4778	4.40	5307	1.00	9471	5.40	0529	27
34	5042	4.40	5247	1.02	.67 9795	5.42	.32 0205	26
35	9.63 5306	4.40	9.95 5186	1.00	9.68 0120	5.40	10.31 9880	25
36	5570	4.40	5126	1.02	0444	5.40	9556	24
37	5834	4.38	5065	1.00	0768	5.40	9232	23
38	6097	4.38	5005	1.02	1092	5.40	8908	22
39	6360	4.38	4944	1.02	1416	5.40	8584	21
40	9.63 6623	4.38	9.95 4883	1.00	9.68 1740	5.38	10.31 8260	20
41	6886	4.37	4823	1.02	2063	5.40	7937	19
42	7148	4.38	4762	1.02	2387	5.38	7613	18
43	7411	4.37	4701	1.02	2710	5.38	7290	17
44	7673	4.37	4640	1.02	3033	5.38	6967	16
45	9.63 7935	4.37	9.95 4579	1.02	9.68 3356	5.38	10.31 6644	15
46	8197	4.35	4518	1.02	3679	5.37	6321	14
47	8458	4.37	4457	1.02	4001	5.38	5999	13
48	8720	4.35	4396	1.02	4324	5.37	5676	12
49	8981	4.35	4335	1.02	4646	5.37	5354	11
50	9.63 9242	4.35	9.95 4274	1.02	9.68 4968	5.37	10.31 5032	10
51	9503	4.35	4213	1.02	5290	5.37	4710	9
52	.63 9764	4.33	4152	1.03	5612	5.37	4388	8
53	.64 0024	4.33	4090	1.02	5934	5.35	4066	7
54	0284	4.33	4029	1.02	6255	5.37	3745	6
55	9.64 0544	4.33	9.95 3968	1.03	9.68 6577	5.35	10.31 3423	5
56	0804	4.33	3906	1.02	6898	5.35	3102	4
57	1064	4.33	3845	1.03	7219	5.35	2781	3
58	1324	4.32	3783	1.02	7540	5.35	2460	2
59	1583	4.32	3722	1.03	7861	5.35	2139	1
60	9.64 1842		9.95 3660		9.68 8182		10.31 1818	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.64 1842	4.32	9.95 3660	1.02	9.68 8182	5.33	10.31 1818	60
1	2101	4.32	3599	1.03	8502	5.32	1498	59
2	2360	4.30	3537	1.03	8823	5.33	1177	58
3	2618	4.32	3475	1.03	9143	5.33	0857	57
4	2877	4.30	3413	1.02	9463	5.33	0537	56
5	9.64 3135	4.30	9.95 3352	1.03	9.68 9783	5.33	10.31 0217	55
6	3393	4.28	3290	1.03	.69 0103	5.33	.30 9897	54
7	3650	4.30	3228	1.03	0423	5.32	9577	53
8	3908	4.28	3166	1.03	0742	5.33	9258	52
9	4165	4.30	3104	1.03	1062	5.32	8938	51
10	9.64 4423	4.28	9.95 3042	1.03	9.69 1381	5.32	10.30 8619	50
11	4680	4.27	2980	1.03	1700	5.32	8300	49
12	4936	4.28	2918	1.05	2019	5.32	7981	48
13	5193	4.28	2855	1.03	2338	5.30	7662	47
14	5450	4.27	2793	1.03	2656	5.32	7344	46
15	9.64 5706	4.27	9.95 2731	1.03	9.69 2975	5.30	10.30 7025	45
16	5962	4.27	2669	1.05	3293	5.32	6707	44
17	6218	4.27	2606	1.03	3612	5.30	6388	43
18	6474	4.25	2544	1.05	3930	5.30	6070	42
19	6729	4.25	2481	1.03	4228	5.30	5752	41
20	9.64 6984	4.27	9.95 2419	1.05	9.69 4566	5.28	10.30 5434	40
21	7240	4.23	2356	1.03	4883	5.30	5117	39
22	7494	4.25	2294	1.05	5201	5.28	4799	38
23	7749	4.25	2231	1.05	5518	5.30	4482	37
24	8004	4.23	2168	1.03	5836	5.28	4164	36
25	9.64 8258	4.23	9.95 2106	1.05	9.69 6153	5.28	10.30 3847	35
26	8512	4.23	2043	1.05	6470	5.28	3530	34
27	8766	4.23	1980	1.05	6787	5.27	3213	33
28	9020	4.23	1917	1.05	7103	5.28	2897	32
29	9274	4.22	1854	1.05	7420	5.27	2580	31
30	9.64 9527	4.23	9.95 1791	1.05	9.69 7736	5.28	10.30 2264	30
31	.64 9781	4.22	1728	1.05	8053	5.27	1947	29
32	.65 0034	4.22	1665	1.05	8369	5.27	1631	28
33	0287	4.20	1602	1.05	8685	5.27	1315	27
34	0539	4.22	1539	1.05	9001	5.25	0999	26
35	9.65 0792	4.20	9.95 1476	1.07	9.69 9316	5.27	10.30 0684	25
36	1044	4.22	1412	1.05	9632	5.25	0368	24
37	1297	4.20	1349	1.05	.69 9947	5.27	.30 0053	23
38	1549	4.18	1286	1.07	.70 0263	5.25	.29 9737	22
39	1800	4.20	1222	1.05	0578	5.25	9422	21
40	9.65 2052	4.20	9.95 1159	1.05	9.70 0893	5.25	10.29 9107	20
41	2304	4.18	1096	1.07	1208	5.25	8792	19
42	2555	4.18	1032	1.07	1523	5.23	8477	18
43	2806	4.18	0968	1.05	1837	5.25	8163	17
44	3057	4.18	0905	1.07	2152	5.23	7848	16
45	9.65 3308	4.17	9.95 0841	1.05	9.70 2466	5.25	10.29 7534	15
46	3558	4.17	0778	1.07	2781	5.23	7219	14
47	3808	4.18	0714	1.07	3095	5.23	6905	13
48	4059	4.17	0650	1.07	3409	5.22	6591	12
49	4309	4.15	0586	1.07	3722	5.23	6278	11
50	9.65 4558	4.17	9.95 0522	1.07	9.70 4036	5.23	10.29 5964	10
51	4808	4.17	0458	1.07	4350	5.22	5650	9
52	5058	4.15	0394	1.07	4663	5.22	5337	8
53	5307	4.15	0330	1.07	4976	5.23	5024	7
54	5556	4.15	0266	1.07	5290	5.22	4710	6
55	9.65 5805	4.15	9.95 0202	1.07	9.70 5603	5.22	10.29 4397	5
56	6054	4.13	0138	1.07	5916	5.20	4084	4
57	6302	4.15	0074	1.07	6228	5.22	3772	3
58	6551	4.13	.95 0010	1.08	6541	5.22	3459	2
59	6799	4.13	.94 9945	1.07	6854	5.20	3146	1
60	9.65 7047		9.94 9881		9.70 7166		10.29 2834	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.65 7047	4.13	9.94 9881	1.08	9.70 7166	5.20	10.29 2834	60
1	7295	4.12	9816	1.07	7478	5.20	2522	59
2	7542	4.13	9752	1.07	7790	5.20	2210	58
3	7790	4.12	9688	1.08	8102	5.20	1898	57
4	8037	4.12	9623	1.08	8414	5.20	1586	56
5	9.65 8284	4.12	9.94 9558	1.07	9.70 8726	5.18	10.29 1274	55
6	8531	4.12	9494	1.08	9037	5.20	0963	54
7	8778	4.12	9429	1.08	9349	5.18	0651	53
8	9025	4.10	9364	1.07	9660	5.18	0340	52
9	9271	4.10	9300	1.08	9971	5.18	.29 0029	51
10	9.65 9517	4.10	9.94 9235	1.08	9.71 0282	5.18	10.28 9718	50
11	.65 9763	4.10	9170	1.08	0593	5.18	9407	49
12	.66 0009	4.10	9105	1.08	0904	5.18	9096	48
13	0255	4.10	9040	1.08	1215	5.17	8785	47
14	0501	4.08	8975	1.08	.70 1525	5.18	8475	46
15	9.66 0746	4.08	9.94 8910	1.08	9.71 1836	5.17	10.28 8164	45
16	0991	4.08	8845	1.08	2146	5.17	7854	44
17	1236	4.08	8780	1.08	2456	5.17	7544	43
18	1481	4.08	8715	1.08	2766	5.17	7234	42
19	1726	4.07	8650	1.10	3076	5.17	6924	41
20	9.66 1970	4.07	9.94 8584	1.08	9.71 3386	5.17	10.28 6614	40
21	2214	4.08	8519	1.08	3696	5.15	6304	39
22	2459	4.07	8454	1.10	4005	5.15	5995	38
23	2703	4.05	8388	1.08	4314	5.17	5686	37
24	2946	4.07	8323	1.10	4624	5.15	5376	36
25	9.66 3190	4.05	9.94 8257	1.08	9.71 4933	5.15	10.28 5067	35
26	3433	4.07	8192	1.10	5242	5.15	4758	34
27	3677	4.05	8126	1.10	5551	5.15	4449	33
28	3920	4.05	8060	1.08	5860	5.13	4140	32
29	4163	4.05	7995	1.10	6168	5.15	3832	31
30	9.66 4406	4.03	9.94 7929	1.10	9.71 6477	5.13	10.28 3523	30
31	4648	4.05	7863	1.10	6785	5.13	3215	29
32	4891	4.03	7797	1.10	7093	5.13	2907	28
33	5133	4.03	7731	1.10	7401	5.13	2599	27
34	5375	4.03	7665	1.08	7709	5.13	2291	26
35	9.66 5617	4.03	9.94 7600	1.12	9.71 8017	5.13	10.28 1983	25
36	5859	4.02	7533	1.10	8325	5.13	1675	24
37	6100	4.03	7467	1.10	8633	5.12	1367	23
38	6342	4.02	7401	1.10	8940	5.13	1060	22
39	6583	4.02	7335	1.10	9248	5.12	0752	21
40	9.66 6824	4.02	9.94 7269	1.10	9.71 9555	5.12	10.28 0445	20
41	7065	4.00	7203	1.12	.71 9862	5.12	.28 0138	19
42	7305	4.02	7136	1.10	.72 0169	5.12	.27 9831	18
43	7546	4.00	7070	1.10	0476	5.12	9524	17
44	7786	4.02	7004	1.12	0783	5.10	9217	16
45	9.66 8027	4.00	9.94 6937	1.10	9.72 1089	5.12	10.27 8911	15
46	8267	3.98	6871	1.12	1396	5.10	8604	14
47	8506	4.00	6804	1.10	1702	5.12	8298	13
48	8746	4.00	6738	1.12	2009	5.12	7991	12
49	8986	3.98	6671	1.12	2315	5.10	7685	11
50	9.66 9225	3.98	9.94 6604	1.10	9.72 2621	5.10	10.27 7379	10
51	9464	3.98	6538	1.12	2927	5.08	7073	9
52	9703	3.98	6471	1.12	3232	5.10	6768	8
53	.66 9942	3.98	6404	1.12	3538	5.10	6462	7
54	.67 0181	3.97	6337	1.12	3844	5.08	6156	6
55	9.67 0419	3.98	9.94 6270	1.12	9.72 4149	5.08	10.27 5851	5
56	0658	3.97	6203	1.12	4454	5.10	5546	4
57	0896	3.97	6136	1.12	4760	5.08	5240	3
58	1134	3.97	6069	1.12	5065	5.08	4935	2
59	1372	3.95	6002	1.12	5370	5.07	4630	1
60	9.67 1609		9.94 5935	1.12	9.72 5674		10.27 4326	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	**9.67 1609**	3.97	**9.94 5935**	1.12	**9.72 5674**	5.08	**10.27 4326**	60
1	1847	3.95	5868	1.13	5979	5.08	4021	59
2	2084	3.95	5800	1.12	6284	5.07	3716	58
3	2321	3.95	5733	1.12	6588	5.07	3412	57
4	2558	3.95	5666	1.13	6892	5.05	3108	56
5	**9.67 2795**	3.95	**9.94 5598**	1.12	**9.72 7197**	5.07	**10.27 2803**	55
6	3032	3.93	5531	1.12	7501	5.07	2499	54
7	3268	3.95	5464	1.13	7805	5.07	2195	53
8	3505	3.93	5396	1.13	8109	5.05	1891	52
9	3741	3.93	5328	1.12	8412	5.07	1588	51
10	**9.67 3977**	3.93	**9.94 5261**	1.13	**9.72 8716**	5.07	**10.27 1284**	50
11	4213	3.92	5193	1.13	9020	5.05	0980	49
12	4448	3.93	5125	1.12	9323	5.05	0677	48
13	4684	3.92	5058	1.13	9626	5.05	0374	47
14	4919	3.93	4990	1.13	.72 9929	5.07	.27 0071	46
15	**9.67 5155**	3.92	**9.94 4922**	1.13	**9.73 0233**	5.03	**10.26 9767**	45
16	5390	3.90	4854	1.13	0535	5.05	9465	44
17	5624	3.92	4786	1.13	0838	5.05	9162	43
18	5859	3.92	4718	1.13	1141	5.05	8859	42
19	6094	3.90	4650	1.13	1444	5.03	8556	41
20	**9.67 6328**	3.90	**9.94 4582**	1.13	**9.73 1746**	5.03	**10.26 8254**	40
21	6562	3.90	4514	1.13	2048	5.05	7952	39
22	6796	3.90	4446	1.15	2351	5.03	7649	38
23	7030	3.90	4377	1.13	2653	5.03	7347	37
24	7264	3.90	4309	1.13	2955	5.03	7045	36
25	**9.67 7498**	3.88	**9.94 4241**	1.15	**9.73 3257**	5.02	**10.26 6743**	35
26	7731	3.88	4172	1.13	3558	5.03	6442	34
27	7964	3.88	4104	1.13	3860	5.03	6140	33
28	8197	3.88	4036	1.15	4162	5.02	5838	32
29	8430	3.88	3967	1.13	4463	5.02	5537	31
30	**9.67 8663**	3.87	**9.94 3899**	1.15	**9.73 4764**	5.03	**10.26 5236**	30
31	8895	3.88	3830	1.15	5066	5.02	4934	29
32	9128	3.87	3761	1.13	5367	5.02	4633	28
33	9360	3.87	3693	1.15	5668	5.02	4332	27
34	9592	3.87	3624	1.15	5969	5.00	4031	26
35	**9.67 9824**	3.87	**9.94 3555**	1.15	**9.73 6269**	5.02	**10.26 3731**	25
36	.68 0056	3.87	3486	1.15	6570	5.00	3430	24
37	0288	3.85	3417	1.15	6870	5.02	3130	23
38	0519	3.85	3348	1.15	7171	5.00	2829	22
39	0750	3.87	3279	1.15	7471	5.00	2529	21
40	**9.68 0982**	3.85	**9.94 3210**	1.15	**9.73 7771**	5.00	**10.26 2229**	20
41	1213	3.83	3141	1.15	8071	5.00	1929	19
42	1443	3.85	3072	1.15	8371	5.00	1629	18
43	1674	3.85	3003	1.15	8671	5.00	1329	17
44	1905	3.83	2934	1.17	8971	5.00	1029	16
45	**9.68 2135**	3.83	**9.94 2864**	1.15	**9.73 9271**	4.98	**10.26 0729**	15
46	2365	3.83	2795	1.15	9570	5.00	0430	14
47	2595	3.83	2726	1.17	.73 9870	4.98	.26 0130	13
48	2825	3.83	2656	1.15	.74 0169	4.98	.25 9831	12
49	3055	3.82	2587	1.17	0468	4.98	9532	11
50	**9.68 3284**	3.83	**9.94 2517**	1.15	**9.74 0767**	4.98	**10.25 9233**	10
51	3514	3.82	2448	1.17	1066	4.98	8934	9
52	3743	3.82	2378	1.17	1365	4.98	8635	8
53	3972	3.82	2308	1.15	1664	4.97	8336	7
54	4201	3.82	2239	1.17	1962	4.98	8038	6
55	**9.68 4430**	3.80	**9.94 2169**	1.17	**9.74 2261**	4.97	**10.25 7739**	5
56	4658	3.82	2099	1.17	2559	4.98	7441	4
57	4887	3.80	2029	1.17	2858	4.97	7142	3
58	5115	3.80	1959	1.17	3156	4.97	6844	2
59	5343	3.80	1889	1.17	3454	4.97	6546	1
60	**9.68 5571**		**9.94 1819**		**9.74 3752**		**10.25 6248**	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.68 5571	3.80	9.94 1819	1.17	9.74 3752	4.97	10.25 6248	60
1	5799	3.80	1749	1.17	4050	4.97	5950	59
2	6027	3.78	1679	1.17	4348	4.95	5652	58
3	6254	3.80	1609	1.17	4645	4.97	5355	57
4	6482	3.78	1539	1.17	4943	4.95	5057	56
5	9.68 6709	3.78	9.94 1469	1.18	9.74 5240	4.97	10.25 4760	55
6	6936	3.78	1398	1.17	5538	4.95	4462	54
7	7163	3.77	1328	1.17	5835	4.95	4165	53
8	7389	3.78	1258	1.18	6132	4.95	3868	52
9	7616	3.78	1187	1.17	6429	4.95	3571	51
10	9.68 7843	3.77	9.94 1117	1.18	9.74 6726	4.95	10.25 3274	50
11	8069	3.77	1046	1.18	7023	4.93	2977	49
12	8295	3.77	0975	1.17	7319	4.95	2681	48
13	8521	3.77	0905	1.18	7616	4.95	2384	47
14	8747	3.75	0834	1.18	7913	4.93	2087	46
15	9.68 8972	3.77	9.94 0763	1.17	9.74 8209	4.93	10.25 1791	45
16	9198	3.75	0693	1.18	8505	4.93	1495	44
17	9423	3.75	0622	1.18	8801	4.93	1199	43
18	9648	3.75	0551	1.18	9097	4.93	0903	42
19	.68 9873	3.75	0480	1.18	9393	4.93	0607	41
20	9.69 0098	3.75	9.94 0409	1.18	9.74 9689	4.93	10.25 0311	40
21	0323	3.75	0338	1.18	.74 9985	4.93	.25 0015	39
22	0548	3.73	0267	1.18	.75 0281	4.92	.24 9719	38
23	0772	3.73	0196	1.18	0576	4.93	9424	37
24	0996	3.73	0125	1.18	0872	4.92	9128	36
25	9.69 1220	3.73	9.94 0054	1.20	9.75 1167	4.92	10.24 8833	35
26	1444	3.73	.93 9982	1.18	1462	4.92	8538	34
27	1668	3.73	9911	1.18	1757	4.92	8243	33
28	1892	3.72	9840	1.20	2052	4.92	7948	32
29	2115	3.73	9768	1.18	2347	4.92	7653	31
30	9.69 2339	3.72	9.93 9697	1.20	9.75 2642	4.92	10.24 7358	30
31	2562	3.72	9625	1.18	2937	4.90	7063	29
32	2785	3.72	9554	1.20	3231	4.92	6769	28
33	3008	3.72	9482	1.20	3526	4.90	6474	27
34	3231	3.70	9410	1.18	3820	4.92	6180	26
35	9.69 3453	3.72	9.93 9339	1.20	9.75 4115	4.90	10.24 5885	25
36	3676	3.70	9267	1.20	4409	4.90	5591	24
37	3898	3.70	9195	1.20	4703	4.90	5297	23
38	4120	3.70	9123	1.18	4997	4.90	5003	22
39	4342	3.70	9052	1.20	5291	4.90	4709	21
40	9.69 4564	3.70	9.93 8980	1.20	9.75 5585	4.88	10.24 4415	20
41	4786	3.68	8908	1.20	5878	4.90	4122	19
42	5007	3.70	8836	1.22	6172	4.88	3828	18
43	5229	3.68	8763	1.20	6465	4.90	3535	17
44	5450	3.68	8691	1.20	6759	4.88	3241	16
45	9.69 5671	3.68	9.93 8619	1.20	9.75 7052	4.88	10.24 2948	15
46	5892	3.68	8547	1.20	7345	4.88	2655	14
47	6113	3.68	8475	1.22	7638	4.88	2362	13
48	6334	3.67	8402	1.20	7931	4.88	2069	12
49	6554	3.68	8330	1.20	8224	4.88	1776	11
50	9.69 6775	3.67	9.93 8258	1.22	9.75 8517	4.88	10.24 1483	10
51	6995	3.67	8185	1.20	8810	4.87	1190	9
52	7215	3.67	8113	1.22	9102	4.88	0898	8
53	7435	3.65	8040	1.22	9395	4.87	0605	7
54	7654	3.67	7967	1.20	9687	4.87	0313	6
55	9.69 7874	3.67	9.93 7895	1.22	9.75 9979	4.88	10.24 0021	5
56	8094	3.65	7822	1.22	.76 0272	4.87	.23 9728	4
57	8313	3.65	7749	1.22	0564	4.87	9436	3
58	8532	3.65	7676	1.20	0856	4.87	9144	2
59	8751	3.65	7604	1.22	1148	4.85	8852	1
60	9.69 8970		9.93 7531		9.76 1439		10.23 8561	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.69 8970	3.65	9.93 7531	1.22	9.76 1439	4.87	10.23 8561	60
1	9189	3.63	7458	1.22	1731	4.87	8269	59
2	9407	3.65	7385	1.22	2023	4.85	7977	58
3	9626	3.63	7312	1.23	2314	4.87	7686	57
4	.69 9844	3.63	7238	1.22	. 2606	4.85	7394	56
5	9.70 0062	3.63	9.93 7165	1.22	9.76 2897	4.85	10.23 7103	55
6	0280	3.63	7092	1.22	3188	4.85	6812	54
7	0498	3.63	7019	1.22	3479	4.85	6521	53
8	0716	3.62	6946	1.23	3770	4.85	6230	52
9	0933	3.63	6872	1.22	4061	4.85	5939	51
10	9.70 1151	3.62	9.93 6799	1.23	9.76 4352	4.85	10.23 5648	50
11	1368	3.62	6725	1.22	4643	4.83	5357	49
12	1585	3.62	6652	1.23	4933	4.85	5067	48
13	1802	3.62	6578	1.22	5224	4.83	4776	47
14	2019	3.62	6505	1.23	5514	4.85	4486	46
15	9.70 2236	3.60	9.93 6431	1.23	9.76 5805	4.83	10.23 4195	45
16	2452	3.62	6357	1.22	6095	4.83	3905	44
17	2669	3.60	6284	1.23	6385	4.83	3615	43
18	2885	3.60	6210	1.23	6675	4.83	3325	42
19	3101	3.60	6136	1.23	6965	4.83	3035	41
20	9.70 3317	3.60	9.93 6062	1.23	9.76 7255	4.83	10.23 2745	40
21	3533	3.60	5988	1.23	7545	4.82	2455	39
22	3749	3.58	5914	1.23	7834	4.83	2166	38
23	3964	3.58	5840	1.23	8124	4.83	1876	37
24	4179	3.60	5766	1.23	8414	4.82	1586	36
25	9.70 4395	3.58	9.93 5692	1.23	9.76 8703	4.82	10.23 1297	35
26	4610	3.58	5618	1.25	8992	4.82	1008	34
27	4825	3.58	5543	1.23	9281	4.83	0719	33
28	5040	3.57	5469	1.23	9571	4.82	0429	32
29	5254	3.58	5395	1.25	.76 9860	4.80	.23 0140	31
30	9.70 5469	3.57	9.93 5320	1.23	9.77 0148	4.82	10.22 9852	30
31	5683	3.58	5246	1.25	0437	4.82	9563	29
32	5898	3.57	5171	1.23	0726	4.82	9274	28
33	6112	3.57	5097	1.25	1015	4.80	8985	27
34	6326	3.55	5022	1.23	1303	4.82	8697	26
35	9.70 6539	3.57	9.93 4948	1.25	9.77 1592	4.80	10.22 8408	25
36	6753	3.57	4873	1.25	1880	4.80	8120	24
37	6967	3.55	4798	1.25	2168	4.82	7832	23
38	7180	3.55	4723	1.23	2457	4.80	7543	22
39	7393	3.55	4649	1.25	2745	4.80	7255	21
40	9.70 7606	3.55	9.93 4574	1.25	9.77 3033	4.80	10.22 6967	20
41	7819	3.55	4499	1.25	3321	4.78	6679	19
42	8032	3.55	4424	1.25	3608	4.80	6392	18
43	8245	3.55	4349	1.25	3896	4.80	6104	17
44	8458	3.53	4274	1.25	4184	4.78	5816	16
45	9.70 8670	3.53	9.93 4199	1.27	9.77 4471	4.80	10.22 5529	15
46	8882	3.53	4123	1.25	4759	4.78	5241	14
47	9094	3.53	4048	1.25	5046	4.78	4954	13
48	9306	3.53	3973	1.25	5333	4.80	4667	12
49	9518	3.53	3898	1.27	5621	4.78	4379	11
50	9.70 9730	3.52	9.93 3822	1.25	9.77 5908	4.78	10.22 4092	10
51	.70 9941	3.53	3747	1.27	6195	4.78	3805	9
52	.71 0153	3.52	3671	1.25	6482	4.77	3518	8
53	0364	3.52	3596	1.27	6768	4.78	3232	7
54	0575	3.52	3520	1.25	7055	4.78	2945	6
55	9.71 0786	3.52	9.93 3445	1.27	9.77 7342	4.77	10.22 2658	5
56	. 0997	3.52	3369	1.27	7628	4.78	2372	4
57	1208	3.52	3293	1.27	7915	4.77	2085	3
58	1419	3.50	3217	1.27	8201	4.78	1799	2
59	1629	3.50	3141	1.25	8488	4.77	1512	1
60	9.71 1839		9.93 3066		9.77 3774		10.22 1226	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.71 1839	3.52	9.93 3066	1.27	9.77 8774	4.77	10.22 1226	60
1	2050	3.50	2990	1.27	9060	4.77	0940	59
2	2260	3.48	2914	1.27	9346	4.77	0654	58
3	2469	3.50	2838	1.27	9632	4.77	0368	57
4	2679	3.50	2762	1.28	.77 9918	4.75	.22 0082	56
5	9.71 2889	3.48	9.93 2685	1.27	9.78 0203	4.77	10.21 9797	55
6	3098	3.50	2609	1.27	0489	4.77	9511	54
7	3308	3.48	2533	1.27	0775	4.75	9225	53
8	3517	3.48	2457	1.28	1060	4.77	8940	52
9	3726	3.48	2380	1.27	1346	4.75	8654	51
10	9.71 3935	3.48	9.93 2304	1.27	9.78 1631	4.75	10.21 8369	50
11	4144	3.47	2228	1.28	1916	4.75	8084	49
12	4352	3.48	2151	1.27	2201	4.75	7799	48
13	4561	3.47	2075	1.28	2486	4.75	7514	47
14	4769	3.48	1998	1.28	.2771	4.75	7229	46
15	9.71 4978	3.47	9.93 1921	1.27	9.78 3056	4.75	10.21 6944	45
16	5186	3.47	1845	1.28	3341	4.75	6659	44
17	5394	3.47	1768	1.28	3626	4.73	6374	43
18	5602	3.45	1691	1.28	3910	4.75	6090	42
19	5809	3.47	1614	1.28	4195	4.73	5805	41
20	9.71 6017	3.45	9.93 1537	1.28	9.78 4479	4.75	10.21 5521	40
21	6224	3.47	1460	1.28	4764	4.73	5236	39
22	6432	3.45	1383	1.28	5048	4.73	4952	38
23	6639	3.45	1306	1.28	5332	4.73	4668	37
24	6846	3.45	1229	1.28	5616	4.73	4384	36
25	9.71 7053	3.43	9.93 1152	1.28	9.78 5900	4.73	10.21 4100	35
26	7259	3.45	1075	1.28	6184	4.73	3816	34
27	7466	3.45	0998	1.28	6468	4.73	3532	33
28	7673	3.43	0921	1.30	6752	4.73	3248	32
29	7879	3.43	0843	1.28	7036	4.72	2964	31
30	9.71 8085	3.43	9.93 0766	1.30	9.78 7319	4.73	10.21 2681	30
31	.8291	3.43	0688	1.28	7603	4.72	2397	29
32	8497	3.43	0611	1.30	7886	4.73	2114	28
33	8703	3.43	0533	1.28	8170	4.72	1830	27
34	8909	3.42	0456	1.30	8453	4.72	1547	26
35	9.71 9114	3.43	9.93 0378	1.30	9.78 8736	4.72	10.21 1264	25
36	9320	3.42	0300	1.28	9019	4.72	0981	24
37	9525	3.42	0223	1.30	9302	4.72	0698	23
38	9730	3.42	0145	1.30	9585	4.72	0415	22
39	.71 9935	3.42	.93 0067	1.30	.78 9868	4.72	.21 0132	21
40	9.72 0140	3.42	9.92 9989	1.30	9.79 0151	4.72	10.20 9849	20
41	0345	3.40	9911	1.30	0434	4.70	9566	19
42	0549	3.42	9833	1.30	0716	4.72	9284	18
43	0754	3.40	9755	1.30	0999	4.70	9001	17
44	0958	3.40	9677	1.30	1281	4.70	8719	16
45	9.72 1162	3.40	9.92 9599	1.30	9.79 1563	4.72	10.20 8437	15
46	1366	3.40	9521	1.32	1846	4.70	8154	14
47	1570	3.40	9442	1.30	2128	4.70	7872	13
48	1774	3.40	9364	1.30	2410	4.70	7590	12
49	1978	3.38	9286	1.32	2692	4.70	7308	11
50	9.72 2181	3.40	9.92 9207	1.30	9.79 2974	4.70	10.20 7026	10
51	2385	3.38	9129	1.32	3256	4.70	6744	9
52	2588	3.38	9050	1.30	3538	4.68	6462	8
53	2791	3.38	8972	1.32	3819	4.70	6181	7
54	2994	3.38	8893	1.30	4101	4.70	5899	6
55	9.72 3197	3.38	9.92 8815	1.32	9.79 4383	4.68	10.20 5617	5
56	3400	3.38	8736	1.32	4664	4.70	5336	4
57	3603	3.37	8657	1.32	4946	4.68	5054	3
58	3805	3.37	8578	1.32	5227	4.68	4773	2
59	4007	3.38	8499	1.32	5508	4.68	4492	1
60	9.72 4210		9.92 8420		9.79 5789		10.20 4211	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.72 4210	3.37	9.92 8420	1.30	9.79 5789	4.68	10.20 4211	60
1	4412	3.37	8342	1.32	6070	4.68	3930	59
2	4614	3.37	8263	1.33	6351	4.68	3649	58
3	4816	3.35	8183	1.32	6632	4.68	3368	57
4	5017	3.37	8104	1.32	6913	4.68	3087	56
5	9.72 5219	3.35	9.92 8025	1.32	9.79 7194	4.67	10.20 2806	55
6	5420	3.37	7946	1.32	7474	4.68	2526	54
7	5622	3.35	7867	1.33	7755	4.68	2245	53
8	5823	3.35	7787	1.32	8036	4.67	1964	52
9	6024	3.35	7708	1.32	8316	4.67	1684	51
10	9.72 6225	3.35	9.92 7629	1.33	9.79 8596	4.68	10.20 1404	50
11	6426	3.33	7549	1.32	8877	4.67	1123	49
12	6626	3.35	7470	1.33	9157	4.67	0843	48
13	6827	3.33	7390	1.33	9437	4.67	0563	47
14	7027	3.35	7310	1.32	9717	4.67	0283	46
15	9.72 7228	3.33	9.92 7231	1.33	9.79 9997	4.67	10.20 0003	45
16	7428	3.33	7151	1.33	.80 0277	4.67	.19 9723	44
17	7628	3.33	7071	1.33	0557	4.65	9443	43
18	7828	3.32	6991	1.33	0836	4.67	9164	42
19	8027	3.33	6911	1.33	1116	4.67	8884	41
20	9.72 8227	3.33	9.92 6831	1.33	9.80 1396	4.65	10.19 8604	40
21	8427	3.32	6751	1.33	1675	4.67	8325	39
22	8626	3.32	6671	1.33	1955	4.65	8045	38
23	8825	3.32	6591	1.33	2234	4.65	7766	37
24	9024	3.32	6511	1.33	2513	4.65	7487	36
25	9.72 9223	3.32	9.92 6431	1.33	9.80 2792	4.67	10.19 7208	35
26	9422	3.32	6351	1.35	3072	4.65	6928	34
27	9621	3.32	6270	1.33	3351	4.65	6649	33
28	.72 9820	3.30	6190	1.33	3630	4.65	6370	32
29	.73 0018	3.32	6110	1.35	3909	4.63	6091	31
30	9.73 0217	3.30	9.92 6029	1.33	9.80 4187	4.65	10.19 5813	30
31	0415	3.30	5949	1.35	4466	4.65	5534	29
32	0613	3.30	5868	1.33	4745	4.63	5255	28
33	0811	3.30	5788	1.35	5023	4.65	4977	27
34	1009	3.28	5707	1.35	5302	4.63	4698	26
35	9.73 1206	3.30	9.92 5626	1.35	9.80 5580	4.65	10.19 4420	25
36	1404	3.30	5545	1.33	5859	4.63	4141	24
37	1602	3.28	5465	1.35	6137	4.63	3863	23
38	1799	3.28	5384	1.35	6415	4.63	3585	22
39	1996	3.28	5303	1.35	6693	4.63	3307	21
40	9.73 2193	3.28	9.92 5222	1.35	9.80 6971	4.63	10.19 3029	20
41	2390	3.28	5141	1.35	7249	4.63	2751	19
42	2587	3.28	5060	1.35	7527	4.63	2473	18
43	2784	3.27	4979	1.37	7805	4.63	2195	17
44	2980	3.28	4897	1.35	8083	4.63	1917	16
45	9.73 3177	3.27	9.92 4816	1.35	9.80 8361	4.62	10.19 1639	15
46	3373	3.27	4735	1.35	8638	4.63	1362	14
47	3569	3.27	4654	1.37	8916	4.62	1084	13
48	3765	3.27	4572	1.35	9193	4.63	0807	12
49	3961	3.27	4491	1.37	9471	4.62	0529	11
50	9.73 4157	3.27	9.92 4409	1.35	9.80 9748	4.62	10.19 0252	10
51	4353	3.27	4328	1.37	.81 0025	4.62	.18 9975	9
52	4549	3.25	4246	1.37	0302	4.63	9698	8
53	4744	3.25	4164	1.35	0580	4.62	9420	7
54	4939	3.27	4083	1.37	0857	4.62	9143	6
55	9.73 5135	3.25	9.92 4001	1.37	9.81 1134	4.60	10.18 8866	5
56	5330	3.25	3919	1.37	1410	4.62	8590	4
57	5525	3.23	3837	1.37	1687	4.62	8313	3
58	5719	3.25	3755	1.37	1964	4.62	8036	2
59	5914	3.25	3673	1.37	2241	4.60	7759	1
60	9.73 6109		9.92 3591		9.81 2517		10.18 7483	0

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.73 6109	3.23	9.92 3591	1.37	9.81 2517	4.62	10.18 7483	60
1	6303	3.25	3509	1.37	2794	4.60	7206	59
2	6498	3.23	3427	1.37	3070	4.62	6930	58
3	6692	3.23	3345	1.37	3347	4.60	6653	57
4	6886	3.23	3263	1.37	3623	4.60	6377	56
5	9.73 7080	3.23	9.92 3181	1.38	9.81 3899	4.62	10.18 6101	55
6	7274	3.22	3098	1.37	4176	4.60	5824	54
7	7467	3.23	3016	1.38	4452	4.60	5548	53
8	7661	3.23	2933	1.37	4728	4.60	5272	52
9	7855	3.22	2851	1.38	5004	4.60	4996	51
10	9.73 8048	3.22	9.92 2768	1.37	9.81 5280	4.58	10.18 4720	50
11	8241	3.22	2686	1.38	5555	4.60	4445	49
12	8434	3.22	2603	1.38	5831	4.60	4169	48
13	8627	3.22	2520	1.37	6107	4.58	3893	47
14	8820	3.22	2438	1.38	6382	4.60	3618	46
15	9.73 9013	3.22	9.92 2355	1.38	9.81 6658	4.58	10.18 3342	45
16	9206	3.20	2272	1.38	6933	4.60	3067	44
17	9398	3.20	2189	1.38	7209	4.58	2791	43
18	9590	3.22	2106	1.38	7484	4.58	2516	42
19	9783	3.20	2023	1.38	7759	4.60	2241	41
20	9.73 9975	3.20	9.92 1940	1.38	9.81 8035	4.58	10.18 1965	40
21	.74 0167	3.20	1857	1.38	8310	4.58	1690	39
22	0359	3.18	1774	1.38	8585	4.58	1415	38
23	0550	3.20	1691	1.40	8860	4.58	1140	37
24	0742	3.20	1607	1.38	9135	4.58	0865	36
25	9.74 0934	3.18	9.92 1524	1.38	9.81 9410	4.57	10.18 0590	35
26	1125	3.18	1441	1.40	9684	4.58	0316	34
27	1316	3.20	1357	1.38	.81 9959	4.58	.18 0041	33
28	1508	3.18	1274	1.40	.82 0234	4.57	.17 9766	32
29	1699	3.17	1190	1.38	0508	4.58	9492	31
30	9.74 1889	3.18	9.92 1107	1.40	9.82 0783	4.57	10.17 9217	30
31	2080	3.18	1023	1.40	1057	4.58	8943	29
32	2271	3.18	0939	1.38	1332	4.57	8668	28
33	2462	3.17	0856	1.40	1606	4.57	8394	27
34	2652	3.17	0772	1.40	1880	4.57	8120	26
35	9.74 2843	3.18	9.92 0688	1.40	9.82 2154	4.58	10.17 7846	25
36	3033	3.17	0604	1.40	2429	4.57	7571	24
37	3223	3.17	0520	1.40	2703	4.57	7297	23
38	3413	3.15	0436	1.40	2977	4.57	7023	22
39	3602	3.17	0352	1.40	3251	4.55	6749	21
40	9.74 3792	3.17	9.92 0268	1.40	9.82 3524	4.57	10.17 6476	20
41	3982	3.15	0184	1.42	3798	4.57	.6202	19
42	4171	3.17	0099	1.40	4072	4.55	5928	18
43	4361	3.15	.92 0015	1.40	4345	4.57	5655	17
44	4550	3.15	.91 9931	1.42	4619	4.57	5381	16
45	9.74 4739	3.15	9.91 9846	1.40	9.82 4893	4.55	10.17 5107	15
46	4928	3.15	9762	1.42	5166	4.55	4834	14
47	5117	3.15	9677	1.40	5439	4.57	4561	13
48	5306	3.13	9593	1.42	5713	4.55	4287	12
49	5494	3.15	9508	1.40	5986	4.55	4014	11
50	9.74 5683	3.13	9.91 9424	1.42	9.82 6254	4.55	10.17 3741	10
51	5871	3.15	9339	1.42	6532	4.55	3468	9
52	6060	3.13	9254	1.42	6805	4.55	3195	8
53	6248	3.13	9169	1.40	7078	4.55	2922	7
54	6436	3.13	9085	1.42	7351	4.55	2649	6
55	9.74 6624	3.13	9.91 9000	1.42	9.82 7624	4.55	10.17 2376	5
56	6812	3.12	8915	1.42	7897	4.55	2103	4
57	6999	3.13	8830	1.42	8170	4.53	1830	3
58	7187	3.12	8745	1.42	8442	4.55	1558	2
59	7374	3.13	8659	1.43	8715	4.53	1285	1
60	9.74 7562		9.91 8574	1.42	9.82 8987		10.17 1013	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″	Cosine.	D.1″	Tang.	D.1″	Cotang.	′
0	9.74 7562	3.12	9.91 8574	1.42	9.82 8987	4.55	10.17 1013	60
1	7749	3.12	8489	1.42	9260	4.53	0740	59
2	7936	3.12	8404	1.43	9532	4.55	0468	58
3	8123	3.12	8318	1.42	.82 9805	4.53	.17 0195	57
4	8310	3.12	8233	1.43	.83 0077	4.53	.16 9923	56
5	9.74 8497	3.10	9.91 8147	1.42	9.83 0349	4.53	10.16 9651	55
6	8683	3.12	8062	1.43	0621	4.53	9379	54
7	8870	3.10	7976	1.42	0893	4.53	9107	53
8	9056	3.12	7891	1.43	1165	4.53	8835	52
9	9243	3.10	7805	1.43	1437	4.53	8563	51
10	9.74 9429	3.10	9.91 7719	1.42	9.83 1709	4.53	10.16 8291	50
11	9615	3.10	7634	1.43	1981	4.53	8019	49
12	9801	3.10	7548	1.43	2253	4.53	7747	48
13	.74 9987	3.08	7462	1.43	2525	4.52	7475	47
14	.75 0172	3.10	7376	1.43	2796	4.53	7204	46
15	9.75 0358	3.08	9.91 7290	1.43	9.83 3068	4.52	10.16 6932	45
16	0543	3.10	7204	1.43	3339	4.53	6661	44
17	0729	3.08	7118	1.43	3611	4.52	6389	43
18	0914	3.08	7032	1.43	3882	4.53	6118	42
19	1099	3.08	6946	1.45	4154	4.52	5846	41
20	9.75 1284	3.08	9.91 6859	1.43	9.83 4425	4.52	10.16 5575	40
21	1469	3.08	6773	1.43	4696	4.52	5304	39
22	1654	3.08	6687	1.45	4967	4.52	5033	38
23	1839	3.07	6600	1.43	5238	4.52	4762	37
24	2023	3.08	6514	1.45	5509	4.52	4491	36
25	9.75 2208	3.07	9.91 6427	1.43	9.83 5780	4.52	10.16 4220	35
26	2392	3.07	6341	1.45	6051	4.52	3949	34
27	2576	3.07	6254	1.45	6322	4.52	3678	33
28	2760	3.07	6167	1.43	6593	4.52	3407	32
29	2944	3.07	6081	1.45	6864	4.50	3136	31
30	9.75 3128	3.07	9.91 5994	1.45	9.83 7134	4.52	10.16 2866	30
31	3312	3.05	5907	1.45	7405	4.50	2595	29
32	3495	3.07	5820	1.45	7675	4.52	2325	28
33	3679	3.07	5733	1.45	7946	4.50	2054	27
34	3862	3.07	5646	1.45	8216	4.52	1784	26
35	9.75 4046	3.05	9.91 5559	1.45	9.83 8487	4.50	10.16 1513	25
36	4229	3.05	5472	1.45	8757	4.50	1243	24
37	4412	3.05	5385	1.47	9027	4.50	0973	23
38	4595	3.05	5297	1.45	9297	4.52	0703	22
39	4778	3.03	5210	1.45	9568	4.50	0432	21
40	9.75 4960	3.05	9.91 5123	1.47	9.83 9838	4.50	10.16 0162	20
41	5143	3.05	5035	1.45	.84 0108	4.50	.15 9892	19
42	5326	3.03	4948	1.47	0378	4.50	9622	18
43	5508	3.03	4860	1.45	0648	4.48	9352	17
44	5690	3.03	4773	1.47	0917	4.50	9083	16
45	9.75 5872	3.03	9.91 4685	1.45	9.84 1187	4.50	10.15 8813	15
46	6054	3.03	4598	1.47	1457	4.50	8543	14
47	6236	3.03	4510	1.47	1727	4.48	8273	13
48	6418	3.03	4422	1.47	1996	4.50	8004	12
49	6600	3.03	4334	1.47	2266	4.48	7734	11
50	9.75 6782	3.02	9.91 4246	1.47	9.84 2535	4.50	10.15 7465	10
51	6963	3.02	4158	1.47	2805	4.48	7195	9
52	7144	3.03	4070	1.47	3074	4.48	6926	8
53	7326	3.02	3982	1.47	3343	4.48	6657	7
54	7507	3.02	3894	1.47	3612	4.50	6388	6
55	9.75 7688	3.02	9.91 3806	1.47	9.84 3882	4.48	10.15 6118	5
56	7869	3.02	3718	1.47	4151	4.48	5849	4
57	8050	3.00	3630	1.48	4420	4.48	5580	3
58	8230	3.02	3541	1.47	4689	4.48	5311	2
59	8411	3.00	3453	1.47	4958	4.48	5042	1
60	9.75 8591		9.91 3365		9.84 5227		10.15 4773	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
9.75 8591	3.02	9.91 3365	1.48	9.84 5227	4.48	10.15 4773	60
8772	3.00	3276	1.48	5496	4.47	4504	59
8952	3.00	3187	1.47	5764	4.48	4236	58
9132	3.00	3099	1.48	6033	4.48	3967	57
9312	3.00	3010	1.47	6302	4.47	3698	56
9.75 9492	3.00	9.91 2922	1.48	9.84 6570	4.48	10.15 3430	55
9672	3.00	2833	1.48	6839	4.48	3161	54
.75 9852	2.98	2744	1.48	7108	4.47	2892	53
.76 0031	3.00	2655	1.48	7376	4.47	2624	52
0211	2.98	2566	1.48	7644	4.48	2356	51
9.76 0390	2.98	9.91 2477	1.48	9.84 7913	4.47	10.15 2087	50
0569	2.98	2388	1.48	8181	4.47	1819	49
0748	2.98	2299	1.48	8449	4.47	1551	48
0927	2.98	2210	1.48	8717	4.48	1283	47
1106	2.98	2121	1.50	8986	4.47	1014	46
9.76 1285	2.98	9.91 2031	1.48	9.84 9254	4.47	10.15 0746	45
1464	2.97	1942	1.48	9522	4.47	0478	44
1642	2.98	1853	1.50	.84 9790	4.45	.15 0210	43
1821	2.97	1763	1.48	.85 0057	4.47	.14 9943	42
1999	2.97	1674	1.50	0325	4.47	9675	41
9.76 2177	2.98	9.91 1584	1.48	9.85 0593	4.47	10.14 9407	40
2356	2.97	1495	1.50	0861	4.47	9139	39
2534	2.97	1405	1.50	1129	4.45	8871	38
2712	2.95	1315	1.48	1396	4.47	8604	37
2889	2.97	1226	1.50	1664	4.45	8336	36
9.76 3067	2.97	9.91 1136	1.50	9.85 1931	4.47	10.14 8069	35
3245	2.95	1046	1.50	2199	4.45	7801	34
3422	2.97	0956	1.50	2466	4.45	7534	33
3600	2.95	0866	1.50	2733	4.47	7267	32
3777	2.95	0776	1.50	3001	4.45	6999	31
9.76 3954	2.95	9.91 0686	1.50	9.85 3268	4.45	10.14 6732	30
4131	2.95	0596	1.50	3535	4.45	6465	29
4308	2.95	0506	1.52	3802	4.45	6198	28
4485	2.95	0415	1.50	4069	4.45	5931	27
4662	2.93	0325	1.50	4336	4.45	5664	26
9.76 4838	2.95	9.91 0235	1.52	9.85 4603	4.45	10.14 5397	25
5015	2.93	0144	1.50	4870	4.45	5130	24
5191	2.93	.91 0054	1.52	5137	4.45	4863	23
5367	2.95	.90 9963	1.50	5404	4.45	4596	22
5544	2.93	9873	1.52	5671	4.45	4329	21
9.76 5720	2.93	9.90 9782	1.52	9.85 5938	4.43	10.14 4062	20
5896	2.93	9691	1.50	6204	4.45	3796	19
6072	2.92	9601	1.52	6471	4.43	3529	18
6247	2.93	9510	1.52	6737	4.45	3263	17
6423	2.92	9419	1.52	7004	4.43	2996	16
9.76 6598	2.93	9.90 9328	1.52	9.85 7270	4.45	10.14 2730	15
6774	2.92	9237	1.52	7537	4.43	2463	14
6949	2.92	9146	1.52	7803	4.43	2197	13
7124	2.93	9055	1.52	8069	4.45	1931	12
7300	2.92	8964	1.52	8336	4.43	1664	11
9.76 7475	2.90	9.90 8873	1.53	9.85 8602	4.43	10.14 1398	10
7649	2.92	8781	1.52	8868	4.43	1132	9
7824	2.92	8690	1.52	9134	4.43	0866	8
7999	2.90	8599	1.53	9400	4.43	.0600	7
8173	2.92	8507	1.52	9666	4.43	0334	6
9.76 8348	2.90	9.90 8416	1.53	9.85 9932	4.43	10.14 0068	5
8522	2.92	8324	1.52	.86 0198	4.43	.13 9802	4
8697	2.90	8233	1.53	0464	4.43	9536	3
8871	2.90	8141	1.53	0730	4.42	9270	2
9045	2.90	8049	1.53	0995	4.43	9005	1
9.76 9219		9.90 7958	1.52	9.86 1261		10.13 8739	0
Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.76 9219	2.90	9.90 7958	1.53	9.86 1261	4.43	10.13 8739	60
1	9393	2.88	7866	1.53	1527	4.42	8473	59
2	9566	2.90	7774	1.53	1792	4.43	8208	58
3	9740	2.88	7682	1.53	2058	4.42	7942	57
4	.76 9913	2.90	7590	1.53	2323	4.43	7677	56
5	9.77 0087	2.88	9.90 7498	1.53	9.86 2589	4.42	10.13 7411	55
6	0260	2.88	7406	1.53	2854	4.42	7146	54
7	0433	2.88	7314	1.53	3119	4.43	6881	53
8	0606	2.88	7222	1.55	3385	4.42	6615	52
9	0779	2.88	7129	1.53	3650	4.42	6350	51
10	9.77 0952	2.88	9.90 7037	1.53	9.86 3915	4.42	10.13 6085	50
11	1125	2.88	6945	1.55	4180	4.42	5820	49
12	1298	2.87	6852	1.53	4445	4.42	5555	48
13	1470	2.88	6760	1.55	4710	4.42	5290	47
14	1643	2.87	6667	1.53	4975	4.42	5025	46
15	9.77 0815	2.87	9.90 6575	1.55	9.86 5240	4.42	10.13 4760	45
16	1987	2.87	6482	1.55	5505	4.42	4495	44
17	2159	2.87	6389	1.55	5770	4.42	4230	43
18	2331	2.87	6296	1.53	6035	4.42	3965	42
19	2503	2.87	6204	1.55	6300	4.40	3700	41
20	9.77 2675	2.87	9.90 6111	1.55	9.86 6564	4.42	10.13 3436	40
21	2847	2.85	6018	1.55	6829	4.42	3171	39
22	3018	2.87	5925	1.55	7094	4.40	2906	38
23	3190	2.85	5832	1.55	7358	4.42	2642	37
24	3361	2.87	5739	1.57	7623	4.40	2377	36
25	9.77 3533	2.85	9.90 5645	1.55	9.86 7887	4.42	10.13 2113	35
26	3704	2.85	5552	1.55	8152	4.40	1848	34
27	3875	2.85	5459	1.55	8416	4.40	1584	33
28	4046	2.85	5366	1.57	8680	4.42	1320	32
29	4217	2.85	5272	1.55	8945	4.40	1055	31
30	9.77 4388	2.83	9.90 5179	1.57	9.86 9209	4.40	10.13 0791	30
31	4558	2.85	5085	1.55	9473	4.40	0527	29
32	4729	2.83	4992	1.57	.86 9737	4.40	.13 0263	28
33	4899	2.85	4898	1.57	.87 0001	4.40	.12 9999	27
34	5070	2.83	4804	1.55	0265	4.40	9735	26
35	9.77 5240	2.83	9.90 4711	1.57	9.87 0529	4.40	10.12 9471	25
36	5410	2.83	4617	1.57	0793	4.40	9207	24
37	5580	2.83	4523	1.57	1057	4.40	8943	23
38	5750	2.83	4429	1.57	1321	4.40	8679	22
39	5920	2.83	4335	1.57	1585	4.40	8415	21
40	9.77 6090	2.82	9.90 4241	1.57	9.87 1849	4.38	10.12 8151	20
41	6259	2.83	4147	1.57	2112	4.40	7888	19
42	6429	2.82	4053	1.57	2376	4.40	7624	18
43	6598	2.83	3959	1.58	3640	4.38	7360	17
44	6768	2.82	3864	1.57	2903	4.40	7097	16
45	9.77 6937	2.82	9.90 3770	1.57	9.87 3167	4.38	10.12 6833	15
46	7106	2.82	3676	1.58	3430	4.40	6570	14
47	7275	2.82	3581	1.57	3694	4.38	6306	13
48	7444	2.82	3487	1.58	3957	4.38	6043	12
49	7613	2.80	3392	1.57	4220	4.40	5780	11
50	9.77 7781	2.82	9.90 3298	1.58	9.87 4484	4.38	10.12 5516	10
51	7950	2.82	3203	1.58	4747	4.38	5253	9
52	8119	2.80	3108	1.57	5010	4.38	4990	8
53	8287	2.80	3014	1.58	5273	4.40	4727	7
54	8455	2.82	2919	1.58	5537	4.38	4463	6
55	9.77 8624	2.80	9.90 2824	1.58	9.87 5800	4.38	10.12 4200	5
56	8792	2.80	2729	1.58	6063	4.38	3937	4
57	8960	2.80	2634	1.58	6326	4.38	3674	3
58	9128	2.78	2539	1.58	6589	4.38	3411	2
59	9295	2.80	2444	1.58	6852	4.37	3148	1
60	9.77 9463		9.90 2349		9.87 7114		10.12 2886	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

TABLE XII.—*Continued.* 142°

Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
9.77 9463	2.80	9.90 2349	1.60	9.87 7114	4.38	10.12 2886	60
9631	2.78	2253	1.58	7377	4.38	2623	59
9798	2.80	2158	1.58	7640	4.38	2360	58
.77 9966	2.78	2063	1.60	7903	4.37	2097	57
.78 0133	2.78	1967	1.58	8165	4.38	1835	56
9.78 0300	2.78	9.90 1872	1.60	9.87 8428	4.38	10.12 1572	55
0467	2.78	1776	1.58	8691	4.37	1309	54
0634	2.78	1681	1.60	8953	4.38	1047	53
0801	2.78	1585	1.58	9216	4.37	0784	52
0968	2.77	1490	1.60	9478	4.38	0522	51
9.78 1134	2.78	9.90 1394	1.60	9.87 9741	4.37	10.12 0259	50
1301	2.78	1298	1.60	.88 0003	4.37	.11 9997	49
1468	2.77	1202	1.60	0265	4.38	9735	48
1634	2.77	1106	1.60	0528	4.37	9472	47
1800	2.77	1010	1.60	0790	4.37	9210	46
9.78 1966	2.77	9.90 0914	1.60	9.88 1052	4.37	10.11 8948	45
2132	2.77	0818	1.60	1314	4.38	8686	44
2298	2.77	0722	1.60	1577	4.37	8423	43
2464	2.77	0626	1.62	1839	4.37	8161	42
2630	2.77	0529	1.60	2101	4.37	7899	41
9.78 2796	2.75	9.90 0433	1.60	9.88 2363	4.37	10.11 7637	40
2961	2.77	0337	1.62	2625	4.37	7375	39
3127	2.75	0240	1.60	2887	4.35	7113	38
3292	2.77	0144	1.62	3148	4.37	6852	37
3458	2.75	.90 0047	1.60	3410	4.37	6590	36
9.78 3623	2.75	9.89 9951	1.62	9.88 3672	4.37	10.11 6328	35
3788	2.75	9854	1.62	3934	4.37	6066	34
3953	2.75	9757	1.62	4196	4.35	5804	33
4118	2.73	9660	1.60	4457	4.37	5543	32
4282	2.75	9564	1.62	4719	4.35	5281	31
9.78 4447	2.75	9.89 9467	1.62	9.88 4980	4.37	10.11 5020	30
4612	2.73	9370	1.62	5242	4.37	4758	29
4776	2.75	9273	1.62	5504	4.35	4496	28
4941	2.73	9176	1.63	5765	4.35	4235	27
5105	2.73	9078	1.62	6026	4.37	3974	26
9.78 5269	2.73	9.89 8981	1.62	9.88 6288	4.35	10.11 3712	25
5433	2.73	8884	1.62	6549	4.37	3451	24
5597	2.73	8787	1.63	6811	4.35	3189	23
5761	2.73	8689	1.62	7072	4.35	2928	22
5925	2.73	8592	1.63	7333	4.35	2667	21
9.78 6089	2.72	9.89 8494	1.62	9.88 7594	4.35	10.11 2406	20
6252	2.73	8397	1.63	7855	4.35	2145	19
6416	2.72	8299	1.62	8116	4.37	1884	18
6579	2.72	8202	1.63	8378	4.35	1622	17
6742	2.73	8104	1.63	8639	4.35	1361	16
9.78 6906	2.72	9.89 8006	1.63	9.88 8900	4.35	10.11 1100	15
7069	2.72	7908	1.63	9161	4.33	0839	14
7232	2.72	7810	1.63	9421	4.35	0579	13
7395	2.70	7712	1.63	9682	4.35	0318	12
7557	2.72	7614	1.63	.88 9943	4.35	.11 0057	11
9.78 7720	2.72	9.89 7516	1.63	9.89 0204	4.35	10.10 9796	10
7883	2.70	7418	1.63	0465	4.33	9535	9
8045	2.72	7320	1.63	0725	4.35	9275	8
8208	2.70	7222	1.65	0986	4.35	9014	7
8370	2.70	7123	1.63	1247	4.33	8753	6
9.78 8532	2.70	9.89 7025	1.65	9.89 1507	4.35	10.10 8493	5
8694	2.70	6926	1.63	1768	4.33	8232	4
8856	2.70	6828	1.65	2028	4.35	7972	3
9018	2.70	6729	1.63	2289	4.33	7711	2
9180	2.70	6631	1.65	2549	4.35	7451	1
9.78 9342		9.89 6532		9.89 2810		10.10 7190	0
Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.78 9342	2.70	9.89 6532	1.65	9.89 2810	4.33	10.10 7190	60
1	9504	2.68	6433	1.63	3070	4.35	6930	59
2	9665	2.70	6335	1.65	3331	4.33	6669	58
3	9827	2.68	6236	1.65	3591	4.33	6409	57
4	9988	2.68	6137	1.65	3851	4.33	6149	56
5	9.79 0149	2.68	9.89 6038	1.65	9.89 4111	4.35	10.10 5889	55
6	0310	2.68	5939	1.65	4372	4.33	5628	54
7	0471	2.68	5840	1.65	4632	4.33	5368	53
8	0632	2.68	5741	1.67	4892	4.33	5108	52
9	0793	2.68	5641	1.65	5152	4.33	4848	51
10	9.79 0954	2.68	9.89 5542	1.65	9.89 5412	4.33	10.10 4588	50
11	1115	2.67	5443	1.67	5672	4.33	4328	49
12	1275	2.68	5343	1.65	5932	4.33	4068	48
13	1436	2.67	5244	1.65	6192	4.33	3808	47
14	1596	2.68	5145	1.67	6452	4.33	3548	46
15	9.79 1757	2.67	9.89 5045	1.67	9.89 6712	4.32	10.10 3288	45
16	1917	2.67	4945	1.65	6971	4.33	3029	44
17	2077	2.67	4846	1.67	7231	4.33	2769	43
18	2237	2.67	4746	1.67	7491	4.33	2509	42
19	2397	2.67	4646	1.67	7751	4.32	2249	41
20	9.79 2557	2.65	9.89 4546	1.67	9.89 8010	4.33	10.10 1990	40
21	2716	2.67	4446	1.67	8270	4.33	1730	39
22	2876	2.65	4346	1.67	8530	4.32	1470	38
23	3035	2.67	4246	1.67	8789	4.33	1211	37
24	3195	2.65	4146	1.67	9049	4.32	0951	36
25	9.79 3354	2.67	9.89 4046	1.67	9.89 9308	4.33	10.10 0692	35
26	3514	2.65	3946	1.67	9568	4.32	0432	34
27	3673	2.65	3846	1.68	.89 9827	4.33	.10 0173	33
28	3832	2.65	3745	1.67	.90 0087	4.32	.09 9913	32
29	3991	2.65	3645	1.68	0346	4.32	9654	31
30	9.79 4150	2.63	9.89 3544	1.67	9.90 0605	4.32	10.09 9395	30
31	4308	2.65	3444	1.68	0864	4.33	9136	29
32	4467	2.65	3343	1.67	1124	4.32	8876	28
33	4626	2.63	3243	1.68	1383	4.32	8617	27
34	4784	2.63	3142	1.68	1642	4.32	8358	26
35	9.79 4943	2.65	9.89 3041	1.68	9.90 1901	4.32	10.09 8099	25
36	5101	2.63	2940	1.68	2160	4.33	7840	24
37	5259	2.63	2839	1.67	2420	4.32	7580	23
38	5417	2.63	2739	1.68	2679	4.32	7321	22
39	5575	2.63	2638	1.70	2938	4.32	7062	21
40	9.79 5733	2.63	9.89 2536	1.68	9.90 3197	4.32	10.09 6803	20
41	5891	2.63	2435	1.68	3456	4.30	6544	19
42	6049	2.62	2334	1.68	3714	4.32	6286	18
43	6206	2.63	2233	1.68	3973	4.32	6027	17
44	6364	2.62	2132	1.70	4232	4.32	5768	16
45	9.79 6521	2.63	9.89 2030	1.68	9.90 4491	4.32	10.09 5509	15
46	6679	2.62	1929	1.70	4750	4.30	5250	14
47	6836	2.62	1827	1.68	5008	4.32	4992	13
48	6993	2.62	1726	1.70	5267	4.32	4733	12
49	7150	2.62	1624	1.68	5526	4.32	4474	11
50	9.79 7307	2.62	9.89 1523	1.70	9.90 5785	4.30	10.09 4215	10
51	7464	2.62	1421	1.70	6043	4.32	3957	9
52	7621	2.60	1319	1.70	6302	4.30	3698	8
53	7777	2.62	1217	1.70	6560	4.32	3440	7
54	7934	2.62	1115	1.70	6819	4.30	3181	6
55	9.79 8091	2.60	9.89 1013	1.70	9.90 7077	4.32	10.09 2923	5
56	8247	2.60	0911	1.70	7336	4.30	2664	4
57	8403	2.62	0809	1.70	7594	4.32	2406	3
58	8560	2.60	0707	1.70	7853	4.30	2147	2
59	8716	2.60	0605	1.70	8111	4.30	1889	1
60	9.79 8872	2.60	9.89 0503	1.70	9.90 8369	4.30	10.09 1631	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.79 8672	2.60	9.89 0503	1.72	9.90 8169	4.32	10.09 1831	60
1	9028	2.60	0400	1.70	8628	4.30	1372	59
2	9184	2.58	0298	1.72	8886	4.30	1114	58
3	9339	2.60	0195	1.70	9144	4.30	0856	57
4	9495	2.60	.89 0093	1.72	9402	4.30	0598	56
5	9.79 9651	2.58	9.88 9990	1.70	9.90 9660	4.30	10.09 0340	55
6	9806	2.60	9888	1.72	.90 9918	4.32	.09 0082	54
7	.79 9962	2.58	9785	1.72	.91 0177	4.30	.08 9823	53
8	.80 0117	2.58	9682	1.72	0435	4.30	9565	52
9	0272	2.58	9579	1.70	0693	4.30	9307	51
10	9.80 0427	2.58	9.88 9477	1.72	9.91 0951	4.30	10.08 9049	50
11	0582	2.58	9374	1.72	1209	4.30	8791	49
12	0737	2.58	9271	1.72	1467	4.30	8533	48
13	0892	2.58	9168	1.73	1725	4.28	8275	47
14	1047	2.57	9064	1.72	1982	4.30	8018	46
15	9.80 1201	2.58	9.88 8961	1.72	9.91 2240	4.30	10.08 7760	45
16	1356	2.58	8858	1.72	2498	4.30	7502	44
17	1511	2.57	8755	1.73	2756	4.30	7244	43
18	1665	2.57	8651	1.72	3014	4.28	6986	42
19	1819	2.57	8548	1.73	3271	4.30	6729	41
20	9.80 1973	2.58	9.88 8444	1.72	9.91 3529	4.30	10.08 6471	40
21	2128	2.57	8341	1.73	3787	4.28	6213	39
22	2282	2.57	8237	1.72	4044	4.30	5956	38
23	2436	2.55	8134	1.73	4302	4.30	5698	37
24	2589	2.57	8030	1.73	4560	4.28	5440	36
25	9.80 2743	2.57	9.88 7926	1.73	9.91 4817	4.30	10.08 5183	35
26	2897	2.55	7822	1.73	5075	4.28	4925	34
27	3050	2.57	7718	1.73	5332	4.30	4668	33
28	3204	2.55	7614	1.73	5590	4.28	4410	32
29	3357	2.57	7510	1.73	5847	4.28	4153	31
30	9.80 3511	2.55	9.88 7406	1.73	9.91 6104	4.30	10.08 3896	30
31	3664	2.55	7302	1.73	6362	4.28	3638	29
32	3817	2.55	7198	1.75	6619	4.30	3381	28
33	3970	2.55	7093	1.73	6877	4.28	3123	27
34	4123	2.55	6989	1.73	7134	4.28	2866	26
35	9.80 4276	2.55	9.88 6885	1.75	9.91 7391	4.28	10.08 2609	25
36	4428	2.55	6780	1.73	7648	4.30	2352	24
37	4581	2.55	6676	1.75	7906	4.28	2094	23
38	4734	2.53	6571	1.75	8163	4.28	1837	22
39	4886	2.55	6466	1.73	8420	4.28	1580	21
40	9.80 5039	2.53	9.88 6362	1.75	9.91 8677	4.28	10.08 1323	20
41	5191	2.53	6257	1.75	8934	4.28	1066	19
42	5343	2.53	6152	1.75	9191	4.28	0809	18
43	5495	2.53	6047	1.75	9448	4.28	0552	17
44	5647	2.53	5942	1.75	9705	4.28	0295	16
45	9.80 5798	2.53	9.88 5837	1.75	9.91 9962	4.28	10.08 0038	15
46	5951	2.53	5732	1.75	.92 0219	4.28	.07 9781	14
47	6103	2.52	5627	1.75	0476	4.28	9524	13
48	6254	2.53	5522	1.77	0733	4.28	9267	12
49	6406	2.52	5416	1.75	0990	4.28	9010	11
50	9.80 6557	2.53	9.88 5311	1.77	9.92 1247	4.27	10.07 8753	10
51	6709	2.52	5205	1.75	1503	4.28	8497	9
52	6860	2.52	5100	1.77	1760	4.28	8240	8
53	7011	2.53	4994	1.75	2017	4.28	7983	7
54	7163	2.52	4889	1.77	2274	4.27	7726	6
55	9.80 7314	2.52	9.88 4783	1.77	9.92 2530	4.28	10.07 7470	5
56	7465	2.50	4677	1.75	2787	4.28	7213	4
57	7615	2.52	4572	1.77	3044	4.27	6956	3
58	7766	2.52	4466	1.77	3300	4.28	6700	2
59	7917	2.50	4360	1.77	3557	4.28	6443	1
60	9.80 8067		9.88 4254		9.92 3814		10.07 6186	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.80 8067	2.52	9.88 4254	1.77	9.92 3814	4.27	10.07 6186	60
1	8218	2.50	4148	1.77	4070	4.28	5930	59
2	8368	2.52	4042	1.77	4327	4.27	5673	58
3	8519	2.50	3936	1.78	4583	4.28	5417	57
4	8669	2.50	3829	1.77	4840	4.27	5160	56
5	9.80 8819	2.50	9.88 3723	1.77	9.92 5096	4.27	10.07 4904	55
6	8969	2.50	3617	1.78	5352	4.28	4648	54
7	9119	2.50	3510	1.77	5609	4.27	4391	53
8	9269	2.50	3404	1.78	5865	4.28	4135	52
9	9419	2.50	3297	1.77	6122	4.27	3878	51
10	9.80 9569	2.48	9.88 3191	1.78	9.92 6378	4.27	10.07 3622	50
11	9718	2.50	3084	1.78	6634	4.27	3366	49
12	.80 9868	2.48	2977	1.77	6890	4.28	3110	48
13	.81 0017	2.50	2871	1.78	7147	4.27	2853	47
14	0167	2.48	2764	1.78	7403	4.27	2597	46
15	9.81 0316	2.48	9.88 2657	1.78	9.92 7659	4.27	10.07 2341	45
16	0465	2.48	2550	1.78	7915	4.27	2085	44
17	0614	2.48	2443	1.78	8171	4.27	1829	43
18	0763	2.48	2336	1.78	8427	4.28	1573	42
19	0912	2.48	2229	1.80	8684	4.27	1316	41
20	9.81 1061	2.48	9.88 2121	1.78	9.92 8940	4.27	10.07 1060	40
21	1210	2.47	2014	1.78	9196	4.27	0804	39
22	1358	2.48	1907	1.80	9452	4.27	0548	38
23	1507	2.47	1799	1.78	9708	4.27	0292	37
24	1655	2.48	1692	1.80	.92 9964	4.27	.07 0036	36
25	9.81 1804	2.47	9.88 1584	1.78	9.93 0220	4.25	10.06 9780	35
26	1952	2.47	1477	1.80	0475	4.27	9525	34
27	2100	2.47	1369	1.80	0731	4.27	9269	33
28	2248	2.47	1261	1.80	0987	4.27	9013	32
29	2396	2.47	1153	1.78	1243	4.27	8757	31
30	9.81 2544	2.47	9.88 1046	1.80	9.93 1499	4.27	10.06 8501	30
31	2692	2.47	0938	1.80	1755	4.25	8245	29
32	2840	2.47	0830	1.80	2010	4.27	7990	28
33	2988	2.45	0722	1.82	2266	4.27	7734	27
34	3135	2.47	0613	1.80	2522	4.27	7478	26
35	9.81 3283	2.45	9.88 0505	1.80	9.93 2778	4.25	10.06 7222	25
36	3430	2.47	0397	1.80	3033	4.27	6967	24
37	3578	2.45	0289	1.82	3289	4.27	6711	23
38	3725	2.45	0180	1.80	3545	4.25	6455	22
39	3872	2.45	.88 0072	1.82	3800	4.27	6200	21
40	9.81 4019	2.45	9.87 9963	1.80	9.93 4056	4.25	10.06 5944	20
41	4166	2.45	9855	1.82	4311	4.27	5689	19
42	4313	2.45	9746	1.82	4567	4.25	5433	18
43	4460	2.45	9637	1.80	4822	4.27	5178	17
44	4607	2.43	9529	1.82	5078	4.25	4922	16
45	9.81 4753	2.45	9.87 9420	1.82	9.93 5333	4.27	10.06 4667	15
46	4900	2.43	9311	1.82	5589	4.25	4411	14
47	5046	2.45	9202	1.82	5844	4.27	4156	13
48	5193	2.43	9093	1.82	6100	4.25	3900	12
49	5339	2.43	8984	1.82	6355	4.27	3645	11
50	9.81 5485	2.45	9.87 8875	1.82	9.93 6611	4.25	10.06 3389	10
51	5632	2.43	8766	1.83	6866	4.25	3134	9
52	5778	2.43	8656	1.82	7121	4.27	2879	8
53	5924	2.42	8547	1.82	7377	4.25	2623	7
54	6069	2.43	8438	1.83	7632	4.25	2368	6
55	9.81 6215	2.43	9.87 8328	1.82	9.93 7887	4.25	10.06 2113	5
56	6361	2.43	8219	1.83	8142	4.27	1858	4
57	6507	2.42	8109	1.83	8398	4.25	1602	3
58	6652	2.43	7999	1.82	8653	4.25	1347	2
59	6798	2.42	7890	1.83	8908	4.25	1092	1
60	9.81 6943		9.87 7780		9.93 9163		10.06 0837	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.81 6943	2.42	9.87 7780	1.83	9.93 9163	4.25	10.06 0837	60
1	7088	2.42	7670	1.83	9418	4.25	0582	59
2	7233	2.43	7560	1.83	9673	4.25	0327	58
3	7379	2.42	7450	1.83	.93 9928	4.25	.06 0072	57
4	7524	2.40	7340	1.83	.94 0183	4.27	.05 9817	56
5	9.81 7668	2.42	9.87 7230	1.83	9.94 0439	4.25	10.05 9561	55
6	7813	2.42	7120	1.83	0694	4.25	9306	54
7	7958	2.42	7010	1.85	0949	4.25	9051	53
8	8103	2.40	6899	1.83	1204	4.25	8796	52
9	8247	2.42	6789	1.85	1459	4.23	8541	51
10	9.81 8392	2.40	9.87 6678	1.83	9.94 1713	4.25	10.05 8287	50
11	8536	2.42	6568	1.85	1968	4.25	8032	49
12	8681	2.40	6457	1.83	2223	4.25	7777	48
13	8825	2.40	6347	1.85	2478	4.25	7522	47
14	8969	2.40	6236	1.85	2733	4.25	7267	46
15	9.81 9113	2.40	9.87 6125	1.85	9.94 2988	4.25	10.05 7012	45
16	9257	2.40	6014	1.83	3243	4.25	6757	44
17	9401	2.40	5904	1.85	3498	4.23	6502	43
18	9545	2.40	5793	1.85	3752	4.25	6248	42
19	9689	2.38	5682	1.85	4007	4.25	5993	41
20	9.81 9832	2.40	9.87 5571	1.87	9.94 4262	4.25	10.05 5738	40
21	.81 9976	2.40	5459	1.85	4517	4.23	5483	39
22	.82 0120	2.38	5348	1.85	4771	4.25	5229	38
23	0263	2.38	5237	1.85	5026	4.25	4974	37
24	0406	2.40	5126	1.87	5281	4.23	4719	36
25	9.82 0550	2.38	9.87 5014	1.85	9.94 5535	4.25	10.05 4465	35
26	0693	2.38	4903	1.87	5790	4.25	4210	34
27	0836	2.38	4791	1.85	6045	4.23	3955	33
28	0979	2.38	4680	1.87	6299	4.25	3701	32
29	1122	2.38	4568	1.87	6554	4.23	3446	31
30	9.82 1265	2.37	9.87 4456	1.87	9.94 6808	4.25	10.05 3192	30
31	1407	2.38	4344	1.87	7063	4.25	2937	29
32	1550	2.38	4232	1.85	7318	4.23	2682	28
33	1693	2.37	4121	1.87	7572	4.25	2428	27
34	1835	2.37	4009	1.88	7827	4.23	2173	26
35	9.82 1977	2.38	9.87 3896	1.87	9.94 8081	4.23	10.05 1919	25
36	2120	2.37	3784	1.87	8335	4.25	1665	24
37	2262	2.37	3672	1.87	8590	4.23	1410	23
38	2404	2.37	3560	1.87	8844	4.25	1156	22
39	2546	2.37	3448	1.88	9099	4.23	0901	21
40	9.82 2688	2.37	9.87 3335	1.87	9.94 9353	4.25	10.05 0647	20
41	2830	2.37	3223	1.88	9608	4.23	0392	19
42	2972	2.37	3110	1.87	.94 9862	4.23	.05 0138	18
43	3114	2.35	2998	1.88	.95 0116	4.25	.04 9884	17
44	3255	2.37	2885	1.88	0371	4.23	9629	16
45	9.82 3397	2.37	9.87 2772	1.88	9.95 0625	4.23	10.04 9375	15
46	3539	2.35	2659	1.87	0879	4.23	9121	14
47	3680	2.35	2547	1.88	1133	4.25	8867	13
48	3821	2.37	2434	1.88	1388	4.23	8612	12
49	3963	2.35	2321	1.88	1642	4.23	8358	11
50	9.82 4104	2.35	9.87 2208	1.88	9.95 1896	4.23	10.04 8104	10
51	4245	2.35	2095	1.90	2150	4.25	7850	9
52	4386	2.35	1981	1.88	2405	4.23	7595	8
53	4527	2.35	1868	1.88	2659	4.23	7341	7
54	4668	2.33	1755	1.90	2913	4.23	7087	6
55	9.82 4808	2.35	9.87 1641	1.88	9.95 3167	4.23	10.04 6833	5
56	4949	2.35	1528	1.90	3421	4.23	6579	4
57	5090	2.33	1414	1.88	3675	4.23	6325	3
58	5230	2.35	1301	1.90	3929	4.23	6071	2
59	5371	2.33	1187	1.90	4183	4.23	5817	1
60	9.82 5511		9.87 1073		9.95 4437		10.04 5563	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.82 5511	2.33	9.87 1073	1.88	9.95 4437	4.23	10.04 5563	60
1	5651	2.33	0960	1.90	4691	4.25	5309	59
2	5791	2.33	0846	1.90	4946	4.23	5054	58
3	5931	2.33	0732	1.90	5200	4.23	4800	57
4	6071	2.33	0618	1.90	5454	4.23	4546	56
5	9.82 6211	2.33	9.87 0504	1.90	9.95 5706	4.22	10.04 4292	55
6	6351	2.33	0390	1.90	5961	4.23	4039	54
7	6491	2.33	0276	1.92	6215	4.23	3785	53
8	6631	2.32	0161	1.90	6469	4.23	3531	52
9	6770	2.33	.87 0047	1.90	6723	4.23	3277	51
10	9.82 6910	2.32	9.86 9933	1.92	9.95 6977	4.23	10.04 3023	50
11	7049	2.33	9818	1.90	7231	4.23	2769	49
12	7189	2.32	9704	1.92	7485	4.23	2515	48
13	7328	2.32	9589	1.92	7739	4.23	2261	47
14	7467	2.32	9474	1.90	7993	4.23	2007	46
15	9.82 7606	2.32	9.86 9360	1.92	9.95 8247	4.22	10.04 1753	45
16	7745	2.32	9245	1.92	8500	4.23	1500	44
17	7884	2.32	9130	1.92	8754	4.23	1246	43
18	8023	2.32	9015	1.92	9008	4.23	0992	42
19	8162	2.32	8900	1.92	9262	4.23	0738	41
20	9.82 8301	2.30	9.86 8785	1.92	9.95 9516	4.22	10.04 0484	40
21	8439	2.32	8670	1.92	.95 9769	4.23	.04 0231	39
22	8578	2.30	8555	1.92	.96 0023	4.23	.03 9977	38
23	8716	2.32	8440	1.93	0277	4.22	9723	37
24	8855	2.30	8324	1.92	0530	4.23	9470	36
25	9.82 8993	2.30	9.86 8209	1.93	9.96 0784	4.23	10.03 9216	35
26	9131	2.30	8093	1.92	1038	4.23	8962	34
27	9269	2.30	7978	1.93	1292	4.22	8708	33
28	9407	2.30	7862	1.92	1545	4.23	8455	32
29	9545	2.30	7747	1.93	1799	4.22	8201	31
30	9.82 9683	2.30	9.86 7631	1.93	9.96 2052	4.23	10.03 7948	30
31	9821	2.30	7515	1.93	2306	4.23	7694	29
32	.82 9959	2.30	7399	1.93	2560	4.22	7440	28
33	.83 0097	2.28	7283	1.93	2813	4.23	7187	27
34	0234	2.30	7167	1.93	3067	4.22	6933	26
35	9.83 0372	2.28	9.86 7051	1.93	9.96 3320	4.23	10.03 6680	25
36	0509	2.28	6935	1.93	3574	4.23	6426	24
37	0646	2.28	6819	1.93	3828	4.22	6172	23
38	0784	2.28	6703	1.95	4081	4.23	5919	22
39	0921	2.28	6586	1.93	4335	4.22	5665	21
40	9.83 1058	2.28	9.86 6470	1.95	9.96 4588	4.23	10.03 5412	20
41	1195	2.28	6353	1.93	4842	4.22	5158	19
42	1332	2.28	6237	1.95	5095	4.23	4905	18
43	1469	2.28	6120	1.93	5349	4.22	4651	17
44	1606	2.27	6004	1.95	5602	4.22	4398	16
45	9.83 1742	2.28	9.86 5887	1.95	9.96 5855	4.23	10.03 4145	15
46	1879	2.27	5770	1.95	6109	4.22	3891	14
47	2015	2.28	5653	1.95	6362	4.23	3638	13
48	2152	2.27	5536	1.95	6616	4.22	3384	12
49	2288	2.28	5419	1.95	6869	4.23	3131	11
50	9.83 2425	2.27	9.86 5302	1.95	9.96 7122	4.22	10.03 2877	10
51	2561	2.27	5185	1.95	7376	4.22	2624	9
52	2697	2.27	5068	1.97	7629	4.23	2371	8
53	2833	2.27	4950	1.95	7883	4.22	2117	7
54	2969	2.27	4833	1.95	8136	4.22	1864	6
55	9.83 3105	2.27	9.86 4716	1.97	9.96 8389	4.23	10.03 1611	5
56	3241	2.27	4598	1.95	8643	4.22	1357	4
57	3377	2.25	4481	1.97	8896	4.22	1104	3
58	3512	2.27	4363	1.97	9149	4.23	0851	2
59	3648	2.25	4245	1.97	9403	4.22	0597	1
60	9.83 3783		9.86 4127		9.96 9656		10.03 0344	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.	′
0	9.83 3783	2.27	9.86 4127	1.95	9.96 9656	4.22	10.03 0344	60
1	3919	2.25	4010	1.97	.96 9909	4.22	.03 0091	59
2	4054	2.25	3892	1.97	.97 0162	4.23	.02 9838	58
3	4189	2.27	3774	1.97	0416	4.22	9584	57
4	4325	2.25	3656	1.97	0669	4.22	9331	56
5	9.83 4460	2.25	9.86 3538	1.98	9.97 0922	4.22	10.03 9078	55
6	4595	2.25	3419	1.97	1175	4.23	8825	54
7	4730	2.25	3301	1.97	1429	4.22	8571	53
8	4865	2.23	3183	1.98	1682	4.22	8318	52
9	4999	2.25	3064	1.97	1935	4.22	8065	51
10	9.83 5134	2.25	9.86 2946	1.98	9.97 2188	4.22	10.03 7812	50
11	5269	2.23	2827	1.97	2441	4.23	7559	49
12	5403	2.25	2709	1.98	2695	4.22	7305	48
13	5538	2.23	2590	1.98	2948	4.22	7052	47
14	5672	2.25	2471	1.97	3201	4.22	6799	46
15	9.83 5807	2.23	9.86 2353	1.98	9.97 3454	4.22	10.03 6546	45
16	5941	2.23	2234	1.98	3707	4.22	6293	44
17	6075	2.23	2115	1.98	3960	4.22	6040	43
18	6209	2.23	1996	1.98	4213	4.22	5787	42
19	6343	2.23	1877	1.98	4466	4.23	5534	41
20	9.83 6477	2.23	9.86 1758	2.00	9.97 4720	4.22	10.03 5280	40
21	6611	2.23	1638	1.98	4973	4.22	5027	39
22	6745	2.22	1519	1.98	5226	4.22	4774	38
23	6878	2.23	1400	2.00	5479	4.22	4521	37
24	7012	2.23	1280	1.98	5732	4.22	4268	36
25	9.83 7146	2.22	9.86 1161	2.00	9.97 5985	4.22	10.03 4015	35
26	7279	2.22	1041	1.98	6238	4.22	3762	34
27	7412	2.23	0922	2.00	6491	4.22	3509	33
28	7546	2.22	0802	2.00	6744	4.22	3256	32
29	7679	2.22	0682	2.00	6997	4.22	3003	31
30	9.83 7812	2.22	9.86 0562	2.00	9.97 7250	4.22	10.03 2750	30
31	7945	2.22	0442	2.00	7503	4.22	2497	29
32	8078	2.22	0322	2.00	7756	4.22	2244	28
33	8211	2.22	0202	2.00	8009	4.22	1991	27
34	8344	2.22	.86 0082	2.00	8262	4.22	1738	26
35	9.83 8477	2.22	9.85 9962	2.00	9.97 8515	4.22	10.03 1485	25
36	8610	2.20	9842	2.02	8768	4.22	1232	24
37	8742	2.22	9721	2.00	9021	4.22	0979	23
38	8875	2.20	9601	2.02	9274	4.22	0726	22
39	9007	2.22	9480	2.00	9527	4.22	0473	21
40	9.83 9140	2.20	9.85 9360	2.02	9.97 9780	4.22	10.03 0220	20
41	9272	2.20	9239	2.00	.98 0033	4.22	.01 9967	19
42	9404	2.20	9119	2.02	0286	4.20	9714	18
43	9536	2.20	8998	2.02	0538	4.22	9462	17
44	9668	2.20	8877	2.02	0791	4.22	9209	16
45	9.83 9800	2.20	9.85 8756	2.02	9.98 1044	4.22	10.01 8956	15
46	.83 9932	2.20	8635	2.02	1297	4.22	8703	14
47	.84 0064	2.20	8514	2.02	1550	4.22	8450	13
48	0196	2.20	8393	2.02	1803	4.22	8197	12
49	0328	2.18	8272	2.02	2056	4.22	7944	11
50	9.84 0459	2.20	9.85 8151	2.03	9.98 2309	4.22	10.01 7691	10
51	0591	2.18	8029	2.02	2562	4.20	7438	9
52	0722	2.20	7908	2.03	2814	4.22	7186	8
53	0854	2.18	7786	2.02	3067	4.22	6933	7
54	0985	2.18	7665	2.03	3320	4.22	6680	6
55	9.84 1116	2.18	9.85 7543	2.02	9.98 3573	4.22	10.01 6427	5
56	1247	2.18	7422	2.03	3826	4.22	6174	4
57	1378	2.18	7300	2.03	4079	4.22	5921	3
58	1509	2.18	7178	2.03	4332	4.20	5668	2
59	1640	2.18	7056	2.03	4584	4.22	5416	1
60	9.84 1771		9.85 6934	2.03	9.98 4837		10.01 5163	0
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.	′

44° TABLE XII.—*Concluded.*

′	Sine.	D.1″.	Cosine.	D.1″.	Tang.	D.1″.	Cotang.
0	9.84 1771	2.18	9.85 6934	2.03	9.98 4837	4.22	10.01 5163
1	1902	2.18	6812	2.03	5090	4.22	4910
2	2033	2.17	6690	2.03	5343	4.22	4657
3	2163	2.18	6568	2.03	5596	4.20	4404
4	2294	2.17	6446	2.05	5848	4.22	4152
5	9.84 2424	2.18	9.85 6323	2.03	9.98 6101	4.22	10.01 3899
6	2555	2.17	6201	2.05	6354	4.22	3646
7	2685	2.17	6078	2.03	6607	4.22	3393
8	2815	2.18	5956	2.05	6860	4.20	3140
9	2946	2.17	5833	2.03	7112	4.22	2888
10	9.84 3076	2.17	9.85 5711	2.05	9.98 7365	4.22	10.01 2635
11	3206	2.17	5588	2.05	7618	4.22	2382
12	3336	2.17	5466	2.05	7871	4.20	2129
13	3466	2.15	5342	2.05	8123	4.22	1877
14	3595	2.17	5219	2.05	8376	4.22	1624
15	9.84 3725	2.17	9.85 5096	2.05	9.98 8629	4.22	10.01 1371
16	3855	2.15	4973	2.05	8882	4.20	1118
17	3984	2.17	4850	2.05	9134	4.22	0866
18	4114	2.15	4727	2.07	9387	4.22	0613
19	4243	2.15	4603	2.05	9640	4.22	0360
20	9.84 4372	2.17	9.85 4480	2.07	9.98 9893	4.20	10.01 0107
21	4502	2.15	4356	2.05	.99 0145	4.22	.00 9855
22	4631	2.15	4233	2.07	0398	4.22	9602
23	4760	2.15	4109	2.05	0651	4.20	9349
24	4889	2.15	3986	2.07	0903	4.22	9097
25	9.84 5018	2.15	9.85 3862	2.07	9.99 1156	4.22	10.00 8844
26	5127	2.15	3738	2.07	1409	4.22	8591
27	5276	2.15	3614	2.07	1662	4.20	8338
28	5405	2.13	3490	2.07	1914	4.22	8086
29	5533	2.15	3366	2.07	2167	4.22	7833
30	9.84 5662	2.13	9.85 3242	2.07	9.99 2420	4.20	10.00 7580
31	5790	2.15	3118	2.07	2672	4.22	7328
32	5919	2.13	2994	2.08	2925	4.22	7075
33	6047	2.13	2869	2.07	3178	4.22	6822
34	6175	2.15	2745	2.08	3431	4.20	6569
35	9.84 6304	2.13	9.85 2620	2.07	9.99 3683	4.22	10.00 6317
36	6432	2.13	2496	2.08	3936	4.22	6064
37	6560	2.13	2371	2.07	4189	4.20	5811
38	6688	2.13	2247	2.08	4441	4.22	5559
39	6816	2.13	2122	2.08	4694	4.22	5306
40	9.84 6944	2.12	9.85 1997	2.08	9.99 4947	4.20	10.00 5053
41	7071	2.13	1872	2.08	5199	4.22	4801
42	7199	2.13	1747	2.08	5452	4.22	4548
43	7327	2.12	1622	2.08	5705	4.20	4295
44	7454	2.13	1497	2.08	5957	4.22	4043
45	9.84 7582	2.12	9.85 1372	2.10	9.99 6210	4.22	10.00 3790
46	7709	2.12	1246	2.08	6463	4.20	3537
47	7836	2.13	1121	2.08	6715	4.22	3285
48	7964	2.12	0996	2.10	6968	4.22	3032
49	8091	2.12	0870	2.08	7221	4.20	2779
50	9.84 8218	2.12	9.85 0745	2.10	9.99 7473	4.22	10.00 2527
51	8345	2.12	0619	2.10	7726	4.22	2274
52	8472	2.12	0493	2.08	7979	4.20	2021
53	8599	2.12	0368	2.10	8231	4.22	1769
54	8726	2.10	0242	2.10	8484	4.22	1516
55	9.84 8852	2.12	9.85 0116	2.10	9.99 8737	4.20	10.00 1263
56	8979	2.12	84 9990	2.10	8989	4.22	1011
57	9106	2.10	9864	2.10	9242	4.22	0758
58	9232	2.12	9738	2.12	9495	4.20	0505
59	9359	2.10	9611	2.10	9.99 9747	4.22	0253
60	9.84 9485		9.84 9485		10.00 0000		10.00 0000
′	Cosine.	D.1″.	Sine.	D.1″.	Cotang.	D.1″.	Tang.

TABLE XIII

′	0° Sine	Cosine	′	′	0° Sine	Cosine	′	′	0° Sine	Cosine	′
0	.00000	1	60	21	.00611	.99998	39	41	.01193	.99993	19
1	.00029	1	59	22	.00640	.99998	38	42	.01222	.99993	18
2	.00058	1	58	23	.00669	.99998	37	43	.01251	.99992	17
3	.00087	1	57	24	.00698	.99998	36	44	.01280	.99992	16
4	.00116	1	56	25	.00727	.99997	35	45	.01309	.99991	15
5	.00145	1	55	26	.00756	.99997	34	46	.01338	.99991	14
6	.00175	1	54	27	.00785	.99997	33	47	.01367	.99991	13
7	.00204	1	53	28	.00814	.99997	32	48	.01396	.99990	12
8	.00233	1	52	29	.00844	.99996	31	49	.01425	.99990	11
9	.00262	1	51	30	.00873	.99996	30	50	.01454	.99989	10
10	.00291	1	50	31	.00902	.99996	29	51	.01483	.99989	9
11	.00320	.99999	49	32	.00931	.99996	28	52	.01513	.99989	8
12	.00349	.99999	48	33	.00960	.99995	27	53	.01542	.99988	7
13	.00378	.99999	47	34	.00989	.99995	26	54	.01571	.99988	6
14	.00407	.99999	46	35	.01018	.99995	25	55	.01600	.99987	5
15	.00436	.99999	45	36	.01047	.99995	24	56	.01629	.99987	4
16	.00465	.99999	44	37	.01076	.99994	23	57	.01658	.99986	3
17	.00495	.99999	43	38	.01105	.99994	22	58	.01687	.99986	2
18	.00524	.99999	42	39	.01134	.99994	21	59	.01716	.99985	1
19	.00553	.99998	41	40	.01164	.99993	20	60	.01745	.99985	0
20	.00582	.99998	40								
′	Cosine	Sine	′	′	Cosine	Sine	′	′	Cosine	Sine	′
	89°				89°				89°		

TABLE XIII.—*Continued*

′	1° Sine	1° Cosine	2° Sine	2° Cosine	3° Sine	3° Cosine	4° Sine	4° Cosine	′
0	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	88°		87°		86°		85°		

TABLE XIII.—*Continued*

′	5° Sine	5° Cosine	6° Sine	6° Cosine	7° Sine	7° Cosine	8° Sine	8° Cosine	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		

TABLE XIII.—*Continued*

′	9° Sine	9° Cosine	10° Sine	10° Cosine	11° Sine	11° Cosine	12° Sine	12° Cosine	′
0	.15643	.98769	.17365	.98481	.19081	.98163	.20791	.97815	60
1	.15672	.98764	.17393	.98476	.19109	.98157	.20820	.97809	59
2	.15701	.98760	.17422	.98471	.19138	.98152	.20848	.97803	58
3	.15730	.98755	.17451	.98466	.19167	.98146	.20877	.97797	57
4	.15758	.98751	.17479	.98461	.19195	.98140	.20905	.97791	56
5	.15787	.98746	.17508	.98455	.19224	.98135	.20933	.97784	55
6	.15816	.98741	.17537	.98450	.19252	.98129	.20962	.97778	54
7	.15845	.98737	.17565	.98445	.19281	.98124	.20990	.97772	53
8	.15873	.98732	.17594	.98440	.19309	.98118	.21019	.97766	52
9	.15902	.98728	.17623	.98435	.19338	.98112	.21047	.97760	51
10	.15931	.98723	.17651	.98430	.19366	.98107	.21076	.97754	50
11	.15959	.98718	.17680	.98425	.19395	.98101	.21104	.97748	49
12	.15988	.98714	.17708	.98420	.19423	.98096	.21132	.97742	48
13	.16017	.98709	.17737	.98414	.19452	.98090	.21161	.97735	47
14	.16046	.98704	.17766	.98409	.19481	.98084	.21189	.97729	46
15	.16074	.98700	.17794	.98404	.19509	.98079	.21218	.97723	45
16	.16103	.98695	.17823	.98399	.19538	.98073	.21246	.97717	44
17	.16132	.98690	.17852	.98394	.19566	.98067	.21275	.97711	43
18	.16160	.98689	.17880	.98389	.19595	.98061	.21303	.97705	42
19	.16189	.98681	.17909	.98383	.19623	.98056	.21331	.97698	41
20	.16218	.98676	.17937	.98378	.19652	.98050	.21360	.97692	40
21	.16246	.98671	.17966	.98373	.19680	.98044	.21388	.97686	39
22	.16275	.98667	.17995	.98368	.19709	.98039	.21417	.97680	38
23	.16304	.98662	.18023	.98362	.19737	.98033	.21445	.97673	37
24	.16333	.98657	.18052	.98357	.19766	.98027	.21474	.97667	36
25	.16361	.98652	.18081	.98352	.19794	.98021	.21502	.97661	35
26	.16390	.98648	.18109	.98347	.19823	.98016	.21530	.97655	34
27	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
28	.16447	.98638	.18166	.98336	.19880	.98004	.21587	.97642	32
29	.16476	.98633	.18195	.98331	.19908	.97987	.21616	.97636	31
30	.16505	.98629	.18224	.98325	.19937	.97992	.21644	.97630	30
31	.16533	.98624	.18252	.98320	.19965	.97987	.21672	.97623	29
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617	28
33	.16591	.98614	.18309	.98310	.20022	.97975	.21729	.97611	27
34	.16620	.98609	.18338	.98304	.20051	.97969	.21758	.97604	26
35	.16648	.98604	.18367	.98299	.20079	.97963	.21786	.97598	25
36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	.97592	24
37	.16706	.98595	.18424	.98288	.20136	.97952	.21843	.97585	23
38	.16734	.98590	.18452	.98283	.20165	.97946	.21871	.97579	22
39	.16763	.98585	.18481	.98277	.20193	.97940	.21899	.97573	21
40	.16792	.98580	.18509	.98272	.20222	.97934	.21928	.97566	20
41	.16820	.98575	.18538	.98267	.20250	.97928	.21956	.97560	19
42	.16849	.98570	.18567	.98261	.20279	.97922	.21985	.97553	18
43	.16878	.98565	.18595	.98256	.20307	.97916	.22013	.97547	17
44	.16906	.98561	.18624	.98250	.20336	.97910	.22041	.97541	16
45	.16935	.98556	.18652	.98245	.20364	.97905	.22070	.97534	15
46	.16964	.98551	.18681	.98240	.20393	.97899	.22098	.97528	14
47	.16992	.98546	.18710	.98234	.20421	.97893	.22126	.97521	13
48	.17021	.98541	.18738	.98229	.20450	.97887	.22155	.97515	12
49	.17050	.98536	.18767	.98223	.20478	.97881	.22183	.97508	11
50	.17078	.98531	.18795	.98218	.20507	.97875	.22212	.97502	10
51	.17107	.98526	.18824	.98212	.20535	.97869	.22240	.97496	9
52	.17136	.98521	.18852	.98207	.20563	.97863	.22268	.97489	8
53	.17164	.98516	.18881	.98201	.20592	.97857	.22297	.97483	7
54	.17193	.98511	.18910	.98196	.20620	.97851	.22325	.97476	6
55	.17222	.98506	.18938	.98190	.20649	.97845	.22353	.97470	5
56	.17250	.98501	.18967	.98185	.20677	.97839	.22382	.97463	4
57	.17279	.98496	.18995	.98179	.20706	.97833	.22410	.97457	3
58	.17308	.98491	.19024	.98174	.20734	.97827	.22438	.97450	2
59	.17336	.98486	.19052	.98168	.20763	.97821	.22467	.97444	1
60	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	80°		79°		78°		77°		

TABLE XIII.—*Continued*

′	13° Sine	13° Cosine	14° Sine	14° Cosine	15° Sine	15° Cosine	16° Sine	16° Cosine	′
0	.22495	.97437	.24192	.97030	.25882	.96593	.27564	.96126	60
1	.22523	.97430	.24220	.97023	.25910	.96585	.27592	.96118	59
2	.22552	.97424	.24249	.97015	.25938	.96578	.27620	.96110	58
3	.22580	.97417	.24277	.97008	.25966	.96570	.27648	.96102	57
4	.22608	.97411	.24305	.97001	.25994	.96562	.27676	.96094	56
5	.22637	.97404	.24333	.96994	.26022	.96555	.27704	.96086	55
6	.22665	.97398	.24362	.96987	.26050	.96547	.27731	.96078	54
7	.22693	.97391	.24390	.96980	.26079	.96540	.27759	.96070	53
8	.22722	.97384	.24418	.96973	.26107	.96532	.27787	.96062	52
9	.22750	.97378	.24446	.96966	.26135	.96524	.27815	.96054	51
10	.22778	.97371	.24474	.96959	.26163	.96517	.27843	.96046	50
11	.22807	.97365	.24503	.96952	.26191	.96509	.27871	.96037	49
12	.22835	.97358	.24531	.96945	.26219	.96502	.27899	.96029	48
13	.22863	.97351	.24559	.96937	.26247	.96494	.27927	.96021	47
14	.22892	.97345	.24587	.96930	.26275	.96486	.27955	.96013	46
15	.22920	.97338	.24615	.96923	.26303	.96479	.27983	.96005	45
16	.22948	.97331	.24644	.96916	.26331	.96471	.28011	.95997	44
17	.22977	.97325	.24672	.96909	.26359	.96463	.28039	.95989	43
18	.23005	.97318	.24700	.96902	.26387	.96456	.28067	.95981	42
19	.23033	.97311	.24728	.96894	.26415	.96448	.28095	.95972	41
20	.23062	.97304	.24756	.96887	.26443	.96440	.28123	.95964	40
21	.23090	.97298	.24784	.96880	.26471	.96433	.28150	.95956	39
22	.23118	.97291	.24813	.96873	.26500	.96425	.28178	.95948	38
23	.23146	.97284	.24841	.96866	.26528	.96417	.28206	.95940	37
24	.23175	.97278	.24869	.96858	.26556	.96410	.28234	.95931	36
25	.23203	.97271	.24897	.96851	.26584	.96402	.28262	.95923	35
26	.23231	.97264	.24925	.96844	.26612	.96394	.28290	.95915	34
27	.23260	.97257	.24954	.96837	.26640	.96386	.28318	.95907	33
28	.23288	.97251	.24982	.96829	.26668	.96379	.28346	.95898	32
29	.23316	.97244	.25010	.96822	.26696	.96371	.28374	.95890	31
30	.23345	.97237	.25038	.96815	.26724	.96363	.28402	.95882	30
31	.23373	.97230	.25066	.96807	.26752	.96355	.28429	.95874	29
32	.23401	.97223	.25094	.96800	.26780	.96347	.28457	.95865	28
33	.23429	.97217	.25122	.96793	.26808	.96340	.28485	.95857	27
34	.23458	.97210	.25151	.96786	.26836	.96332	.28513	.95849	26
35	.23486	.97203	.25179	.96778	.26864	.96324	.28541	.95841	25
36	.23514	.97196	.25207	.96771	.26892	.96316	.28569	.95832	24
37	.23542	.97189	.25235	.96764	.26920	.96308	.28597	.95824	23
38	.23571	.97182	.25263	.96756	.26948	.96301	.28625	.95816	22
39	.23599	.97176	.25291	.96749	.26976	.96293	.28652	.95807	21
40	.23627	.97169	.25320	.96742	.27004	.96285	.28680	.95799	20
41	.23656	.97162	.25348	.96734	.27032	.96277	.28708	.95791	19
42	.23684	.97155	.25376	.96727	.27060	.96269	.28736	.95782	18
43	.23712	.97148	.25404	.96719	.27088	.96261	.28764	.95774	17
44	.23740	.97141	.25432	.96712	.27116	.96253	.28792	.95766	16
45	.23769	.97134	.25460	.96705	.27144	.96246	.28820	.95757	15
46	.23797	.97127	.25488	.96697	.27172	.96238	.28847	.95749	14
47	.23825	.97120	.25516	.96690	.27200	.96230	.28875	.95740	13
48	.23853	.97113	.25545	.96682	.27228	.96222	.28903	.95732	12
49	.23882	.97106	.25573	.96675	.27256	.96214	.28931	.95724	11
50	.23910	.97100	.25601	.96667	.27284	.96206	.28959	.95715	10
51	.23938	.97093	.25629	.96660	.27312	.96198	.28987	.95707	9
52	.23966	.97086	.25657	.96653	.27340	.96190	.29015	.95698	8
53	.23995	.97079	.25685	.96645	.27368	.96182	.29042	.95690	7
54	.24023	.97072	.25713	.96638	.27396	.96174	.29070	.95681	6
55	.24051	.97065	.25741	.96630	.27424	.96166	.29098	.95673	5
56	.24079	.97058	.25769	.96623	.27452	.96158	.29126	.95664	4
57	.24108	.97051	.25798	.96615	.27480	.96150	.29154	.95656	3
58	.24136	.97044	.25826	.96608	.27508	.96142	.29182	.95647	2
59	.24164	.97037	.25854	.96600	.27536	.96134	.29209	.95639	1
60	.24192	.97030	.25882	.96593	.27564	.96126	.29237	.95630	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	76°		75°		74°		73°		

TABLE XIII.—*Continued*

′	17° Sine	17° Cosine	18° Sine	18° Cosine	19° Sine	19° Cosine	20° Sine	20° Cosine	′
0	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	60
1	.29265	.95622	.30929	.95097	.32584	.94542	.34229	.93959	59
2	.29293	.95613	.30957	.95088	.32612	.94533	.34257	.93949	58
3	.29321	.95605	.30985	.95079	.32639	.94523	.34284	.93939	57
4	.29348	.95596	.31012	.95070	.32667	.94514	.34311	.93929	56
5	.29376	.95588	.31040	.95061	.32694	.94504	.34339	.93919	55
6	.29404	.95579	.31068	.95052	.32722	.94495	.34366	.93909	54
7	.29432	.95571	.31095	.95043	.32749	.94485	.34393	.93899	53
8	.29460	.95562	.31123	.95033	.32777	.94476	.34421	.93889	52
9	.29487	.95554	.31151	.95024	.32804	.94466	.34448	.93879	51
10	.29515	.95545	.31178	.95015	.32832	.94457	.34475	.93869	50
11	.29543	.95536	.31206	.95006	.32859	.94447	.34503	.93859	49
12	.29571	.95528	.31233	.94997	.32887	.94438	.34530	.93849	48
13	.29599	.95519	.31261	.94988	.32914	.94428	.34557	.93839	47
14	.29626	.95511	.31289	.94979	.32942	.94418	.34584	.93829	46
15	.29654	.95502	.31316	.94970	.32969	.94409	.34612	.93819	45
16	.29682	.95493	.31344	.94961	.32997	.94399	.34639	.93809	44
17	.29710	.95485	.31372	.94952	.33024	.94390	.34666	.93799	43
18	.29737	.95476	.31399	.94943	.33051	.94380	.34694	.93789	42
19	.29765	.95467	.31427	.94933	.33079	.94370	.34721	.93779	41
20	.29793	.95459	.31454	.94924	.33106	.94361	.34748	.93769	40
21	.29821	.95450	.31482	.94915	.33134	.94351	.34775	.93759	39
22	.29849	.95441	.31510	.94906	.33161	.94342	.34803	.93748	38
23	.29876	.95433	.31537	.94897	.33189	.94332	.34830	.93738	37
24	.29904	.95424	.31565	.94888	.33216	.94322	.34857	.93728	36
25	.29932	.95415	.31593	.94878	.33244	.94313	.34884	.93718	35
26	.29960	.95407	.31620	.94869	.33271	.94303	.34912	.93708	34
27	.29987	.95398	.31648	.94860	.33298	.94293	.34939	.93698	33
28	.30015	.95389	.31675	.94851	.33326	.94284	.34966	.93688	32
29	.30043	.95380	.31703	.94842	.33353	.94274	.34993	.93677	31
30	.30071	.95372	.31730	.94832	.33381	.94264	.35021	.93667	30
31	.30098	.95363	.31758	.94823	.33408	.94254	.35048	.93657	29
32	.30126	.95354	.31786	.94814	.33436	.94245	.35075	.93647	28
33	.30154	.95345	.31813	.94805	.33463	.94235	.35102	.93637	27
34	.30182	.95337	.31841	.94795	.33490	.94225	.35130	.93626	26
35	.30209	.95328	.31868	.94786	.33518	.94215	.35157	.93616	25
36	.30237	.95319	.31896	.94777	.33545	.94206	.35184	.93606	24
37	.30265	.95310	.31923	.94768	.33573	.94196	.35211	.93596	23
38	.30292	.95301	.31951	.94758	.33600	.94186	.35239	.93585	22
39	.30320	.95293	.31979	.94749	.33627	.94176	.35266	.93575	21
40	.30348	.95284	.32006	.94740	.33655	.94167	.35293	.93565	20
41	.30376	.95275	.32034	.94730	.33682	.94157	.35320	.93555	19
42	.30403	.95266	.32061	.94721	.33710	.94147	.35347	.93544	18
43	.30431	.95257	.32089	.94712	.33737	.94137	.35375	.93534	17
44	.30459	.95248	.32116	.94702	.33764	.94127	.35402	.93524	16
45	.30486	.95240	.32144	.94693	.33792	.94118	.35429	.93514	15
46	.30514	.95231	.32171	.94684	.33819	.94108	.35456	.93503	14
47	.30542	.95222	.32199	.94674	.33846	.94098	.35484	.93493	13
48	.30570	.95213	.32227	.94665	.33874	.94088	.35511	.93483	12
49	.30597	.95204	.32254	.94656	.33901	.94078	.35538	.93472	11
50	.30625	.95195	.32282	.94646	.33929	.94068	.35565	.93462	10
51	.30653	.95186	.32309	.94637	.33956	.94058	.35592	.93452	9
52	.30680	.95177	.32337	.94627	.33983	.94049	.35619	.93441	8
53	.30708	.95168	.32364	.94618	.34011	.94039	.35647	.93431	7
54	.30736	.95159	.32392	.94609	.34038	.94029	.35674	.93420	6
55	.30763	.95150	.32419	.94599	.34065	.94019	.35701	.93410	5
56	.30791	.95142	.32447	.94590	.34093	.94009	.35728	.93400	4
57	.30819	.95133	.32474	.94580	.34120	.93999	.35755	.93389	3
58	.30846	.95124	.32502	.94571	.34147	.93989	.35782	.93379	2
59	.30874	.95115	.32529	.94561	.34175	.93979	.35810	.93368	1
60	.30902	.95106	.32557	.94552	.34202	.93969	.35837	.93358	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	72°		71°		70°		69°		

TABLE XIII.—*Continued*

′	21° Sine	21° Cosine	22° Sine	22° Cosine	23° Sine	23° Cosine	24° Sine	24° Cosine	′
0	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	68°		67°		66°		65°		

TABLE XIII.—*Continued*

′	25° Sine	25° Cosine	26° Sine	26° Cosine	27° Sine	27° Cosine	28° Sine	28° Cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		

TABLE XIII.—*Continued*

′	29° Sine	29° Cosine	30° Sine	30° Cosine	31° Sine	31° Cosine	32° Sine	32° Cosine	′
0	.48481	.87462	.50000	.86603	.51504	.85717	.52992	.84805	60
1	.48506	.87448	.50025	.86588	.51529	.85702	.53017	.84789	59
2	.48532	.87434	.50050	.86573	.51554	.85687	.53041	.84774	58
3	.48557	.87420	.50076	.86559	.51579	.85672	.53066	.84759	57
4	.48583	.87406	.50101	.86544	.51604	.85657	.53091	.84743	56
5	.48608	.87391	.50126	.86530	.51628	.85642	.53115	.84728	55
6	.48634	.87377	.50151	.86515	.51653	.85627	.53140	.84712	54
7	.48659	.87363	.50176	.86501	.51678	.85612	.53164	.84697	53
8	.48684	.87349	.50201	.86486	.51703	.85597	.53189	.84681	52
9	.48710	.87335	.50227	.86471	.51728	.85582	.53214	.84666	51
10	.48735	.87321	.50252	.86457	.51753	.85567	.53238	.84650	50
11	.48761	.87306	.50277	.86442	.51778	.85551	.53263	.84635	49
12	.48786	.87292	.50302	.86427	.51803	.85536	.53288	.84619	48
13	.48811	.87278	.50327	.86413	.51828	.85521	.53312	.84604	47
14	.48837	.87264	.50352	.86398	.51852	.85506	.53337	.84588	46
15	.48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	.48888	.87235	.50403	.86369	.51902	.85476	.53386	.84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
18	.48938	.87207	.50453	.86340	.51952	.85446	.53435	.84526	42
19	.48964	.87193	.50478	.86325	.51977	.85431	.53460	.84511	41
20	.48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	.49014	.87164	.50528	.86295	.52026	.85401	.53509	.84480	39
22	.49040	.87150	.50553	.86281	.52051	.85385	.53534	.84464	38
23	.49065	.87136	.50578	.86266	.52076	.85370	.53558	.84448	37
24	.49090	.87121	.50603	.86251	.52101	.85355	.53583	.84433	36
25	.49116	.87107	.50628	.86237	.52126	.85340	.53607	.84417	35
26	.49141	.87093	.50654	.86222	.52151	.85325	.53632	.84402	34
27	.49166	.87079	.50679	.86207	.52175	.85310	.53656	.84386	33
28	.49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	.49217	.87050	.50729	.86178	.52225	.85279	.53705	.84355	31
30	.49242	.87036	.50754	.86163	.52250	.85264	.53730	.84339	30
31	.49268	.87021	.50779	.86148	.52275	.85249	.53754	.84324	29
32	.49293	.87007	.50804	.86133	.52299	.85234	.53779	.84308	28
33	.49318	.86993	.50829	.86119	.52324	.85218	.53804	.84292	27
34	.49344	.86978	.50854	.86104	.52349	.85203	.53828	.84277	26
35	.49369	.86964	.50879	.86089	.52374	.85188	.53853	.84261	25
36	.49394	.86949	.50904	.86074	.52399	.85173	.53877	.84245	24
37	.49419	.86935	.50929	.86059	.52423	.85157	.53902	.84230	23
38	.49445	.86921	.50954	.86045	.52448	.85142	.53926	.84214	22
39	.49470	.86906	.50979	.86030	.52473	.85127	.53951	.84198	21
40	.49495	.86892	.51004	.86015	.52498	.85112	.53975	.84182	20
41	.49521	.86878	.51029	.86000	.52522	.85096	.54000	.84167	19
42	.49546	.86863	.51054	.85985	.52547	.85081	.54024	.84151	18
43	.49571	.86849	.51079	.85970	.52572	.85066	.54049	.84135	17
44	.49596	.86834	.51104	.85956	.52597	.85051	.54073	.84120	16
45	.49622	.86820	.51129	.85941	.52621	.85035	.54097	.84104	15
46	.49647	.86805	.51154	.85926	.52646	.85020	.54122	.84088	14
47	.49672	.86791	.51179	.85911	.52671	.85005	.54146	.84072	13
48	.49697	.86777	.51204	.85896	.52696	.84989	.54171	.84057	12
49	.49723	.86762	.51229	.85881	.52720	.84974	.54195	.84041	11
50	.49748	.86748	.51254	.85866	.52745	.84959	.54220	.84025	10
51	.49773	.86733	.51279	.85851	.52770	.84943	.54244	.84009	9
52	.49798	.86719	.51304	.85836	.52794	.84928	.54269	.83994	8
53	.49824	.86704	.51329	.85821	.52819	.84913	.54293	.83978	7
54	.49849	.86690	.51354	.85806	.52844	.84897	.54317	.83962	6
55	.49874	.86675	.51379	.85792	.52869	.84882	.54342	.83946	5
56	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83930	4
57	.49924	.86646	.51429	.85762	.52918	.84851	.54391	.83915	3
58	.49950	.86632	.51454	.85747	.52943	.84836	.54415	.83899	2
59	.49975	.86617	.51479	.85732	.52967	.84820	.54440	.83883	1
60	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	60°		59°		58°		57°		

TABLE XIII.—*Continued*

′	33° Sine	33° Cosine	34° Sine	34° Cosine	35° Sine	35° Cosine	36° Sine	36° Cosine	′
0	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	60
1	.54488	.83851	.55943	.82887	.57381	.81899	.58802	.80885	59
2	.54513	.83835	.55968	.82871	.57405	.81882	.58826	.80867	58
3	.54537	.83819	.55992	.82855	.57429	.81865	.58849	.80850	57
4	.54561	.83804	.56016	.82839	.57453	.81848	.58873	.80833	56
5	.54586	.83788	.56040	.82822	.57477	.81832	.58896	.80816	55
6	.54610	.83772	.56064	.82806	.57501	.81815	.58920	.80799	54
7	.54635	.83756	.56088	.82790	.57524	.81798	.58943	.80782	53
8	.54659	.83740	.56112	.82773	.57548	.81782	.58967	.80765	52
9	.54683	.83724	.56136	.82757	.57572	.81765	.58990	.80748	51
10	.54708	.83708	.56160	.82741	.57596	.81748	.59014	.80730	50
11	.54732	.83692	.56184	.82724	.57619	.81731	.59037	.80713	49
12	.54756	.83676	.56208	.82708	.57643	.81714	.59061	.80696	48
13	.54781	.83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	.54805	.83645	.56256	.82675	.57691	.81681	.59108	.80662	46
15	.54829	.83629	.56280	.82659	.57715	.81664	.59131	.80644	45
16	.54854	.83613	.56305	.82643	.57738	.81647	.59154	.80627	44
17	.54878	.83597	.56329	.82626	.57762	.81631	.59178	.80610	43
18	.54902	.83581	.56353	.82610	.57786	.81614	.59201	.80593	42
19	.54927	.83565	.56377	.82593	.57810	.81597	.59225	.80576	41
20	.54951	.83549	.56401	.82577	.57833	.81580	.59248	.80558	40
21	.54975	.83533	.56425	.82561	.57857	.81563	.59272	.80541	39
22	.54999	.83517	.56449	.82544	.57881	.81546	.59295	.80524	38
23	.55024	.83501	.56473	.82528	.57904	.81530	.59318	.80507	37
24	.55048	.83485	.56497	.82511	.57928	.81513	.59342	.80489	36
25	.55072	.83469	.56521	.82495	.57952	.81496	.59365	.80472	35
26	.55097	.83453	.56545	.82478	.57976	.81479	.59389	.80455	34
27	.55121	.83437	.56569	.82462	.57999	.81462	.59412	.80438	33
28	.55145	.83421	.56593	.82446	.58023	.81445	.59436	.80420	32
29	.55169	.83405	.56617	.82429	.58047	.81428	.59459	.80403	31
30	.96194	.83389	.56641	.82413	.58070	.81412	.59482	.80386	30
31	.55218	.83373	.56665	.82396	.58094	.81395	.59506	.80368	29
32	.55242	.83356	.56689	.82340	.58118	.81378	.59529	.80351	28
33	.55266	.83340	.56713	.82363	.58141	.81361	.59552	.80334	27
34	.55291	.83324	.56736	.82347	.58165	.81344	.59576	.80316	26
35	.55315	.83308	.56760	.82330	.58189	.81327	.59599	.80299	25
36	.55339	.83292	.56784	.82314	.58212	.81310	.59622	.80282	24
37	.55363	.83276	.56808	.82297	.58236	.81293	.59646	.80264	23
38	.55388	.83260	.56832	.82281	.58260	.81276	.59669	.80247	22
39	.55412	.83244	.56856	.82264	.58283	.81259	.59693	.80230	21
40	.55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	.55460	.83212	.56904	.82231	.58330	.81225	.59739	.80195	19
42	.55484	.83195	.56928	.82214	.58354	.81208	.59763	.80178	18
43	.55509	.83179	.56952	.82198	.58378	.81191	.59786	.80160	17
44	.55533	.83163	.56976	.82181	.58401	.81174	.59809	.80143	16
45	.55557	.83147	.57000	.82165	.58425	.81157	.59832	.80125	15
46	.55581	.83131	.57024	.82148	.58449	.81140	.59856	.80108	14
47	.55605	.83115	.57047	.82132	.58472	.81123	.59879	.80091	13
48	.55630	.83098	.57071	.82115	.58496	.81106	.59902	.80073	12
49	.55654	.83082	.57095	.82098	.58519	.81089	.59926	.80056	11
50	.55678	.83066	.57119	.82082	.58543	.81072	.59949	.80038	10
51	.55702	.83050	.57143	.82065	.58567	.81055	.59972	.80021	9
52	.55726	.83034	.57167	.82048	.58590	.81038	.59995	.80003	8
53	.55750	.83017	.57191	.82032	.58614	.81021	.60019	.79986	7
54	.55775	.83001	.57215	.82015	.58637	.81004	.60042	.79968	6
55	.55799	.82985	.57238	.81999	.58661	.80987	.60065	.79951	5
56	.55823	.82969	.57262	.81982	.58684	.80970	.60089	.79934	4
57	.55847	.82953	.57286	.81965	.58708	.80953	.60112	.79916	3
58	.55871	.82936	.57310	.81949	.58731	.80936	.60135	.79899	2
59	.55895	.82920	.57334	.81932	.58755	.80919	.60158	.79881	1
60	.55919	.82904	.57358	.81915	.58779	.80902	.60182	.79864	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	56°		55°		54°		53°		

TABLE XIII.—*Continued*

′	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	40° Sine	40° Cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	.76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	.60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	.60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	.62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	52°		51°		50°		49°		

TABLE XIII.—*Concluded*

′	41° Sine	41° Cosine	42° Sine	42° Cosine	43° Sine	43° Cosine	44° Sine	44° Cosine	′
0	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	48°		47°		46°		45°		

TABLE XIV

′	0° Tan.	0° Co-tan.	1° Tan.	1° Co-tan.	2° Tan.	2° Co-tan.	3° Tan.	3° Co-tan.	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.750	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.870	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.920	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00814	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04832	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	89°		88°		87°		86°		

TABLE XIV.—*Continued*

′	4° Tan.	4° Co-tan.	5° Tan.	5° Co-tan.	6° Tan.	6° Co-tan.	7° Tan.	7° Co-tan.	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	85°		84°		83°		82°		

TABLE XIV.—*Continued*

′	8°		9°		10°		11°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18383	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91235	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	81°		80°		79°		78°		

TABLE XIV.—*Continued*

′	12° Tan.	12° Co-tan.	13° Tan.	13° Co-tan.	14° Tan.	14° Co-tan.	15° Tan.	15° Co-tan.	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27920	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03075	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	77°		76°		75°		74°		

TABLE XIV.—*Continued*

′	16° Tan.	16° Co-tan.	17° Tan.	17° Co-tan.	18° Tan.	18° Co-tan.	19° Tan.	19° Co-tan.	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35019	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33160	3.01489	.35117	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94590	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	78°		72°		71°		70°		

TABLE XIV.—*Continued*

′	20°		21°		22°		23°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38862	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	69°		68°		67°		66°		

TABLE XIV.—*Continued*

′	24°		25°		26°		27°		
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45537	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07322	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97680	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52984	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	65°		64°		63°		62°		

TABLE XIV.—*Continued*

	28°		29°		30°		31°		
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1..6209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65534	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56500	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76630	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	
	61°		60°		59°		58°		

TABLE XIV.—*Continued*

′	32°		33°		34°		35°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	CO-TAN.	TAN.	CO-TAN	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	57°		56°		55°		54°		

TABLE XIV.—*Continued*

′	36°		37°		38°		39°		′
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan	′
	53°		52°		51°		50°		

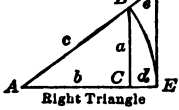
TABLE XIV.—*Continued*

40°		41°		42°		43°		
TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	′
.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
.85307	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
49°		48°		47°		46°		

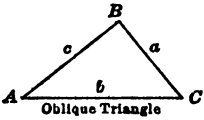
TABLE XIV.—*Concluded*

′	44° TAN.	44° CO-TAN.	′	′	44° TAN.	44° CO-TAN.	′	′	44° TAN.	44° CO-TAN.	′
0	.96569	1.03553	60	21	.97756	1.02295	39	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	.99016	1.00994	17
3	.96738	1.03372	57	24	.97927	1.02117	36	44	.99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	14
6	.96907	1.03192	54	27	.98098	1.01939	33	47	.99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	.99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	.99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	29	51	.99478	1.00525	9
11	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.97246	1.02832	48	33	.98441	1.01583	27	53	.99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	.97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	.97473	1.02593	44	37	.98671	1.01347	23	57	.99826	1.00175	3
17	.97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	.97643	1.02414	41	40	.98843	1.01170	20	60	1	1	0
20	.97700	1.02355	40								
′	45° CO-TAN.	45° TAN.	′	′	45° CO-TAN.	45° TAN.	′	′	45° CO-TAN.	45° TAN.	′

TABLE XV.—TRIGONOMETRIC FORMULAS



Right Triangle



Oblique Triangle

RIGHT TRIANGLES

$$\sin A = \frac{a}{c} = \cos B \qquad \sec A = \frac{c}{b} = \operatorname{cosec} B$$

$$\cos A = \frac{b}{c} = \sin B \qquad \operatorname{cosec} A = \frac{c}{a} = \sec B$$

$$\tan A = \frac{a}{b} = \cot B \qquad \operatorname{vers} A = \frac{c-b}{c} = \frac{d}{c}$$

$$\cot A = \frac{b}{a} = \tan B \qquad \operatorname{exsec} A = \frac{e}{c}$$

$$a = c \sin A = c \cos B = b \tan A = b \cot B = \sqrt{c^2 - b^2}$$

$$b = c \cos A = c \sin B = a \cot A = a \tan B = \sqrt{c^2 - a^2}$$

$$c = \frac{a}{\sin A} = \frac{a}{\cos B} = \frac{b}{\sin B} = \frac{b}{\cos A} = \frac{d}{\operatorname{vers} A} = \frac{e}{\operatorname{exsec} A} = \sqrt{a^2 + b^2}$$

$$d = c \operatorname{vers} A \qquad e = c \operatorname{exsec} A$$

OBLIQUE TRIANGLES

Given	Sought	Formulas
A, B, a	b, c	$b = \frac{a}{\sin A} \cdot \sin B \qquad c = \frac{a}{\sin A} \cdot \sin (A + B)$
A, a, b	B, c	$\sin B = \frac{\sin A}{a} \cdot b \qquad c = \frac{a}{\sin A} \cdot \sin C$
C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2}C$
	$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \cdot \tan \frac{1}{2}(A + B)$
a, b, c	A	If $s = \frac{1}{2}(a + b + c)$, $\sin \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}$
		$\cos \frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$, $\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
		$\sin A = 2\frac{\sqrt{s(s-a)(s-b)(s-c)}}{bc}$
		$\operatorname{vers} A = \frac{2(s-b)(s-c)}{bc}$
	area	$area = \sqrt{s(s-a)(s-b)(s-c)}$
C, a, b	*area*	$area = \frac{1}{2}ab \sin C$
A, B, C, a	*area*	$area = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$

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